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Subband Adaptive Filter Exploiting Sparsity of System

Young-Seok Choi

Abstract—This paper presents a normalized subband adaptive filtering (NSAF) algorithm to cope with the sparsity condition of an underlying system in the context of compressive sensing. By regularizing a weighted l_1-norm of the filter taps estimate onto the cost function of the NSAF and utilizing a subgradient analysis, the update recursion of the l_1-norm constraint NSAF is derived. Considering two distinct weighted l_1-norm regularization cases, two versions of the l_1-norm constraint NSAF are presented. Simulation results clearly indicate the superior performance of the proposed l_1-norm constraint NSAFs comparing with the classical NSAF.

Keywords—Subband adaptive filtering, sparsity constraint, weighted l_1-norm.

I. Introduction

OVER the past few decades, the relative simplicity and good performance of the normalized least mean square (NLMS) algorithm have made it a popular choice for adaptive filtering applications. However, its convergence performance is significantly deteriorated in case when correlated input signals are involved [1], [2]. To tackle this issue, adaptive filtering in the subband has been recently developed, referred to as subband adaptive filtering (SAF) [3]–[6]. Its distinct feature is based on the property that the LMS-type adaptive filters converge faster for white input signals than colored ones [1], [2]. Thus, carrying out a pre-whitening on colored input signals, it results in the accelerated convergence of an adaptive filter. In spite of the virtue of the SAF, the use of the classical SAF has been hampered due to the structural issues such as aliasing and band-edge effects since the classical SAF adapts the filter weights independently at each band [3]. By incorporating the fullband weight model, the recently developed SAF schemes successfully address the structural problems [4], [5]. More recently, the use of multiple-constraints optimization problem based on the principle of minimal disturbance leads to the normalized SAF (NSAF), which possesses similar update recursion with those in [4], [5], allowing the accelerated convergence rate over the NLMS.

It is known that in case when identifying a sparse system which is common in practical environment, adaptive filtering exhibits poor convergence performance [7]–[9]. In this paper, to address this problem in the context of the NSAF, the sparsity constraint NSAF which exploit the sparsity condition in an underlying systems to be identified is presented.

Y.-S. Choi is with the Department of Electronic Engineering, Gangneung-Wonju National University, 7 Jukheon-gil, Gangneung, 210-702 Republic of Korea (phone: +82-33-640-2429; fax: +82-33-646-0740; e-mail: yschoi@gwnu.ac.kr).

Recently, compressive sensing, an emerging signal processing framework, has been allowing adaptive filtering to utilize the sparsity property [7]–[10]. Along with this line, this study presents a framework of the sparsity constraint NSAFs in a manner of regularizing a weighted l_1-norm of the filter taps estimate onto the cost function of the NSAF. By choosing the distinct weighting matrices for a weighted l_1-norm regularization, two stochastic gradient based l_1-norm constrained NSAF algorithms are derived: First, the l_1-norm NSAF (l_1-NSAF) is derived by utilizing the identity matrix as a weighting matrix. Second, the reweighted l_1-norm NSAF (l_1-RNSAF) which makes use of a current estimate of the system is developed. Through various simulations, the resulting l_1-norm constraint NSAFs have proven their superiority over the classical NSAF, especially when the sparsity of the underlying system gets severe.

II. Sparsity Constrained NSAF

Consider a desired signal $d(n)$ that arise from the system identification model

$$d(n) = \mathbf{u}(n)\mathbf{w}^{\circ} + v(n), \tag{1}$$

where $\mathbf{w}^{\circ}$ is a column vector for the impulse response of an unknown system that we wish to estimate, $v(n)$ accounts for measurement noise with zero mean and variance σ_v^2 and $\mathbf{u}(n)$ denotes the $1 \times M$ input vector,

$$\mathbf{u}(n) = [u(n) \;\; u(n-1) \;\; \cdots u(n-M+1)]. \tag{2}$$

A. Conventional NSAF

Fig. 1 shows the structure of the NSAF, where the desired signal $d(n)$ and output signal $y(n)$ are partitioned into N subbands by the analysis filters $H_0(z), H_1(z), \ldots, H_{N-1}(z)$. The resulting subband signals, $d_i(n)$ and $y_i(n)$ for $i = 0, 1, \ldots, N-1$, are critically decimated to a lower sampling rate commensurate with their bandwidth. Here, the variable n to index the original sequences, and k to index the decimated sequences are used for all signals. Then, the decimated filter output signal at each subband is defined as $y_{i,D}(k) = \mathbf{u}_i(k)\mathbf{w}(k)$, where $\mathbf{u}_i(k)$ is $1 \times M$ row vector such that

$$\mathbf{u}_i(k) = [u_i(kN), u_i(kN-1), \ldots, u_i(kN-M+1)]$$

and $\mathbf{w}(k) = [w_0(k), w_1(k), \ldots, w_{M-1}(k)]^T$ denotes an estimate for $\mathbf{w}^{\circ}$ with length M. Thus the decimated subband error signal is given by

$$e_{i,D}(k) = d_{i,D}(k) - y_{i,D}(k) = d_{i,D}(k) - \mathbf{u}_i(k)\mathbf{w}(k), \tag{3}$$

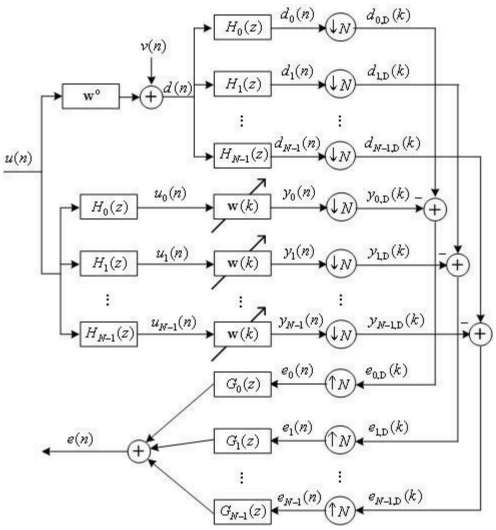

Fig. 1 Subband structure with the analysis filter and synthesis filter and the subband desired signals, subband filter outputs, and subband error signals

where $d_{i,D}(k) = d_i(kN)$ is the decimated desired signal at each subband.

In [6], the authors have formulated the Lagrangian based multiple-constraint optimization problem, which is formulated as

$$\begin{aligned} J_{\mathrm{NSAF}}(k) &= \|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2 \\ &+ \sum_{i=0}^{N-1} \lambda_i [d_{i,D}(k) - \mathbf{u}_i(k)\mathbf{w}(k+1)], \end{aligned} \tag{4}$$

where λ_i for $i = 0, 1, \ldots, N-1$ denotes the Lagrange multipliers. solving the cost function (4), the update recursion of the NSAF algorithm is given by [6]

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \sum_{i=0}^{N-1} \frac{\mathbf{u}_i^T(k)}{\|\mathbf{u}_i(k)\|^2} e_{i,D}(k), \tag{5}$$

where μ is the step-size parameter.

B. Derivation of l_1-Norm Constraint NSAF

To exploit the sparsity condition with the concept of compressive sensing, a weighted l_1-norm of the filter weight estimate is regularized on the cost function of the NSAF, being formulated as

$$\begin{aligned} &J_{l_1-\mathrm{NSAF}}(k) = \|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2 \\ &+ \sum_{i=0}^{N-1} \lambda_i [d_{i,D}(k) - \mathbf{u}_i(k)\mathbf{w}(k+1)] + \gamma \|\mathbf{\Pi} \mathbf{w}(k)\|_1. \end{aligned} \tag{6}$$

where $\|\mathbf{\Pi}\mathbf{w}(k)\|$ accounts for the weighted l_1-norm of the filter weight vector $\mathbf{w}(k)$ and is written as

$$\|\mathbf{\Pi}\mathbf{w}(k)\|_1 = \sum_{m=0}^{M-1} \pi_m |w_m(k)|, \tag{7}$$

where $\mathbf{\Pi}$ is a $M \times M$ weighting matrix whose diagonal elements are π_m and other elements equal to zero, and $w_m(k)$ denotes the mth tap weight of $\mathbf{w}(k)$, $m = 0, 1, \ldots, M-1$. In addition, γ is a positive valued parameter which plays a role in compromising the error related term and the weighted l_1-norm regularization in right-hand side of (6).

To find the optimal weight vector $\mathbf{w}(k+1)$ which minimizes the cost function (6), the derivative of (6) with respect to $\mathbf{w}(k+1)$ is taken and set to zero. Note that the weighted l_1-norm regularization term, i.e., $\|\mathbf{\Pi}\mathbf{w}(k)\|_1$ is not differentiable at any point in case when $w_m(k)$ equals zero.

To deal with this issue, a subgradient analysis [11] is incorporated, providing a proper subgradient of non-differentiable function, here, $\|\mathbf{\Pi}\mathbf{w}(k)\|_1$. Thus, taking the derivative of (6) with respect to the weight vector $\mathbf{w}(k+1)$ and letting the derivative to zero, it leads to

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{1}{2} \sum_{i=0}^{N-1} \lambda_i \mathbf{u}_i^T(k) - \frac{\gamma}{2} \nabla_{\mathbf{w}}^s \|\mathbf{\Pi}\mathbf{w}(k)\|_1, \tag{8}$$

where $\nabla_{\mathbf{w}}^s f(\cdot)$ denotes a subgradient vector of a function $f(\cdot)$ with respect to $bfw(k+1)$. A possible subgradient vector $\nabla_{\mathbf{w}}^s \|\mathbf{\Pi}\mathbf{w}(k)\|_1$ can be obtained as [11]

$$\nabla_{\mathbf{w}}^s \|\mathbf{\Pi}\mathbf{w}(k)\|_1 = \mathbf{\Pi}^T \mathrm{sgn}(\mathbf{\Pi}\mathbf{w}(k)) = \mathbf{\Pi}\mathrm{sgn}(\mathbf{w}(k)), \tag{9}$$

since $\mathbf{\Pi}$ is assumed as a diagonal matrix with positive-valued elements. Substituting (9) into (8), it is given by

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{1}{2} \sum_{i=0}^{N-1} \lambda_i \mathbf{u}_i^T(k) - \frac{\gamma}{2} \mathbf{\Pi}\mathrm{sgn}(\mathbf{w}(k)), \tag{10}$$

Substituting (10) into the multiple constraints of the NSAF, i.e., $d_{i,D}(k) = \mathbf{u}_i(k)\mathbf{w}(k+1)$, $i = 0, 1, \ldots, N-1$ and rewriting as a matrix form, it leads to

$$\begin{aligned} \mathbf{\Lambda} &= 2[\mathbf{U}(k)\mathbf{U}^T(k)]^{-1} \mathbf{e}_D(k) \\ &+ \gamma [\mathbf{U}(k)\mathbf{U}^T(k)]^{-1} \mathbf{U}(k) \mathbf{\Pi}\mathrm{sgn}(\mathbf{w}(k)), \end{aligned} \tag{11}$$

where $\mathbf{\Lambda} = [\lambda_0, \lambda_1, \ldots, \lambda_{N-1}]^T$ is the $N \times 1$ Lagrange vector,

$$\mathbf{U}(k) = \begin{bmatrix} \mathbf{u}_0(k) \\ \vdots \\ \mathbf{u}_{N-1}(k) \end{bmatrix}, \ \mathbf{e}_D(k) = \begin{bmatrix} e_{0,D}(k) \\ \vdots \\ e_{N-1,D}(k) \end{bmatrix}.$$

By neglecting the off-diagonal elements of $\mathbf{U}(k)\mathbf{U}^T(k)$ [6], (11) can be simplified to

$$\lambda_i = 2 \frac{e_{i,D}(k)}{||\mathbf{u}_i(k)||^2} + \gamma \frac{\mathbf{u}_i(k)}{||\mathbf{u}_i(k)||^2} \mathbf{\Pi}\mathrm{sgn}(\mathbf{w}(k)), \tag{12}$$

for $i = 0, 1, \ldots, N-1$.

Consequently, combining (10) and (12), the update recursion of the weighted l_1-norm constraint NSAF is given by

$$\begin{aligned} &\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \sum_{i=0}^{N-1} \Big[\frac{\mathbf{u}_i^T(k)}{||\mathbf{u}_i(k)||^2} e_{i,D}(k) \\ &+ \frac{1}{2} \gamma \frac{\mathbf{u}_i(k)}{||\mathbf{u}_i(k)||^2} \mathbf{\Pi}\mathrm{sgn}(\mathbf{w}(k)) \mathbf{u}_i^T(k) \Big] - \frac{\mu\gamma}{2} \mathbf{\Pi}\mathrm{sgn}(\mathbf{w}(k)), \end{aligned} \tag{13}$$

where μ is the step-size parameter.

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TABLE I
COMPUTATIONAL COMPLEXITY

	NSAF	l_1-NSAF	l_1-RNSAF
Multiplications	$3M + 3NL$	$6M + 3NL$	$7M + 3NL$
Divisions	1	2	$2 + M/N$

C. Choosing the Weighting Matrix for l_1-Norm Constraint NSAF

Here, by choosing the weighting matrix $\mathbf{\Pi}$, two versions of the l_1-norm constraint NSAF are developed: First, the use of the identity matrix as the weighting matrix, i.e., $\mathbf{\Pi} = \mathbf{I}_M$, results in the following update recursion

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \sum_{i=0}^{N-1} \Big[\frac{\mathbf{u}_i^T(k)}{||\mathbf{u}_i(k)||^2} e_{i,D}(k) + \frac{1}{2}\gamma \frac{\mathbf{u}_i(k)}{||\mathbf{u}_i(k)||^2} \mathrm{sgn}(\mathbf{w}(k))\mathbf{u}_i^T(k) \Big] - \frac{\mu\gamma}{2}\mathrm{sgn}(\mathbf{w}(k)), \quad (14)$$

which is referred to as the l_1-norm NSAF (l_1-NSAF).

Second, to approximate the actual sparsity condition of an underlying system, i.e., l_0-norm of the system, the weights of $\mathbf{\Pi}$ need to be chosen inversely proportional to magnitude of the actual tap values of the system. However, since the actual tap values of the system is unknown, the current filter taps estimate is utilized instead of the actual tap values which is referred to as the reweighting scheme [12], as follows:

$$\pi_m(k) = \frac{1}{|w_m(k)| + \epsilon} \quad \text{for } m = 0, 1, \ldots, M-1, \quad (15)$$

where $w_m(k)$ denotes the mth tap weight of the $\mathbf{w}(k)$ and ϵ is a small positive value to avoid singularity in the case when $|w_m(k)| = 0$. Then, the weighting matrix $\mathbf{\Pi}$ consists of the values of $\pi_m(k)$ as the mth diagonal elements and has a time-varying feature. Finally, the update recursion of the reweighted l_1-norm NSAF (l_1-RNSAF) is given by

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \sum_{i=0}^{N-1} \Big[\frac{\mathbf{u}_i^T(k)}{||\mathbf{u}_i(k)||^2} e_{i,D}(k) + \frac{1}{2}\gamma \frac{\bar{\mathbf{u}}_i(k)}{||\mathbf{u}_i(k)||^2} \mathrm{sgn}(\mathbf{w}(k))\mathbf{u}_i^T(k) \Big] - \frac{\mu\gamma}{2} \frac{\mathrm{sgn}(\mathbf{w}(k))}{|\mathbf{w}(k)| + \epsilon}, \quad (16)$$

where $\bar{\mathbf{u}}_i(k) = \mathbf{u}_i(k)\mathbf{\Pi}$ and the vector division operation in last term accounts for a component-wise division.

Table I lists the number of multiplications and divisions of the NSAF [6], l_1-NSAF, and l_1-RNSAF per iteration. As shown in Table I, the use of l_1-norm constraint leads to an increase in computation.

III. SIMULATION RESULTS

The performance of the proposed l_1-norm constraint NSAFs is validated by carrying out computer simulations in a system identification scenario in which the unknown channel is randomly generated. The length of the unknown system is $M = 128$ in experiments and P of them have non-zero values. Then, the degree of sparsity is denoted as $S = P/M$. The non-zero valued taps are positioned randomly and their values are taken from a Gaussian distribution $\mathcal{N}(0, 1/P)$. The adaptive filter and the unknown system are assumed to have the same number of taps. The input signals are obtained by filtering a white, zero-mean, Gaussian random sequence $x(i)$ through a first-order system $G(z) = 1/(1 - 0.9z^{-1})$. The signal-to-noise ratio (SNR) is calculated by

$$\mathrm{SNR} = 10 \log_{10} \left(\frac{E[y^2(i)]}{E[v^2(i)]} \right),$$

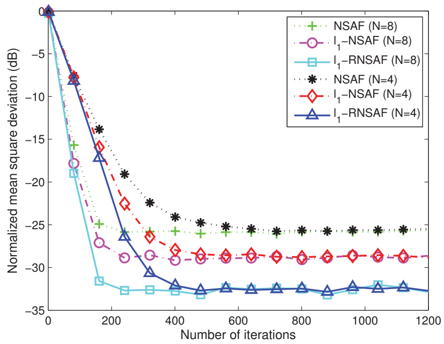

Fig. 2 Normalized MSD curves of the NSAF, l_1-NSAF, and l_1-RNSAF ($N = 4$ and 8)

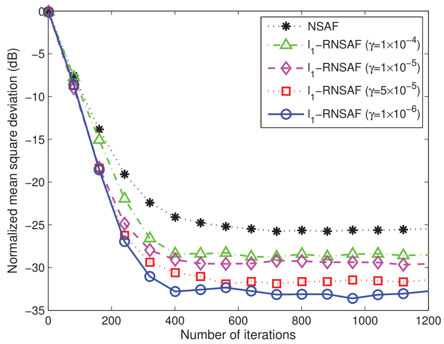

Fig. 3 Normalized MSD curves of the NSAF and l_1-RNSAF for various γ values ($N = 4$)

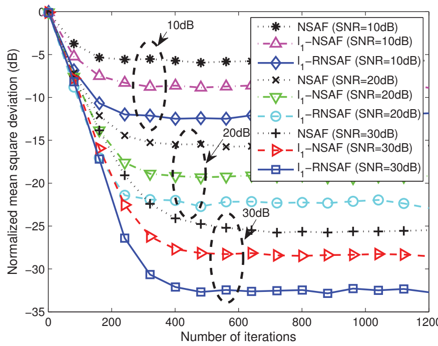

Fig. 4 Normalized MSD curves of the NSAF, l_1-NSAF, and l_1-RNSAF under various SNR conditions (SNR = 10, 20, and 30dB)

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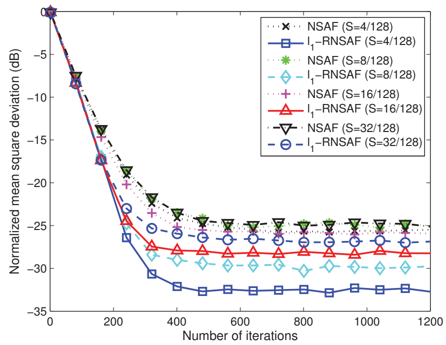

Fig. 5 Normalized MSD curves of the NSAF, l_1-NSAF, and l_1-RNSAF under various sparsity conditions ($S = 4/128$, $8/128$, $16/128$, and $32/128$)

where $y(i) = \mathbf{u}_i \mathbf{w}^\circ$. The measurement noise $v(i)$ is added to $y(i)$ such that SNR $= 10, 20$, and 30dB. In order to compare the convergence performance, the normalized mean square deviation (MSD),

$$\text{Normalized MSD} = \frac{E\|\mathbf{w}^\circ - \mathbf{w}_i\|^2}{\|\mathbf{w}^\circ\|^2},$$

is taken and averaged over 50 independent trials. The cosine-modulated filter banks [13] with the subband numbers of $N = 4$ and 8 are used in the simulations. The prototype filter of length $L = 32$ is used. The step-size is set to $\mu = 0.5$ for the NSAF, l_1-NSAF, and l_1-RNSAF. In addition, $\epsilon = 0.1$ is chosen for the l_1-RNSAF.

Fig. 2 shows the normalized MSD curves of the NSAF, l_1-NSAF, and l_1-RNSAF in cases of $N = 4$ and 8. For the l_1-NSAF and l_1-RNSAF, $\gamma = 1 \times 10^{-4}$ for $N = 8$ and $\gamma = 3 \times 10^{-5}$ for $N = 4$ are chosen, respectively. As shown in Fig. 2, the l_1-RNSAF not only outperforms the conventional NSAF and l_1-NSAF, but also the l_1-NSAF has better performance than the NSAF in term of the convergence rate and the steady-state misadjustment.

In Fig. 3, the normalized MSD curves of the NSAF and l_1-RNSAF for different γ values are illustrated. For different γ values ($\gamma = 1 \times 10^{-4}$, 1×10^{-5}, 5×10^{-5}, and 1×10^{-6}), the l_1-RNSAF are superior to the NSAF, indicating that the l_1-RNSAF is not excessively sensitive to γ. The analysis for an optimal γ value remains as a future work.

Next, the performance of the proposed l_1-norm constraint NSAFs are compared to the NSAF under different SNR conditions. Fig. 4 depicts the normalized MSD curves of the NSAF, l_1-NSAF, and l_1-RNSAF under SNR $= 10$, 20, and 30dB, respectively. The values of γ are set to 5×10^{-4}, 5×10^{-5}, and 3×10^{-5} for SNR $= 10$, 20, and 30dB, respectively. It is clear that both the l_1-NSAF and l_1-RNSAF are superior to the NSAF under different SNR cases. Furthermore, the l_1-RNSAF performs well compared to l_1-NSAF in cases when both low and high SNR conditions.

In Fig. 5, the convergence properties of the NSAF and l_1-RNSAF is compared under various sparsity conditions of an underlying system. With the same length of the system, i.e., $M = 128$, different sparsity conditions ($S = 4/128$, $8/128$, $16/128$, and $32/128$) are considered. The values of γ are set to 3×10^{-5} for the l_1-RNSAF in all sparsity cases. Fig. 5 shows that the NSAF is independent from the sparsity condition. On the other hand, the results indicate that the more sparse the underlying system, the better the l_1-RNSAF.

IV. Conclusion

A framework of the NSAF with sparsity constraint has been presented in the context of compressive sensing. By incorporating a weighted l_1-norm regularization in the cost function, the proposed l_1-norm constraint NSAF has exploited the sparsity condition of an underlying system. By choosing the distinct weighting matrices which are thought to be different l_1-norm regularization, two l_1-norm constraint NSAFs, i.e., l_1-NSAF and l_1-RNSAF, have been developed. The simulation results indicated that the proposed l_1-NSAF and l_1-RNSAF improved convergence performance.

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