

Comparative Study of Dynamic Effect on Analysis Approaches for Circular Tanks Using Codal Provisions

P. Deepak Kumar, Aishwarya Alok, P. R. Maiti

Abstract—Liquid storage tanks have become widespread during the recent decades due to their extensive usage. Analysis of liquid containing tanks is known to be complex due to hydrodynamic force exerted on tank which makes the analysis a complex one. The objective of this research is to carry out analysis of liquid domain along with structural interaction for various geometries of circular tanks considering seismic effects. An attempt has been made to determine hydrodynamic pressure distribution on the tank wall considering impulsive and convective components of liquid mass. To get a better picture, a comparative study of Draft IS 1893 Part 2, ACI 350.3 and Eurocode 8 for Circular Shaped Tank has been performed. Further, the differences in the magnitude of shear and moment at base as obtained from static (IS 3370 IV) and dynamic (Draft IS 1892 Part 2) analysis of ground supported circular tank highlight the need for us to mature from the old code to a newer code, which is more accurate and reliable.

Keywords—Liquid filled containers, Circular Tanks, IS 1893 (Part 2), Seismic analysis, Sloshing.

I. INTRODUCTION

Liquid storage tanks are one of the most critical lifeline structures which are extensively used in water distribution systems and in industries for storing toxic and flammable liquids. The dynamic interaction between fluid and structure is of significant concern for such structures. The dynamic characteristics of the structure and consequently its response to transient and cyclic excitation are changed due to such interaction. Therefore, accurate modeling of these diverse systems with the inclusion of fluid-structure interaction becomes necessary for analysis of such structures.

Seismic analysis of liquid-containing tanks differs from buildings in two ways: first, during seismic excitation, liquid inside the tank exerts hydrodynamic force on tank walls and base. Second, liquid-containing tanks are generally less ductile and have lower redundancy as compared to buildings [1].

It has been found that hydrodynamic pressure in a flexible tank can be significantly higher than the corresponding rigid container due to the interaction effects between flexible

P. Deepak Kumar is an undergraduate student at Civil Engineering Department, Indian Institute of Technology (Banaras Hindu University), India (Corresponding author to provide phone: 91-7052546556; e-mail: pdeepak.kumar.civ12@iitbhu.ac.in).

Aishwarya Alok was with Indian Institute of Technology (Banaras Hindu University), (e-mail: aishwarya.alok@gmail.com).

P. R. Maiti is with the Civil Engineering Department, Indian Institute of Technology (Banaras Hindu University) Varanasi, India (e-mail: prmaiti.civ@iitbhu.ac.in).

structure and contained liquid. The hydrodynamic pressure induced by earthquake can usually be separated into impulsive and convective terms. The impulsive component is governed by the interaction between tank wall and liquid and is highly dependent on the flexibility of the wall while the convective component is induced by slosh waves.

Sloshing is the dynamic load acting over a tank structure as a result of the fluid motion with free surface confined inside a tank. The clear understanding of sloshing characteristics is essential for the determination of the required freeboard to prevent overflow of the contaminated cooling water, and for the estimation of hydrodynamic pressure on the pool and submerged components such as racks and fuel assemblies. The sloshing motion can affect the stability of the free-standing spent fuels during earthquakes. The sloshing characteristics in a storage pool may vary considerably depending upon the size and location of the stored spent fuel. The liquid sloshing can result in a highly localized pressure on the tank walls (and roofs, if present) which is highly dependent on the tank configuration and seismic characteristics of the applied load.

As a part of this research work, design charts have been generated and used to study the effect of geometry of tank on design seismic forces and sloshing. The focus of this research work is primarily to perform a comparative study of various codes on liquid containing tanks and this highlights the need for us to mature from the old code to newer code which is more accurate and reliable.

II. DYNAMIC MODELING

Seismic analysis of liquid containing tanks requires special considerations which account for the hydrodynamic forces exerted by the fluid on tank wall. Evaluation of these hydrodynamic forces requires suitable dynamic modeling of tank liquid system, which is rather complex. However, availability of mechanical models (analogues) of tanks has considerably simplified the analysis. These mechanical models, convert the tank-liquid system into an equivalent spring-mass system. Design codes use these mechanical models to evaluate seismic response of tanks.

Eurocode 8 [2] mentions mechanical model of [3] as an acceptable procedure for rigid circular tanks. For flexible circular tanks, models of [4] and [5] are described along with the procedure of [6]. The procedure given in NZSEE [7] guidelines is also described in Eurocode 8 for evaluating impulsive and convective mass of horizontal circular tank.

The circular tank-liquid system can be idealized as spring-mass model, which considerably simplifies the evaluation of hydrodynamic forces. In this mechanical model, it is recognized that vibrating fluid inside the container has two components, one that moves in unison with the tank (called impulsive component) and another one which undergoes sloshing motion (called convective component). The impulsive mass of liquid, m_i is rigidly attached to tank wall at height h_i (or h^*_i) and convective mass, m_c is attached to the tank wall at height h_c (or h^*_c) by a spring of stiffness K_c as shown in the Fig. 1. It may be noted that heights h_i and h_c are used when base pressure is not considered and if base pressure is included then corresponding heights are denoted by h^*_i and h^*_c respectively.

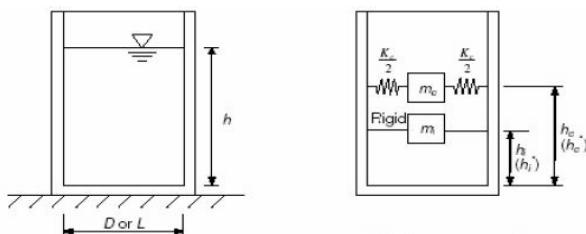


Fig. 1 Spring Mass Model for Ground Supported Circular Tanks [9]

The impulsive and the convective components have periods associated with them that are generally far apart. The total approximate response of the system can be estimated by the square-root-of-the-sum-of-the-squares (SRSS) combination of the responses of the two components [8]-[10]. Except Eurocode 8, all the codes suggest SRSS (square root of sum of square) rule to combine impulsive and convective forces. Eurocode 8 suggests use of absolute summation rule. Malhotra through numerical analysis of large number of tanks has proved that SRSS rule is better than absolute summation. For evaluating the impulsive force, mass of tank wall and roof is also considered along with impulsive fluid mass. ACI 350.3 [11] and Eurocode 8 suggest a reduction factor to suitably reduce the mass of tank wall. Such a reduction factor was suggested by [4] to compensate the conservativeness in the evaluation of impulsive force.

III. PARAMETRIC STUDY

The seismic responses of a ground supported liquid filled tank as shown in Fig. 2 are primarily influenced by its geometrical properties. According to various international codes such as Eurocode 8, the ratio of liquid height to the inner lateral dimension of tank defines the parameters of the dynamic model of the liquid storage tank. Thus a comparative analysis of the seismic response of tanks with various geometrical properties as mentioned in Table I was conducted. Constants considered for calculation are listed in Table II. Constant volume has been taken in the various iterations, since the main idea of the study was to investigate the influence of the geometry of tank on its dynamic responses. As far as possible realistic data input has been taken with slight exceptions in the case of wall and base slab thickness. The

seismic responses have been analyzed on Indian conditions only. The results of this parametric study were represented graphically.

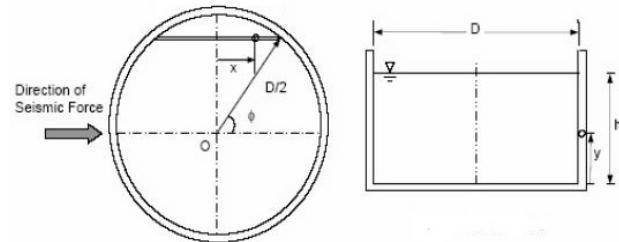


Fig. 2 Plan and Sectional Elevation of Tank [9]

TABLE I CHANGE IN ITERATION FOR SAME VOLUME					
Sr. No.	Iteration	Volume (l)	Diameter (m)	Height (m)	Free board (m)
Case 1					
1	1	50000	4.65	3.3	0.3
2	2	50000	4.25	3.83	0.3
3	3	50000	3.75	4.83	0.3
4	4	50000	3.5	5.5	0.3
5	5	50000	3.25	6.4	0.3
Case 2					
6	1	100000	6.5	3.3	0.3
7	2	100000	6	3.83	0.3
8	3	100000	5.75	4.15	0.3
9	4	100000	5.5	4.5	0.3
10	5	100000	5	5.45	0.3
Case 3					
11	1	200000	8.5	3.82	0.3
12	2	200000	8	4.28	0.3
13	3	200000	7.5	4.9	0.3
14	4	200000	7	5.5	0.3
15	5	200000	6.5	6.4	0.3

TABLE II
CONSTANTS CONSIDERED FOR CALCULATION

Seismic Zone	III
Importance Factor	1.5
Response Reduction Factor	2
Base Condition	Fixed Base
Concrete	M20
Wall Thickness	250 mm
Base Thickness	400

IV. RESULTS AND DISCUSSIONS

A. Comparative Analysis of Seismic Response of Tanks with Various Geometrical Properties

It is observed from Figs. 3-5 with the increase in h/d ratio it was observed that m_i increases but m_c decreases.

It is observed from Figs. 6-8 that with increasing h/d ratio T_c decreases but T_i remains fairly straight with very little increment. Also the values of T_c are much higher than T_i .

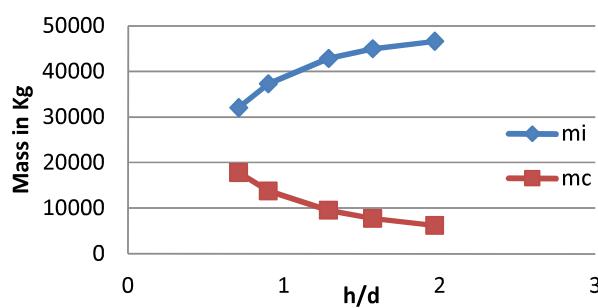


Fig. 3 Variation of Impulsive and Convective Mass with h/d (50 KL)

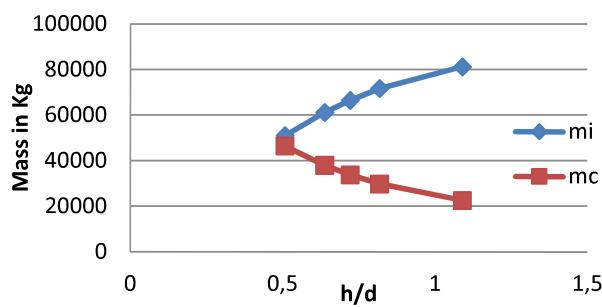


Fig. 4 Variation of Impulsive and Convective Mass with h/d (100 KL)

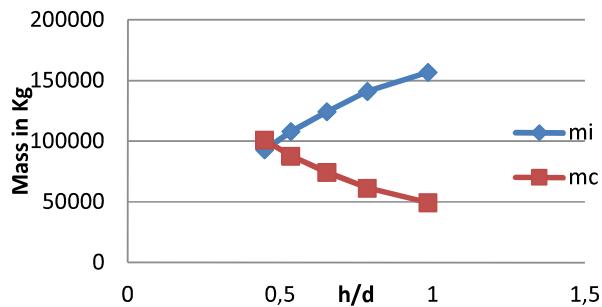


Fig. 5 Variation of Impulsive and Convective Mass with h/d (200 KL)

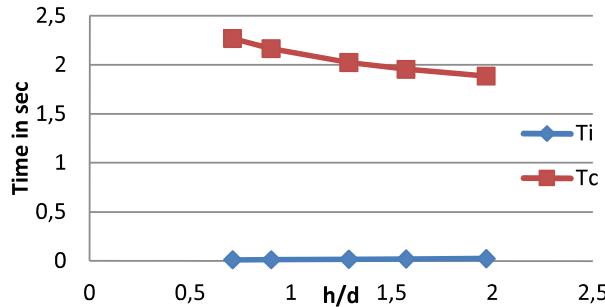


Fig. 6 Variation of Impulsive and Convective Time Period with h/d (50 KL)

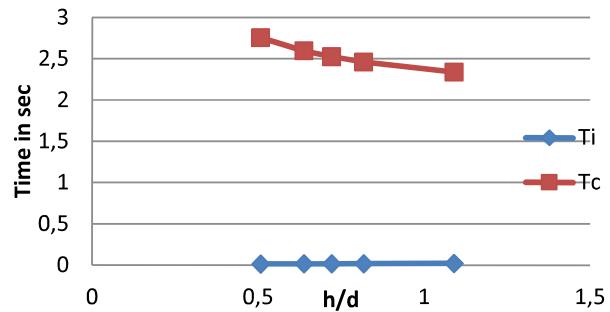


Fig. 7 Variation of Impulsive and Convective Time Period with h/d (100 KL)

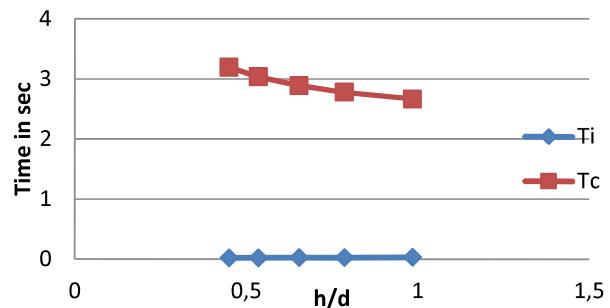


Fig. 8 Variation of Impulsive and Convective Time Period with h/d (200 KL)

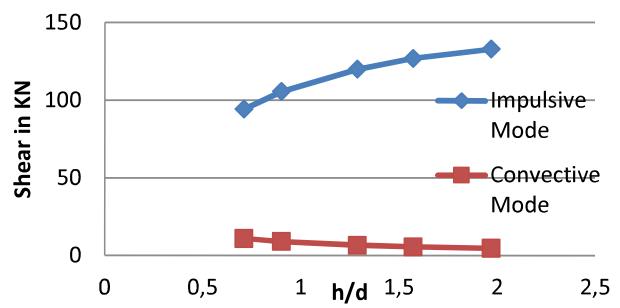


Fig. 9 Variation of Impulsive and Convective Component of Base Shear with h/d (50 KL)

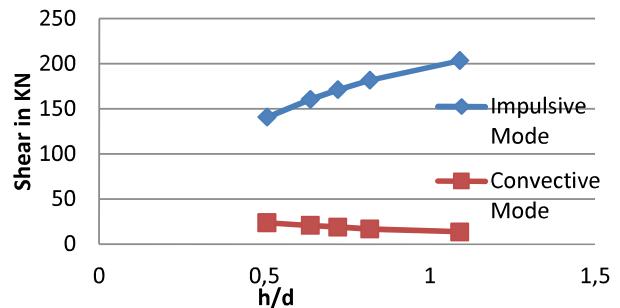


Fig. 10 Variation of Impulsive and Convective Component of Base Shear with h/d (100 KL)

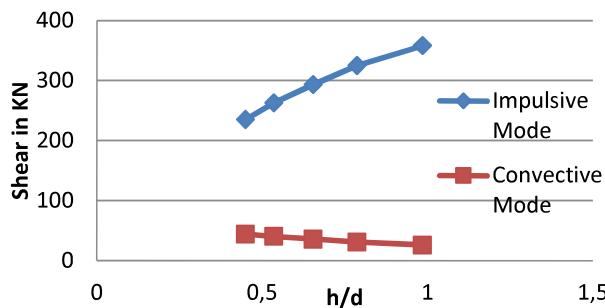


Fig. 11 Variation of Impulsive and Convective Component of Base Shear with h/d (200 KL)

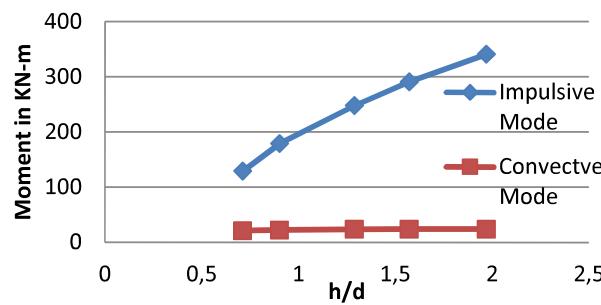


Fig. 12 Variation of Impulsive and Convective Component of Base Moment Mass with h/d (50 KL)

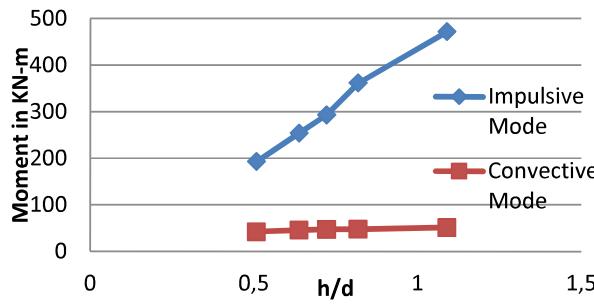


Fig. 13 Variation of Impulsive and Convective Component of Base Moment Mass with h/d (100 KL)

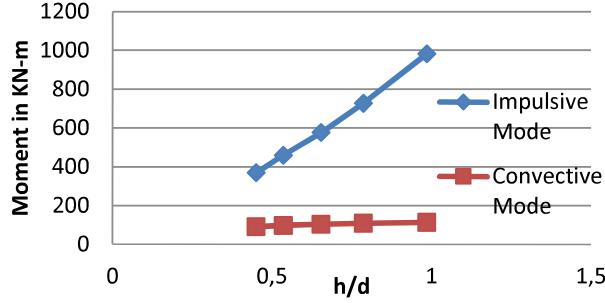


Fig. 14 Variation of Impulsive and Convective Component of Base Moment Mass with h/d (200 KL)

It is observed from Figs. 9-11, that the convective component of base shear is much less than impulsive

component. Also convective component's contribution decreases with increase in h/d , whereas, the impulsive component increases both with h/d and also with larger volume.

It is observed from Figs. 12-14, the M_i increases sharply with h/d and also its value is higher for larger volume of liquid stored. M_c is much lower when compared to M_i also its value increases at very small rate.

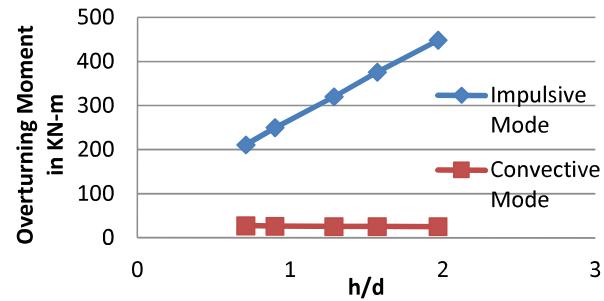


Fig. 15 Variation of Impulsive and Convective Overturning Moment with h/d (50 KL)

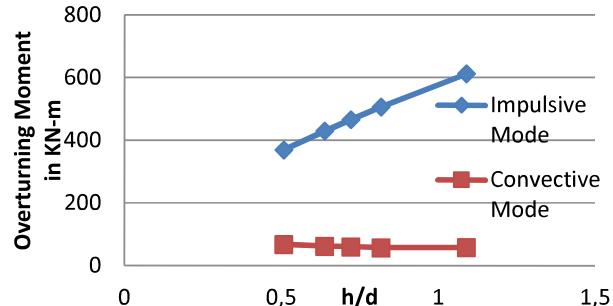


Fig. 16 Variation of Impulsive and Convective Overturning Moment with h/d (100 KL)

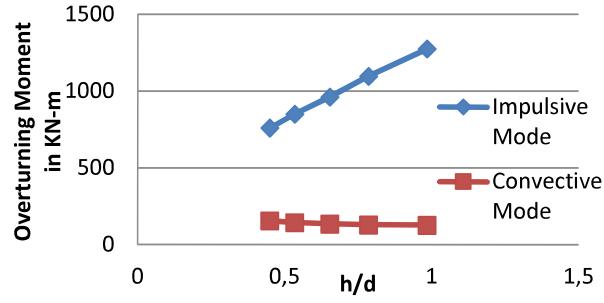


Fig. 17 Variation of Impulsive and Convective Overturning Moment with h/d (200 KL)

It is observed from Fig. 15-17, M^*_i increases sharply with h/d ratio and its value is much higher compared to M^*_c . Also M^*_c decreases with h/d with the effect more pronounced at higher volume of liquid.

It is observed from Fig. 18-20 that the maximum sloshing height decreases with h/d . In Fig. 19, a sharp declination is

observed between h/d 0.6 and 0.8. Another observation made was the change in curvature as volume of liquid stored is increased.

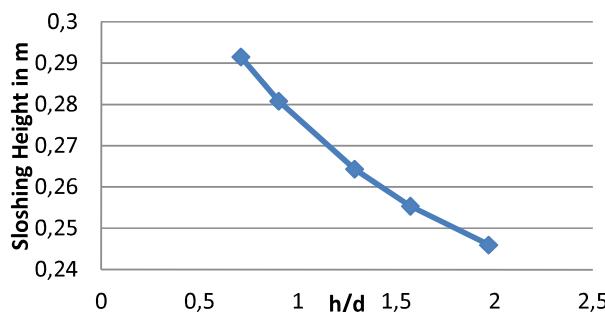


Fig. 18 Variation of Maximum Sloshing Height with h/d (50 KL)

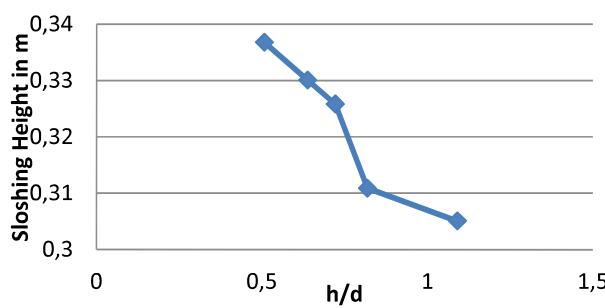


Fig. 19 Variation of Maximum Sloshing Height with h/d (100 KL)

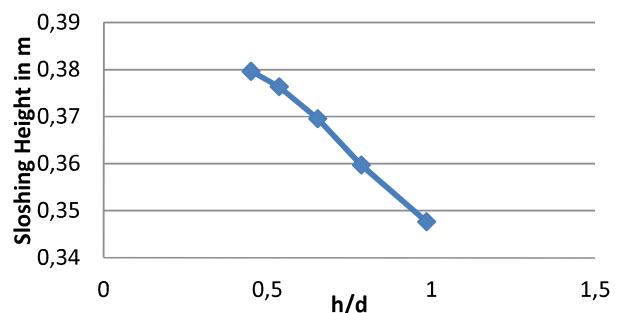


Fig. 20 Variation of Maximum Sloshing Height with h/d (200 KL)

B. Comparative Study of Base Shear and Moment for IS 3370(IV) and Draft IS 1893 Part 2 for Circular Tank

With the parameters of the model remaining same moment and shear at base were calculated based upon the relevant tables of IS 3370 IV -1967. The objective of this study is to highlight the differences in the magnitude of shear and moment at base as obtained from static (IS 3370 IV) and dynamic (Draft IS 1893 Part 2) analysis of ground supported tanks. From Table III, it has been observed that values obtained as per Draft IS 1893 Part 2 are considerably higher than those obtained by IS 3370 IV -1967 highlighting the need for us to mature from the old code to newer code which is more accurate and safe.

TABLE III
SHEAR AND MOMENT AT BASE WITH H/L AS PER IS 3370 PART IV AND IS 1893 PART 2

Capacity (l)	h/d	MOMENT AT BASE (KN-m)		SHEAR AT BASE (KN)	
		IS 3370	IS 1893 PART 2	IS 3370	IS 1893 PART 2
100000	0.507692	124.6906	197.6895	20.3643	142.67
100000	0.638333	135.4837	258.598	23.616929	161.88
100000	0.721739	135.4974	297.307	25.14485	172.2245
100000	0.818182	160.5203	400.6245876	28.755	198.4743645
100000	1.09	210.944	619.5999224	39.2073	257.4066383
200000	0.449412	252.9226	379.49466	27.287788	239.21
200000	0.535	261.9405	469.9146	33.339488	266.0433
200000	0.653333	271.5221	586.34	33.8541	295.54
200000	0.785714	321.8092	802.3998721	39.93	350.9529046
200000	0.984615	444.0798	1245.714257	52.8384	439.6818455

TABLE IV
 T_i AND T_c WITH H/L AS PER IS 1893 PART 2, ACI 350.3 AND EUROCODE 8

Capacity (l)	h/d	Time Period			
		ACI 350.3	Eurocode 8	IS 1893 PART 2	ACI 350.3
50000	0.709677	0.011	0.03984541	0.012333	2.27
50000	0.901176	0.01313	0.04514629	0.013811	2.16694
50000	1.288	0.01728	0.059095	0.017815	2.02559
50000	1.571429	0.0208	0.07059919	0.021412	1.9567
50000	1.969231	0.025167	0.08440157	0.033833	1.886051
100000	0.507692	0.0142	0.04813565	0.015162	2.75589
100000	0.638333	0.015593	0.05258724	0.016371	2.5955
100000	0.721739	0.01652	0.05574218	0.017238	2.526122
100000	0.818182	0.01772	0.05958447	0.018308	2.46158
100000	1.09	0.021337	0.07273243	0.022078	2.339
200000	0.449412	0.019467	0.06497602	0.02056	3.19698
200000	0.535	0.02064	0.06880019	0.021618	3.03528
					3.014
					3.01641

TABLE V
BASE SHEAR WITH H/D AS PER IS 1893 PART 2, ACI 350.3 AND EUROCODE 8

Capacity (l)	h/d	Base Shear					
		Impulsive Component			Convective Component		
		ACI 350.3	Eurocode 8	IS 1893 PART 2	ACI 350.3	Eurocode 8	IS 1893 PART 2
50000	0.709677	94.289	94.289782	94.43	10.96	10.9902044	11.01
50000	0.901176	105.69	105.690183	105.9	8.95	8.96508387	9.06
50000	1.288	120.011	120.01187	120.23	6.58	6.58388762	6.88
50000	1.571429	126.98	126.982159	127.01	5.545	5.54215946	5.75
50000	1.969231	133	133.832270	133.45	4.6149	4.61353107	4.83
100000	0.50769	140.71	140.71411	140.98	23.609	23.7512895	23.901
100000	0.63833	160.58	160.58890	161.05	20.51	20.4986432	21.01
100000	0.72173	171.198	171.19876	171.83	18.77	18.8976894	19.22
100000	0.81818	181.741	181.74422	181.98	16.549	17.0341220	16.87
100000	1.09	203.57	203.12950	204.09	13.505	13.5041779	13.912
200000	0.44941	235.097	235.09741	235.89	44.2263	43.964496	44.6314
200000	0.535	262.935	262.93597	263.35	40.54969	40.836078	40.9654
200000	0.65333	293.350	293.35030	293.96	35.98386	35.9742469	36.1254
200000	0.78571	325.119	325.11986	325.88	30.975	31.08764	31.231
200000	0.98461	358.038	358.03846	358.74	25.981	25.99340	26.321

C. Comparative Study of Draft IS 1893 Part 2, ACI 350.3 and Eurocode 8 for Circular Shaped Tank

The geometry details considered were as per Table I. In this analysis only parameters of the proposed model of the relevant

codes were considered. Once the model parameters were obtained, they were used for analysis in Indian conditions. The constants used are listed in Table II.

TABLE VI
M_I AND M_C WITH H/L AS PER IS 1893 PART 2, ACI 350.3 AND EUROCODE 8

Capacity (l)	h/d	Moment at Base					
		Impulsive Component			Convective Component		
		ACI 350.3	Eurocode 8	IS 1893 PART 2	ACI 350.3	Eurocode 8	IS 1893 PART 2
50000	0.709677	128.843	128.8436324	129.013	20.878	20.9298174	21.014
50000	0.901176	178.949	178.9497522	179.212	22.13	22.1750391	22.451
50000	1.288	247.92	247.9297101	248.221	23.26	23.2488851	23.512
50000	1.571429	291.24	291.2427523	291.641	23.58	23.5717156	23.86
50000	1.969231	340.89	340.893996	341.112	23.708	23.7008	24.002
100000	0.507692	193.122	193.12206	193.854	42.25	42.5106	42.631
100000	0.638333	254.48	254.4825519	254.961	45.97	45.9244	46.214
100000	0.721739	293.48	293.4865903	293.954	47.552	47.8679	47.964
100000	0.818182	361.68	361.6861085	362.012	47.768	49.1051	48.214
100000	1.09	471.88	471.8842354	472.123	51.26	51.2641	51.861
200000	0.449412	368.62	368.6276661	369.141	90.1974	89.6634	90.856
200000	0.535	459.68	459.6805581	460.02	97.5405	98.2295	97.996
200000	0.653333	577.0628	577.062864	577.827	103.9	103.8725	104.451
200000	0.785714	727.654	727.654999	723.014	109.237	109.634	109.874
200000	0.984615	981.722	981.7222977	982.31	113.6125	113.666	114.012

TABLE VII
IMPULSIVE AND CONVECTIVE OVERTURNING MOMENT WITH H/L AS PER IS 1893 PART 2, ACI 350.3 AND EUROCODE 8

Capacity (l)	h/d	Overturning Moment					
		Impulsive Component			Convective Component		
		ACI 350.3	Eurocode 8	IS 1893 PART 2	ACI 350.3	Eurocode 8	IS 1893 PART 2
50000	0.7096774	211.578	211.578255	211.987	28.01	28.08196375	29.101
50000	0.9011765	250.13	250.261655	250.631	26.708	26.75235106	27.301
50000	1.288	320.511	320.511678	320.945	26.05	26.0420078	26.941
50000	1.5714286	376.497	376.497866	376.997	25.848	25.83371406	26.784
50000	1.9692308	448.773	448.773	449.265	25.56	25.55524	26.631
100000	0.5076923	369.504	369.5045	369.912	67.52	67.9256	68.461
100000	0.6383333	429.58	429.5867	430.123	62.004	61.94255	62.984
100000	0.7217391	466.08	466.081	466.987	60.138	60.53775	60.993
100000	0.8181818	506.799	506.7994	507.65	57.33	59.00779	58.124
100000	1.09	612.375	612.375	613.321	56.44	57.53972	58.012
200000	0.4494118	759.557	759.5578	760.891	154.989	154.0714	155.845
200000	0.535	851.1638	851.1638	852.014	143.077	144.0882	143.962
200000	0.6533333	961.183	961.183984	962.145	134.558	134.52211	135.621
200000	0.7857143	1096.056	1096.051	1097.13	129.095	129.8788	130.841
200000	0.9846154	1274.896	1274.897	1275.93	127.065	127.1251	127.984

From Tables IV-VIII following observations were made:

- Impulsive time period obtained from Eurocode 8 are higher than ACI 350.3 and Draft IS 1893 Part 2 with values obtained from ACI 350.3 being slightly lower than Draft IS 1893 Part 2.
- Convective time period obtained is nearly constant for all the codes.

- Since Draft IS 1893 Part 2 specifies that for T_i less than 0.1 sec the value S_a/g be taken as 2.5 for 5% damping the values of impulsive components of base shear, moment at base and overturning moment are all nearly equal.
- Also since T_c is nearly the same in all cases the convective components of base shear, moment at base and overturning moment are all nearly equal with Draft IS 1893 Part 2 giving slightly higher values.

TABLE VIII
MAXIMUM SLOSHING HEIGHT

Capacity (l)	h/d	Sloshing Height(m)		
		ACI 350.3	Eurocode 8	IS 1893 PART 2
50000	0.709677	0.29155	0.245506	0.3015
50000	0.901176	0.28084	0.236295	0.2931
50000	1.288	0.26436	0.221961	0.27891
50000	1.571429	0.2554	0.214435	0.26541
50000	1.969231	0.246	0.206762	0.2546
100000	0.507692	0.3368	0.284612	0.3401
100000	0.638333	0.3301	0.277018	0.3351
100000	0.721739	0.32584	0.27485	0.3324
100000	0.818182	0.310911	0.268808	0.3247
100000	1.09	0.3051	0.256299	0.3124
200000	0.449412	0.37967	0.317035	0.3841
200000	0.535	0.37637	0.318386	0.37912
200000	0.653333	0.36954	0.310335	0.37412
200000	0.785714	0.359714	0.303256	0.36711
200000	0.984615	0.3477	0.292225	0.3501

V.CONCLUDING REMARKS

From the analysis of a simple model of ground supported circular liquid storage tank taking the obligatory provisions of Draft of IS 1893 Part 2 the following conclusion may be drawn:

- The contribution of impulsive component of liquid mass is always greater than the convective component of liquid mass in terms of base shear, base moment and overturning moment for all capacities of tanks under external excitation acting on the tank liquid system.
- Impulsive period of vibrations, base shear force and overturning moment increase almost linearly with increase of height to lateral dimension ratio, whereas convective period of vibrations approaches a constant value above height to lateral dimension ratio greater than 1.2 for both circular shaped tanks.
- Maximum sloshing height decreases with height to lateral dimension ratio, and it is found to vary for different capacities of tank. The effect of water sloshing must be included in the analysis. Free board to be provided in the tank should be based on maximum value of sloshing wave height. If sufficient free board is not provided, roof structure should be designated to resist the uplift pressure due to sloshing of water.
- Response obtained by static method to dynamic method differs considerably for similar geometry and conditions of liquid storage tank. Also, even if we consider two cases for same capacity of tank, change in geometric features of a container can show considerable in the response of

ground supported water tank. A larger capacity tank exhibits greater values of both impulsive and convective components of liquid mass and its related effects.

- Provisions mentioned in the Draft of IS 1893 Part 2 are quite comprehensive and simple to use as is the case for ACI 350.3. Most of the provisions stipulated in the draft have been derived from ACI 350.3 and Eurocode 8. The results obtained are satisfactory and quite similar with those obtained as per international code provisions.

REFERENCES

- [1] O. R. Jaiswal, D.C. Rai and S.K. Jain, "Review of code provisions on seismic analysis of liquid storage tanks: a review" Report No. IITK-GSDMA-EQ-04-V1.0, Indian Institute of Technology, Kanpur. (2004).
- [2] Eurocode 8, "Design provisions for earthquake resistance of structures, Part 1- General rules and Part 4 – Silos, tanks and pipelines", European Committee for Standardization, Brussels. (1998).
- [3] Veletsos, A.S. and Yang, J.Y. "Earthquake Response of Liquid Storage Tanks, in Advances in Civil Engineering through Engineering Mechanics", Proceedings of the Second Engineering Mechanics Specialty Conference, ASCE/EMD Specialty Conference, Raleigh, NC, pp. 1-24, (1977).
- [4] Veletsos, A. S., "Seismic response and design of liquid storage tanks", Standards for the seismic design of oil and gas pipeline systems, Technical Council on Lifeline Earthquake Engineering, ASCE, N.Y., 255-370, 443-461. (1984)
- [5] M. A. Haroun, G. W. Housner, "Dynamic characteristics of liquid storage tanks", ASCE 108, 783-799. (1982).
- [6] P.K. Malhotra, T. Wenk, and M. Wieland, "Simple procedure for seismic analysis of liquid storage tanks", Structural Engineering, IABSE, Vol. 10, No.3, 197-201, (2000).
- [7] NZS 3106, "Code of practice for concrete structures for the storage of liquids", Standards Association of New Zealand, Wellington, (1986). E. H. Miller, "A note on reflector arrays (Periodical style—Accepted for publication)," IEEE Trans. Antennas Propagat., to be published.

- [8] Barros, R.C. "Seismic Analysis and Design of Bottom Supported Anchored Metallic Tanks", Edições INEGI, ISBN: 978-972-8826-18-5, pp. 1-160, Porto, Portugal, (2008).
- [9] "IITK-GSDMA: Guidelines for Seismic Design of Liquid Storage Tanks, Provisions with Commentary on the Indian seismic code 'Indian Standard IS 1893 (Part 1): 2002'", Indian Institute of Technology Kanpur, Gujarat State Disaster Management Authority, (2005).
- [10] Barros, R.C. "On the Seismic Response of Anchored Tanks: Methodologies for Finite Element Analysis and Parametric Study for Design Codes, in Civil Engineering Computations: Tools and Techniques", Ed.: B.H.V. Topping, Chapter 17, 391-447, Saxe-Coburg Publications, Stirlingshire, UK. (2007).
- [11] ACI 350.3, "Seismic design of liquid containing concrete structures", An American Concrete Institute Standard, (2001).