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Solving the Nonlinear Heat Conduction in a Spherical Coordinate with Electrical Simulation

A. M. Gheitaghy, H. Saffari, G. Q. Zhang

Abstract**—**Numerical approach based on the electrical simulation method is proposed to solve a nonlinear transient heat conduction problem with nonlinear boundary for a spherical body. This problem represents a strong nonlinearity in both the governing equation for temperature dependent thermal property and the boundary condition for combined convective and radiative cooling. By analysing the equivalent electrical model using the electrical circuit simulation program HSPICE, transient temperature and heat flux distributions at sphere can be obtained easily and fast. The solutions clearly illustrate the effect of the radiation-conduction parameter N_{rc}, the Biot number and the linear coefficient of temperature dependent conductivity and heat capacity. On comparing the results with corresponding numerical solutions, the accuracy and efficiency of this computational method is found to be good.

Keywords**—**Convective boundary, radiative boundary, electrical simulation method, nonlinear heat conduction, spherical coordinate.

I. Introduction

THE cooling or heating of a solid by a combination of thermal radiation and free or forced convection has received little attention in the literature. Interactions of radiation with convection become important in various high-temperature applications such as combustion (fires, furnaces and rocket nozzles), nuclear reactions (solar emission, nuclear weapons), ablating material to protect from high external temperatures, aerodynamic heating of spaceships and satellites, glass manufacturing, solid oxide fuel cells and solar energy collectors [1]. Furthermore, if there is a wide temperature difference within a medium, the assumption of constant thermal property is not appropriate. This variation in thermal property becomes important in the case of most nonmetallic materials subjected to intermediate or high temperature differences.

An earlier transient analysis for the radiative cooling of a sphere of a high-temperature gas was performed by Viskanta and Lall [2]. The gas was grey, without scattering, and heat conduction was not included. Viskanta and Merriam [3] investigated heat transfer by combined conduction and radiation between concentric spheres separated by radiating medium. Bayazitoglu and Suryanarayana [4] considered transient heat transfer in an opaque sphere surrounded by a translucent concentric spherical layer. The internal sphere was opaque, and heat transfer within it occurred only by conduction. The temperatures were obtained numerically with an explicit finite difference method. Tsai and Özisik [5] studied the interaction of transient, combined conduction and radiation in an isotropically scattering, homogeneous solid sphere. Thynell [6] used the Galerkin method to consider a steady-state heat transfer by simultaneous conduction and radiation in a linearly anisotropic, homogeneous solid sphere. Trabelsi et al. [7] extended the Galerkin method to study the combined conduction and radiation heat transfer between two concentric spheres separated by a participating isotropically scattering medium.

Haji-Sheikh and Sparrow [8] used the Monte Carlo technique to obtain solutions for a plate subjected to simultaneous boundary convection and radiation. Davies discussed the cooling of a plate by thermal radiation using the heat balance integral technique [9]. Parang et al. [10] studied the problem of inward solidification of a liquid in cylindrical and spherical geometries due to combined convective and radiative cooling by the regular perturbation method. Sunden [11] presented numerical solutions based on the finite difference method of the thermal response of a composite slab subjected to a time-varying incident heat flux on one side and combined convective and radiative cooling on the other side. Kessler [12] analyzed multishell spherical systems which are heated in their inner part and under convection and radiation in their outer part. Different geometrical thickness of the spherical shells and temperature dependent thermal material properties were assumed. Su [13] investigated the transient radiative cooling of a spherical body by using improved lumped models. Here, the distributed model of a spherical body is considered. Homotopy analysis method was presented to treat such complicated nonlinearity [14], [15]. This method does not depend on the existence of small or large parameters in the studied problem such as the perturbation methods and Adomian decomposition method. In their work, the constant heat capacity per volume was assumed, but here it's the linear dependent to temperature is assumed to investigate the effect of it.

In this paper, an approach called the network simulation method, according to electrical analogy is proposed as a means of solving the heat conduction problem. Electrical analogy has been used for the first time by Paschkis to solve the unsteady-state and unidirectional heat conduction equation in a plate [16]. Ever since, equivalent electrical models, so-called thermal networks, seem a relatively simple, but sufficiently accurate and powerful tool for simulating real

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thermal systems. Aim of this work is introducing this method to spherical coordinate and also, for solving the equivalent circuit the HSPICE program is proposed. For demonstration the reliability and robustness of this method, a solid sphere with temperature dependent thermal property under convective and radiative boundary condition is investigated and results are verified with reference [14]. This problem has a strong nonlinearity in both governing equation and boundary condition, and so encounters difficulties in obtaining exact solution.

II. Mathematical Formulation

Let us consider a spherical body of radius r_o, initially at a uniform temperature T_0. At $t = 0$, the spherical body is suddenly exposed to an environment of a constant fluid temperature T_f and a constant radiation sink temperature T_s. It is assumed that the spherical body is homogeneous, isotropic and opaque. Thermal conductivity k and heat capacity ρc_p are assumed linear function of temperature.

The mathematical formulation of the problem is given by

$$\rho c_p(T)\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 k(T)\frac{\partial T}{\partial r}\right], \text{in } r < r_o \tag{1}$$

with initial and boundary conditions taken as

$$T_{(r,0)} = T_0 \tag{2a}$$

$$-k(T)\frac{\partial T_{(r_o,t)}}{\partial r} = h\left(T_{(r_o,t)} - T_{\mathrm{f}}\right) + \varepsilon\sigma\left({T_{(r_o,t)}}^4 - {T_s}^4\right) \tag{2b}$$

$$\frac{\partial T_{(0,t)}}{\partial r} = 0 \tag{2c}$$

where T denotes the temperature, t the time, r the spatial coordinate, h the convective heat transfer coefficient, ε the surface emissivity, and σ the Stefan–Boltzmann constant, respectively.

Introducing the concept of the adiabatic surface temperature T_a by

$$hT_{\mathrm{f}} + \varepsilon\sigma {T_s}^4 = hT_a + \varepsilon\sigma {T_a}^4 \tag{3}$$

Combining (3) with (2b), the corresponding boundary condition can be rewritten as

$$-k(T)\frac{\partial T_{(r_o,t)}}{\partial r} = h\left(T_{(r_o,t)} - T_a\right) + \varepsilon\sigma\left({T_{(r_o,t)}}^4 - {T_a}^4\right) \tag{4}$$

Also, using the dimensionless parameters

$$\theta = \frac{T}{T_0}, \eta = \frac{r}{r_o}, Fo = \frac{\alpha_0 t}{{r_o}^2}, Q = \frac{q r_o}{k_0 T_i}, Bi = \frac{h r_o}{k_0}, N_{rc} = \frac{\varepsilon\sigma r_o {T_i}^3}{k_0}, \alpha_0 = \frac{k_0}{\rho c_{p_0}}, \lambda = \frac{k}{k_0}, \gamma = \frac{\rho c_p}{\rho c_{p_0}} \tag{5}$$

in which k_0 is a reference thermal conductivity, ρc_{p_0} is a reference heat capacity per unit volume and α_0 is a reference thermal diffusivity. Without loss of generality, we consider the case in which the thermal conductivity and heat capacity per unit volume vary linearly with the temperature, given by

$$k = k_0(1 + bT), \rho c_p = \rho c_{p_0}(1 + eT) \tag{6}$$

The dimensionless form of thermal property can be written as

$$\lambda(\theta) = 1 + \beta\theta, \gamma(\theta) = 1 + \xi\theta \text{ where } \beta = \frac{bT_0}{k_0}, \xi = \frac{eT_0}{\rho c_{p_0}} \tag{7}$$

The above mathematical formulations can now be rewritten in dimensionless form as

$$(1 + \xi\theta)\frac{\partial\theta}{\partial Fo} = \frac{1}{\eta^2}\frac{\partial}{\partial\eta}\left[(1 + \beta\theta)\eta^2\frac{\partial\theta}{\partial\eta}\right] \tag{8}$$

subject to the initial and boundary conditions

$$\theta(\eta, 0) = 1 \tag{9a}$$

$$-(1 + \beta\theta)\frac{\partial\theta_{(1,Fo)}}{\partial\eta} = Bi\left(\theta_{(1,Fo)} - \theta_a\right) + N_{rc}\left({\theta_{(1,Fo)}}^4 - {\theta_a}^4\right) \tag{9b}$$

$$\frac{\partial\theta(0, Fo)}{\partial\eta} = 0 \tag{9c}$$

It can be seen that the problem is governed by five dimensionless parameters, θ_a, β, ξ, Bi and N_{rc}. The radiation–conduction parameter, N_{rc}, is conceptually analog to the Biot number, *Bi*, which is the governing parameter for convective cooling.

III. Electrical Network Simulation Method

The analogy existing between electrical and thermal quantities is well known. The variables heat flux *(q)* and temperature *(T)* are equivalent to the variables electric current *(I)* and voltage *(V)* in this analogy. The network simulation method according to this analogy is a numerical technique whose accuracy, efficiency and reliability have already been proven in modeling differential equations specially heat transfer equations in Cartesian coordinate [17]-[19]. Numbers of networks are connected in series to make up the whole medium and boundary conditions are added by special electrical devices.

Divide the physical domain $0 < r < r_o$ into a finite number of cells with thickness Δr_i, creating sort of a computational domain. An energy balance in a typical cell, i = 1, 2, 3, ..., n, may be expressed as

$$q_{i-\Delta} = \left(\rho v c_p\right)_i \frac{dT_i}{dt} + q_{i+\Delta} \tag{10}$$

where $q_{i-\Delta}$ and $q_{i+\Delta}$ signify the respective heat flux densities entering and leaving cell i, as indicated in Fig. 1 and v_i is a volume of cell i. Equation (10) may be interpreted as Kirchhoff's current law (implying energy conservation), where the temperature, *T*, is a continuous single-valued dependent variable that satisfies Kirchhoff's voltage law and the heat, *q*, is analogous with current. At this point, each term of (10) is conveniently redefined by way of an electric current.

This operation leads to

$$I_{i-\Delta} = I_{i,c} + I_{i+\Delta} \text{ where } I_{i,c} = C_i \frac{dV_i}{dt} \tag{11}$$

By considering the equivalent electric network between nodes as '**T**' form, *RC* trees are *RC* circuits with capacitors from all nodes to ground, no floating capacitors, no resistor loops, and no resistors to ground. According to similarity between (10) and (11), $I_{i,c}$ is an entrance current to the capacitor branch with capacitance

$$C_i = \frac{4}{3}\pi\left({r_{i+\Delta}}^3 - {r_{i-\Delta}}^3\right) * \left(\rho c_p\right)_i \tag{12}$$

For finding the resistance on each branch, structural Fourier equation in spherical coordinates is used as

$$q(r,t) = -k(T) * 4\pi r^2 * \nabla T(r,t) \tag{13}$$

With integrating to (13) in $i - \Delta$ to i

$$\frac{q_{i-\Delta}}{4\pi}\int_{r_{i-\Delta}}^{r_i} \frac{dr}{r^2} = -k(T_i)\int_{T_{i-\Delta}}^{T_i} dT \tag{14}$$

And considering constant entrance heat flux $q_{i-\Delta}$ in this domain and dependency of k to T_i, we have

$$\frac{1}{4\pi k(T_i)}\left(\frac{1}{r_{i-\Delta}} - \frac{1}{r_i}\right) = \frac{T_{i-\Delta} - T_i}{q_{i-\Delta}} \tag{15}$$

In this equation, $T_{i-\Delta}$ and T_i denote the respective temperatures at the left extreme and at the centre of cell i. According to similarity between (15) and Ohm's law, resistance in entrance branch can be obtained as

$$R_{i-\Delta} = \frac{r_i - r_{i-\Delta}}{4\pi k(T_i) * r_i * r_{i-\Delta}} \tag{16}$$

By repetition of this process for output branch, the resistance on another branch can be obtained as

$$R_{i+\Delta} = \frac{r_{i+\Delta} - r_i}{4\pi k(T_i) * r_i * r_{i+\Delta}} \tag{17}$$

It can be concluded that the resistance of a spherical shell is proportional to the difference of the inverse of the inside and outside radii; and the capacitance is proportional to the difference of the third powers of the radii limiting the shell. Accordingly, the interconnection between the two resistors $R_{i-\Delta}$ and $R_{i+\Delta}$ and the capacitor C_i is illustrated in the electric circuit of Fig. 1. The generalization of these elements produces *n* cells, which are connected in series to form the complete network model.

For the completion of the electric network model, the final requirement is pertinent to incorporation of the initial and boundary conditions. The initial condition is easily adjusted by charging the capacitors to the initial temperature. Resistor of infinite value, R_{inf}(an open circuit), is connected to the center node of sphere to cope with the symmetry boundary condition. A voltage source with magnitude of θ_a connected at variable resistance R_s to handle the dominant convective and radiation boundary condition. The magnitude of R_s is define with equation:

$$R_{rad+conv} = \frac{1}{Bi + N_{rc}(V_{n+\Delta} + \theta_a)\left({V_{n+\Delta}}^2 + {\theta_a}^2\right)} \tag{18}$$

where $V_{n+\Delta}$ is a voltage of surface node.

Once the electric network model has been set up, the numerical treatment of the analog electric circuit can be easily done with the computer code HSPICE. This program is a member of the SPICE family which can calculate the behavior of analog circuits with relatively high speed and accuracy. This program uses the modified nodal analysis, as the equation formulation method, and Gaussian elimination with associated LU factorization for the solution of linear systems. This program also uses the Newton-Raphson algorithm as a nonlinear equation solution method, and the simple-limiting method of Colon as a limiting algorithm [20].

IV. Results and Discussion

Electrical simulation method has several advantages over the numerical techniques. This method relies on discrete spatial intervals and real continuous time. Hence, the discretization error is quantified by the spatial interval Δx exclusively without the intervention of the time interval Δt; thus, can be easily controlled and does not require convergence criteria. Also, all types of linear and nonlinear boundary conditions imposed on the linear or nonlinear heat equation may be treated easily and in a few seconds run time. An unparalleled advantage is its suitability to composite plane walls, because the boundary conditions of temperature continuity and heat flux continuity across the material interfaces are satisfied in an electric sense. Heat flows across any section and both spatial and temporal distributions of temperature are obtained simultaneously, without mathematical complexities in a really low run time of HSPICE program. This electrical model is constructed based upon central finite difference spatial discretion of the heat flow equation.

The temperature field $\theta(\eta, Fo)$ and the heat flux density field $Q(\eta, Fo)$ in a body sphere have been accurately determined by the code HSPICE. To facilitate the numerical computations, we assigned values of one to $T_i = r_o = \alpha_0$. Certainly, a sensitivity analysis of the number of cells forming the computational domain is a mandatory step in numerical calculus. Our analyses were carried out by taking 100 compartments, enough to ensure good accuracy and low CPU times (typically, less than 3seconds, on an IBM ThinkPad, Pentium 4, 1.66 GHz, 1GB RAM).

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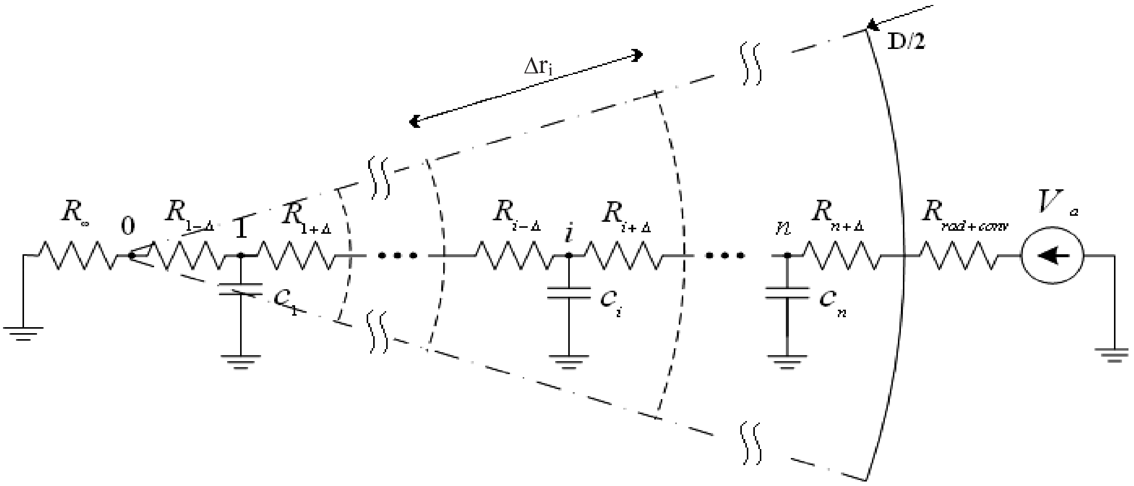

Fig. 1 Electrical network elements for solid sphere with radiation and convection boundary condition

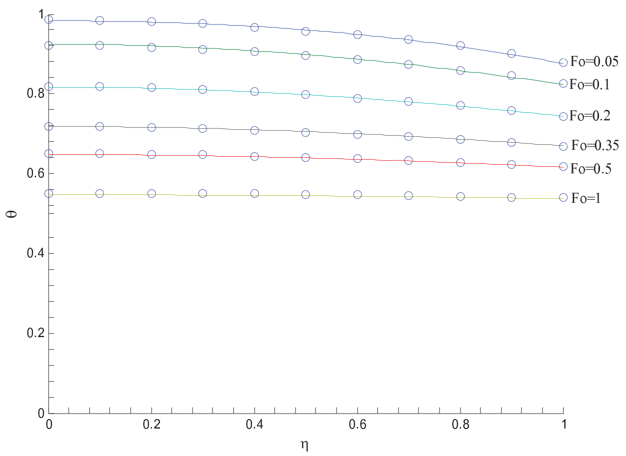

Fig. 2 Comparison of the NSM results with analytic approximation with $\beta = 1, \xi = 0, N_{rc} = .25, \theta_a = .5, Bi = .5$ at the dimensionless time $Fo = 0.05, 0.1, 0.2, 0.35, 0.5, 1$. Solid line: NSM results; open circle: analytic approximation

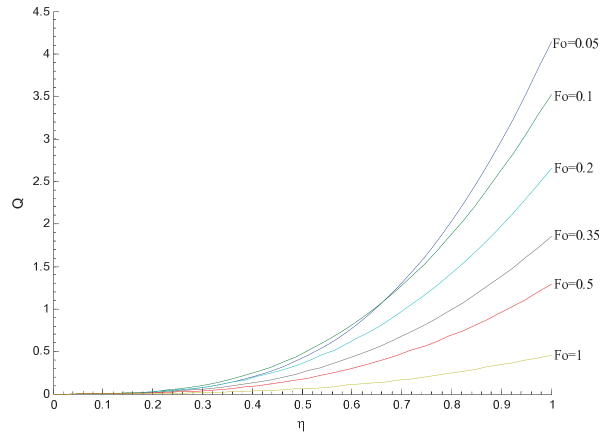

Fig. 3 Spatial variation in the heat flux with $\beta = 1, \xi = 0, N_{rc} = 0.25$, $\theta_a = 0.5$, $Bi = 0.5$ at the dimensionless time $Fo = 0.05, 0.1, 0.2, 0.35, 0.5, 1$

Figs. 2 and 3 investigates the electrical network solution for the spatial variation of dimensionless temperature and heat flux at different dimensionless time Fo, for the parametric values $\beta = 1$, $\xi = 0$, $N_{rc} = 0.25$, $\theta_a = 0.5$, $Bi = 0.5$. Results from Fig. 2 shows that the electrical network solutions have a good agreement with the solution of 30th order Homotopy solution that shows with the open circle. By increasing in time, the temperature reach to ambient temperature and the variation of temperature and therefore the heat flux in Fig. 3 are decreased.

Fig. 4 describes the temporal variations in the temperature on the surface of the spherical body for different values of the Biot number Bi=0.5, 1, 2 and the values $\beta = 1$, $\xi = 0$, $N_{rc} = 0$, $\theta_a = 0$. This case agrees with the approximate solution for the dimensionless time. This figure shows that the temperature decreases as the Biot number enlarges, and also, decays more quickly for large values of the Biot number.

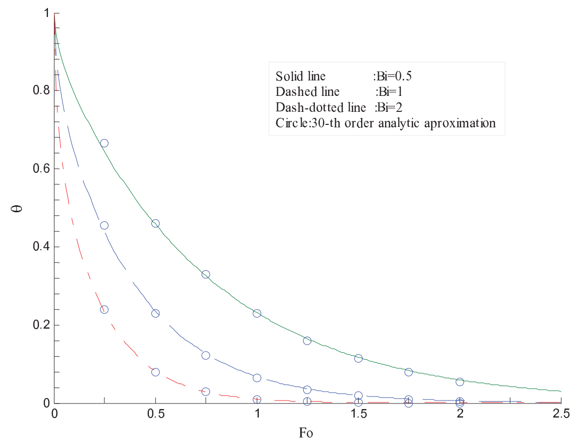

Fig. 4 Comparison between NSM result and analytic approximation at the boundary for different values of Bi when $\beta = 1, \xi = 0, N_{rc} = 0, \theta_a = 0$

Fig. 5 describes the temporal variations in the temperature on the surface of the spherical body, at the boundary $\eta = 1$ for different values of the radiation-conduction parameter ($N_{rc} = 0$, 0.25, 0.5, 1, 2) and the value $\beta = 1$, $Bi = 0.5$, $\theta_a = 0.5$. It also indicates that the temperature on the surface decays more quickly for larger value of the radiation–conduction parameter N_{rc}.

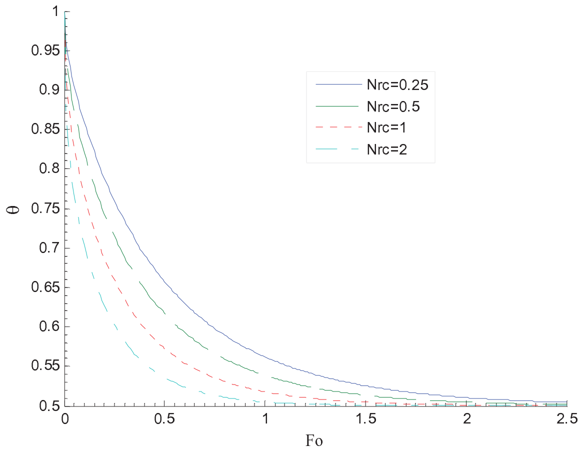

Fig. 5 Temporal variation in temperature on the surface of spherical body for different radiation conduction parameter $N_{rc} = 0.25, 0.5, 1, 2$ with $\beta = 1, \xi = 0, Bi = 0.5, \theta_a = 0.5$

Fig. 6 views the spatial variations in the temperature of the spherical body without radiation ($N_{rc} = 0$) through different values of dimensionless times with $\beta = 1$, $\xi = 0$, $\theta_a = 0$, $Bi = 1$. All of these results verify the validity of the electrical simulation method for the unsteady nonlinear heat transfer problems.

Figs. 7 and 8 show the effect of variable heat capacity with the linear coefficients $\xi = 0$, 0.5, 1, 2 on temporal temperature and heat flux profile on the surface of spherical body, respectively. By increasing coefficient of heat capacity, the temperature and heat flux is increased.

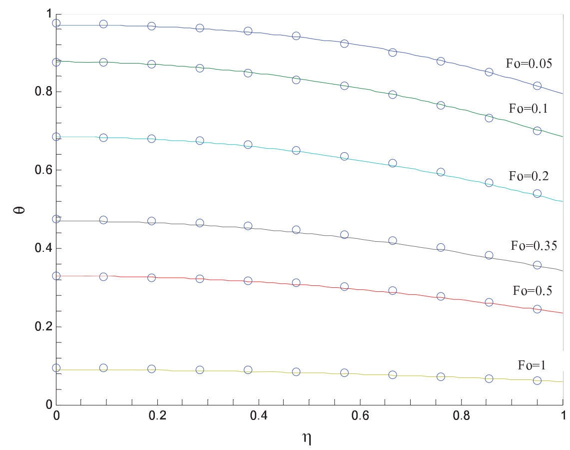

Fig. 6 Temperature comparison between NSM result and analytic approximation with $\beta = 1, \xi = 0, N_{rc} = 0, \theta_a = 0, Bi = 1$ at the dimensionless time $Fo = 0.05, 0.1, 0.2, 0.35, 0.5, 1$. Solid line: NSM results; open circle: analytic approximation

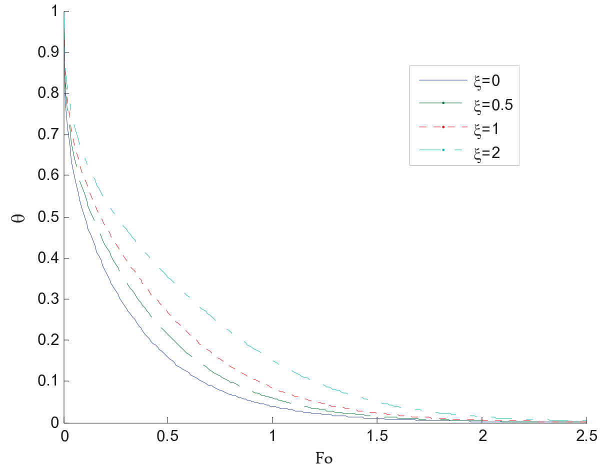

Fig. 7 Temporal variation in temperature on the surface of spherical body for different heat capacity linear coefficient $\xi = 0, 0.5, 1, 2$ with $\beta = 0, Bi = 1, N_{rc} = 1, \theta_a = 0$

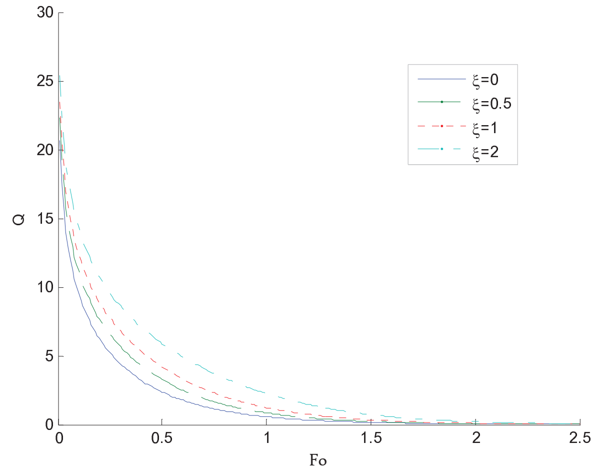

Fig. 8 Temporal variation in heat flux on the surface of spherical body for different heat capacity linear coefficient $\xi = 0, 0.5, 1, 2$ with $\beta = 0, Bi = 1, N_{rc} = 1, \theta_a = 0$

V. Conclusion

This paper treated the application of the electrical simulation method for a spherical body with variable thermal properties under the combined convective and radiative cooling. All the conditions of the problem make it highly non-linear. This numerical approach based on the analogy between electrical circuit and heat transfer gives us a tool to solving unsteady nonlinear heat transfer problems in spherical coordinate. Following the steps of the network simulation, a network model has been designed, which runs numerically with appropriate circuit simulation software to give the transient responses. The treatment proved the ability and flexibility of this method to handle such kinds of problems for different bases. The obtained solutions give accurate spatial and temporal variations in the temperature, which indicate that the temperature on the surface of the body decays rapidly for large values of the Biot number, *Bi* and the radiation-conduction parameter, N_{rc}. Moreover, by increasing the linear coefficient of heat capacity the temperature and heat flux increased. Adversely, by increasing the linear coefficient of conductivity the temperature and heat flux decreased.

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