International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:9, No:12, 2015

Membership Surface and Arithmetic Operations of Imprecise Matrix

Dhruba Das

Abstract—In this paper, a method has been developed to construct the membership surfaces of row and column vectors and arithmetic operations of imprecise matrix. A matrix with imprecise elements would be called an imprecise matrix. The membership surface of imprecise vector has been already shown based on Randomness-Impreciseness Consistency Principle. The Randomness-Impreciseness Consistency Principle leads to defining a normal law of impreciseness using two different laws of randomness. In this paper, the author has shown row and column membership surfaces and arithmetic operations of imprecise matrix and demonstrated with the help of numerical example.

Keywords—Imprecise number, Imprecise vector, Membership surface, Imprecise matrix.

I. Introduction

BARUAH ([1]-[4]) has defined an imprecise number $X = [a, b, c]$ with membership function

$$\mu_{X|Y}(x,y) = \begin{cases} L(x) & a \le x \le b, p \le y \le r \\ R(x) & b \le x \le c, p \le y \le r \\ 0 & \text{otherwise.} \end{cases}$$

$L(x)$ being a continuous non-decreasing function in the interval $[a, b]$, and $R(x)$ being a continuous non-increasing function in the interval $[b, c]$, with $L(a) = R(c) = 0$ and $L(b) = R(b) = 1$. A continuous non-decreasing function of this type is also called a distribution function with reference to Lebesgue-Stieltjes measure ([5]). Das and Baruah ([6]-[8]) have shown the construction method of the membership surface of imprecise vector. If (X, Y) is an imprecise vector, where X and Y both are imprecise numbers represented by $X = [a, b, c]$ and $Y = [p, q, r]$ respectively and if the membership function of X and Y be

$$\mu_{X|Y}(x,y) = \begin{cases} L(x) & a \le x \le b, p \le y \le r \\ R(x) & b \le x \le c, p \le y \le r \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Y|X}(x,y) = \begin{cases} L(y) & a \le x \le c, p \le y \le q \\ R(y) & a \le x \le c, q \le y \le r, \\ 0 & \text{otherwise.} \end{cases}$$

Then the membership surface of the imprecise vector (X, Y) can be obtained [3] as:

$$\mu_{X,Y}(x,y) = \begin{cases} L(x)L(y) & a \le x \le b, p \le y \le q \\ L(x)R(y) & a \le x \le b, q \le y \le r \\ R(x)L(y) & b \le x \le c, p \le y \le q \\ R(x)R(y) & b \le x \le c, q \le y \le r \\ 0 & \text{otherwise.} \end{cases}$$

D. Das is with the Department of Statistics, Gauhati University, Assam, India, PIN 781014 (e-mail: dhrubadas@gauhati.ac.in).

Here it is assumed that X and Y are independently distributed. In a matrix all rows and columns are nothing but row vectors and column vectors. So, we may obtain the row membership surfaces and column membership surfaces of an imprecise matrix. If the elements of the matrix are imprecise then we can construct the membership surfaces of row vectors and column vectors of that matrix.

II. Membership Surfaces of Row and Column Vectors of an Imprecise Matrix

Consider a 2×2 matrix

$$\begin{pmatrix} X & Y \\ Z & W \end{pmatrix}$$

where all the elements of the matrix X, Y, Z and W are imprecise. Assume that all the elements X, Y, Z and W are triangular imprecise numbers represented by $[x_1, x_2, x_3], [y_1, y_2, y_3], [z_1, z_2, z_3]$ and $[w_1, w_2, w_3]$ respectively. We may consider (X, Y) and (Z, W) as row vectors and (X, Z) and (Y, W) as column vectors. The membership functions of X, Y, Z and W for row vectors are as:

$$\mu_{X|Y}(x,y) = \begin{cases} \dfrac{x - x_1}{x_2 - x_1} & x_1 \le x \le x_2, \\ & y_1 \le y \le y_3 \\ \dfrac{x - x_3}{x_2 - x_3} & x_2 \le x \le x_3, \\ & y_1 \le y \le y_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Y|X}(x,y) = \begin{cases} \dfrac{y - y_1}{y_2 - y_1} & x_1 \le x \le x_3, \\ & y_1 \le y \le y_2 \\ \dfrac{y - y_3}{y_2 - y_3} & x_1 \le x \le x_3, \\ & y_2 \le y \le y_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z|W}(z,w) = \begin{cases} \dfrac{z - z_1}{z_2 - z_1} & w_1 \le w \le w_3, \\ & z_1 \le z \le z_2 \\ \dfrac{z - z_3}{z_2 - z_3} & w_1 \le w \le w_3, \\ & z_2 \le z \le z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{W|Z}(z,w) = \begin{cases} \dfrac{w - w_1}{w_2 - w_1} & w_1 \le w \le w_2, \\ & z_1 \le z \le z_3 \\ \dfrac{w - w_3}{w_2 - w_3} & w_2 \le w \le w_3, \\ & z_1 \le z \le z_3 \\ 0 & \text{otherwise.} \end{cases}$$

Now the membership surfaces of row vectors of the imprecise matrix will be as:

$$\mu_R(x,y) = \begin{cases} \frac{(x-x_1)(y-y_1)}{(x_2-x_1)(y_2-y_1)} & x_1 \le x \le x_2, y_1 \le y \le y_2 \\ \frac{(x-x_1)(y-y_3)}{(x_2-x_1)(y_2-y_3)} & x_1 \le x \le x_2, y_2 \le y \le y_3 \\ \frac{(x-x_3)(y-y_1)}{(x_2-x_3)(y_2-y_1)} & x_2 \le x \le x_3, y_1 \le y \le y_2 \\ \frac{(x-x_3)(y-y_3)}{(x_2-x_3)(y_2-y_3)} & x_2 \le x \le x_3, y_2 \le y \le y_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_R(z,w) = \begin{cases} \frac{(z-z_1)(w-w_1)}{(z_2-z_1)(w_2-w_1)} & z_1 \le z \le z_2, w_1 \le w \le w_2 \\ \frac{(z-z_1)(w-w_3)}{(z_2-z_1)(w_2-w_3)} & z_1 \le z \le z_2, w_2 \le w \le w_3 \\ \frac{(z-z_3)(w-w_1)}{(z_2-z_3)(w_2-w_1)} & z_2 \le z \le z_3, w_1 \le w \le w_2 \\ \frac{(z-z_3)(w-w_3)}{(z_2-z_3)(w_2-w_3)} & z_2 \le z \le z_3, w_2 \le w \le w_3 \\ 0 & \text{otherwise.} \end{cases}$$

Again, the membership functions of X, Y, Z and W for column vectors are as:

$$\mu_{X|Z}(x,z) = \begin{cases} \frac{x-x_1}{x_2-x_1} & x_1 \le x \le x_2, z_1 \le z \le z_3 \\ \frac{x-x_3}{x_2-x_3} & x_2 \le x \le x_3, z_1 \le z \le z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Y|W}(y,w) = \begin{cases} \frac{y-y_1}{y_2-y_1} & y_1 \le y \le y_2, w_1 \le w \le w_3 \\ \frac{y-y_3}{y_2-y_3} & y_2 \le y \le y_3, w_1 \le w \le w_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z|X}(z,x) = \begin{cases} \frac{z-z_1}{z_2-z_1} & x_1 \le x \le x_3, z_1 \le z \le z_2 \\ \frac{z-z_3}{z_2-z_3} & x_1 \le x \le x_3, z_2 \le z \le z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{W|Y}(w,y) = \begin{cases} \frac{w-w_1}{w_2-w_1} & w_1 \le w \le w_2, y_1 \le y \le y_3 \\ \frac{w-w_3}{w_2-w_3} & w_2 \le w \le w_3, y_1 \le y \le y_3 \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, the membership surfaces of column vectors of the imprecise matrix will be as:

$$\mu_C(x,z) = \begin{cases} \frac{(x-x_1)(z-z_1)}{(x_2-x_1)(z_2-z_1)} & x_1 \le x \le x_2, z_1 \le z \le z_2 \\ \frac{(x-x_1)(z-z_3)}{(x_2-x_1)(z_2-z_3)} & x_1 \le x \le x_2, z_2 \le z \le z_3 \\ \frac{(x-x_3)(z-z_1)}{(x_2-x_3)(z_2-z_1)} & x_2 \le x \le x_3, z_1 \le z \le z_2 \\ \frac{(x-x_3)(z-z_3)}{(x_2-x_3)(z_2-z_3)} & x_2 \le x \le x_3, z_2 \le z \le z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(y,w) = \begin{cases} \frac{(y-y_1)(w-w_1)}{(y_2-y_1)(w_2-w_1)} & y_1 \le y \le y_2, w_1 \le w \le w_2 \\ \frac{(y-y_1)(w-w_3)}{(y_2-y_1)(w_2-w_3)} & y_1 \le y \le y_2, w_2 \le w \le w_3 \\ \frac{(y-y_3)(w-w_1)}{(y_2-y_3)(w_2-w_1)} & y_2 \le y \le y_3, w_1 \le w \le w_2 \\ \frac{(y-y_3)(w-w_3)}{(y_2-y_3)(w_2-w_3)} & y_2 \le y \le y_3, w_2 \le w \le w_3 \\ 0 & \text{otherwise.} \end{cases}$$

In the same way, for a 3×3 matrix

$$\begin{pmatrix} X & Y & P \\ Z & W & Q \\ M & N & R \end{pmatrix}$$

where all the elements of the matrix X, Y, P, Z, W, Q, M, N and R are imprecise. Assume that all the elements X, Y, P, Z, W, Q, M, N and R are triangular imprecise numbers represented by $[x_1, x_2, x_3]$, $[y_1, y_2, y_3]$,$[p_1, p_2, p_3]$, $[z_1, z_2, z_3]$,$[w_1, w_2, w_3]$, $[q_1, q_2, q_3]$, $[m_1, m_2, m_3]$, $[n_1, n_2, n_3]$ and $[r_1, r_2, r_3]$ respectively. We may consider as $(X, Y, P), (Z, W, Q)$ and (M, N, R) as row vectors and $(X, Z, M), (Y, W, N)$ and (P, Q, R) as column vectors. The membership functions of X, Y, P, Z, W, Q, M, N and R for row vectors are as:

$$\mu_{X|Y,P}(x,y,p) = \begin{cases} \frac{x-x_1}{x_2-x_1} & x_1 \le x \le x_2, y_1 \le y \le y_3, \\ & p_1 \le p \le p_3 \\ \frac{x-x_3}{x_2-x_3} & x_2 \le x \le x_3, y_1 \le y \le y_3, \\ & p_1 \le p \le p_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Y|X,P}(x,y,p) = \begin{cases} \frac{y-y_1}{y_2-y_1} & x_1 \le x \le x_3, y_1 \le y \le y_2, \\ & p_1 \le p \le p_3 \\ \frac{y-y_3}{y_2-y_3} & x_1 \le x \le x_3, y_2 \le y \le y_3, \\ & p_1 \le p \le p_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{P|X,Y}(x,y,p) = \begin{cases} \frac{p-p_1}{p_2-p_1} & x_1 \le x \le x_3, y_1 \le y \le y_3, \\ & p_1 \le p \le p_2 \\ \frac{p-p_3}{p_2-p_3} & x_1 \le x \le x_3, y_1 \le y \le y_3, \\ & p_2 \le p \le p_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z|W,Q}(z,w,q) = \begin{cases} \frac{z-z_1}{z_2-z_1} & z_1 \le z \le z_2, w_1 \le w \le w_3, \\ & q_1 \le q \le q_3 \\ \frac{z-z_3}{z_2-z_3} & z_2 \le z \le z_3, w_1 \le w \le w_3, \\ & q_1 \le q \le q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{W|Q,Z}(z,w,q) = \begin{cases} \frac{w-w_1}{w_2-w_1} & z_1 \le z \le z_3, w_1 \le w \le w_2, \\ & q_1 \le q \le q_3 \\ \frac{w-w_3}{w_2-w_3} & z_1 \le z \le z_3, w_2 \le w \le w_3, \\ & q_1 \le q \le q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Q|Z,W}(z,w,q) = \begin{cases} \frac{q-q_1}{q_2-q_1} & z_1 \le z \le z_3, w_1 \le w \le w_3, \\ & q_1 \le q \le q_2 \\ \frac{q-q_3}{q_2-q_3} & z_1 \le z \le z_3, w_1 \le w \le w_3, \\ & q_2 \le q \le q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{M|N,R}(m,n,r) = \begin{cases} \frac{m-m_1}{m_2-m_1} & m_1 \le m \le m_2, n_1 \le n \le n_3, \\ & r_1 \le r \le r_3 \\ \frac{m-m_3}{m_2-m_3} & m_2 \le m \le m_3, n_1 \le n \le n_3, \\ & r_1 \le r \le r_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{N|M,R}(m,n,r) = \begin{cases} \frac{n-n_1}{n_2-n_1} & m_1 \le m \le m_3, n_1 \le n \le n_2, \\ & r_1 \le r \le r_3 \\ \frac{n-n_3}{n_2-n_3} & m_1 \le m \le m_3, n_2 \le n \le n_3, \\ & r_1 \le r \le r_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{R|M,N}(m,n,r)=\begin{cases}\dfrac{r-r_1}{r_2-r_1} & m_1\le m\le m_3, n_1\le n\le n_3,\\ & r_1\le r\le r_2\\ \dfrac{r-r_3}{r_2-r_3} & m_1\le m\le m_3, n_1\le n\le n_3,\\ & r_2\le r\le r_3\\ 0 & \text{otherwise.}\end{cases}$$

Then the membership surfaces of row vectors of the imprecise matrix will be as:

$$\mu_R(x,y,p)=$$
$$\begin{cases}\dfrac{(x-x_1)(y-y_1)(p-p_1)}{(x_2-x_1)(y_2-y_1)(p_2-p_1)} & x_1\le x\le x_2,\\ & y_1\le y\le y_2, p_1\le p\le p_2\\ \dfrac{(x-x_1)(y-y_3)(p-p_1)}{(x_2-x_1)(y_2-y_3)(p_2-p_1)} & x_1\le x\le x_2,\\ & y_2\le y\le y_3, p_1\le p\le p_2\\ \dfrac{(x-x_3)(y-y_1)(p-p_1)}{(x_2-x_3)(y_2-y_1)(p_2-p_1)} & x_2\le x\le x_3,\\ & y_1\le y\le y_2, p_1\le p\le p_2\\ \dfrac{(x-x_3)(y-y_3)(p-p_1)}{(x_2-x_3)(y_2-y_3)(p_2-p_1)} & x_2\le x\le x_3,\\ & y_2\le y\le y_3, p_1\le p\le p_2\\ \dfrac{(x-x_1)(y-y_1)(p-p_3)}{(x_2-x_1)(y_2-y_1)(p_2-p_3)} & x_1\le x\le x_2,\\ & y_1\le y\le y_2, p_2\le p\le p_3\\ \dfrac{(x-x_1)(y-y_3)(p-p_3)}{(x_2-x_1)(y_2-y_3)(p_2-p_3)} & x_1\le x\le x_2,\\ & y_2\le y\le y_3, p_2\le p\le p_3\\ \dfrac{(x-x_3)(y-y_1)(p-p_3)}{(x_2-x_3)(y_2-y_1)(p_2-p_3)} & x_2\le x\le x_3,\\ & y_1\le y\le y_2, p_2\le p\le p_3\\ \dfrac{(x-x_3)(y-y_3)(p-p_3)}{(x_2-x_3)(y_2-y_3)(p_2-p_3)} & x_2\le x\le x_3,\\ & y_2\le y\le y_3, p_2\le p\le p_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_R(z,w,q)=$$
$$\begin{cases}\dfrac{(z-z_1)(w-w_1)(q-q_1)}{(z_2-z_1)(w_2-w_1)(q_2-q_1)} & z_1\le z\le z_2,\\ & w_1\le w\le w_2, q_1\le q\le q_2\\ \dfrac{(z-z_1)(w-w_3)(q-q_1)}{(z_2-z_1)(w_2-w_3)(q_2-q_1)} & z_1\le z\le z_2,\\ & w_2\le w\le w_3, q_1\le q\le q_2\\ \dfrac{(z-z_3)(w-w_1)(q-q_1)}{(z_2-z_3)(w_2-w_1)(q_2-q_1)} & z_2\le z\le z_3,\\ & w_1\le w\le w_2, q_1\le q\le q_2\\ \dfrac{(z-z_3)(w-w_3)(q-q_1)}{(z_2-z_3)(w_2-w_3)(q_2-q_1)} & z_2\le z\le z_3,\\ & w_2\le w\le w_3, q_1\le q\le q_2\\ \dfrac{(z-z_1)(w-w_1)(q-q_3)}{(z_2-z_1)(w_2-w_1)(q_2-q_3)} & z_1\le z\le z_2,\\ & w_1\le w\le w_2, q_2\le q\le q_3\\ \dfrac{(z-z_1)(w-w_3)(q-q_3)}{(z_2-z_1)(w_2-w_3)(q_2-q_3)} & z_1\le z\le z_2,\\ & w_2\le w\le w_3, q_2\le q\le q_3\\ \dfrac{(z-z_3)(w-w_1)(q-q_3)}{(z_2-z_3)(w_2-w_1)(q_2-q_3)} & z_2\le z\le z_3,\\ & w_1\le w\le w_2, q_2\le q\le q_3\\ \dfrac{(z-z_3)(w-w_3)(q-q_3)}{(z_2-z_3)(w_2-w_3)(q_2-q_3)} & z_2\le z\le z_3,\\ & w_2\le w\le w_3, q_2\le q\le q_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_R(m,n,r)=$$
$$\begin{cases}\dfrac{(m-m_1)(n-n_1)(r-r_1)}{(m_2-m_1)(n_2-n_1)(r_2-r_1)} & m_1\le m\le m_2,\\ & n_1\le n\le n_2, r_1\le r\le r_2\\ \dfrac{(m-m_1)(n-n_3)(r-r_1)}{(m_2-m_1)(n_2-n_3)(r_2-r_1)} & m_1\le m\le m_2,\\ & n_2\le n\le n_3, r_1\le r\le r_2\\ \dfrac{(m-m_3)(n-n_1)(r-r_1)}{(m_2-m_3)(n_2-n_1)(r_2-r_1)} & m_2\le m\le m_3,\\ & n_1\le n\le n_2, r_1\le r\le r_2\\ \dfrac{(m-m_3)(n-n_3)(r-r_1)}{(m_2-m_3)(n_2-n_3)(r_2-r_1)} & m_2\le m\le m_3,\\ & n_2\le n\le n_3, r_1\le r\le r_2\\ \dfrac{(m-m_1)(n-n_1)(r-r_3)}{(m_2-m_1)(n_2-n_1)(r_2-r_3)} & m_1\le m\le m_2,\\ & n_1\le n\le n_2, r_2\le r\le r_3\\ \dfrac{(m-m_1)(n-n_3)(r-r_3)}{(m_2-m_1)(n_2-n_3)(r_2-r_3)} & m_1\le m\le m_2,\\ & n_2\le n\le n_3, r_2\le r\le r_3\\ \dfrac{(m-m_3)(n-n_1)(r-r_3)}{(m_2-m_3)(n_2-n_1)(r_2-r_3)} & m_2\le m\le m_3,\\ & n_1\le n\le n_2, r_2\le r\le r_3\\ \dfrac{(m-m_3)(n-n_3)(r-r_3)}{(m_2-m_3)(n_2-n_3)(r_2-r_3)} & m_2\le m\le m_3,\\ & n_2\le n\le n_3, r_2\le r\le r_3\\ 0 & \text{otherwise.}\end{cases}$$

The membership functions of X, Z, M, Y, W, N, P, Q and R for column vectors are as:

$$\mu_{X|M,Z}(x,m,z)=\begin{cases}\dfrac{x-x_1}{x_2-x_1} & x_1\le x\le x_2,\\ & m_1\le m\le m_3, z_1\le z\le z_3\\ \dfrac{x-x_3}{x_2-x_3} & x_2\le x\le x_3,\\ & m_1\le m\le m_3, z_1\le z\le z_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_{Z|X,M}(x,m,z)=\begin{cases}\dfrac{z-z_1}{z_2-z_1} & z_1\le z\le z_2,\\ & x_1\le x\le x_3, m_1\le m\le m_3\\ \dfrac{z-z_3}{z_2-z_3} & z_2\le z\le z_3,\\ & x_1\le x\le x_3, m_1\le m\le m_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_{M|X,Z}(x,m,z)=\begin{cases}\dfrac{m-m_1}{m_2-m_1} & m_1\le m\le m_2,\\ & x_1\le x\le x_3, z_1\le z\le z_3\\ \dfrac{m-m_3}{m_2-m_3} & m_2\le m\le m_3,\\ & x_1\le x\le x_3, z_1\le z\le z_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_{Y|N,W}(y,n,w)=\begin{cases}\dfrac{y-y_1}{y_2-y_1} & y_1\le y\le y_2,\\ & n_1\le n\le n_3, w_1\le w\le w_3\\ \dfrac{y-y_3}{y_2-y_3} & y_2\le y\le y_3,\\ & n_1\le n\le n_3, w_1\le w\le w_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_{W|N,Y}(w,n,y)=\begin{cases}\dfrac{w-w_1}{w_2-w_1} & n_1\le n\le n_3,\\ & w_1\le w\le w_2, y_1\le q\le q_3\\ \dfrac{w-w_3}{w_2-w_3} & n_1\le n\le n_3,\\ & w_2\le w\le w_3, y_1\le q\le q_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_{N|M,Y}(m,n,y)=\begin{cases}\dfrac{n-n_1}{n_2-n_1} & m_1\le m\le m_3,\\ & n_1\le n\le n_2, y_1\le y\le y_3\\ \dfrac{n-n_3}{n_2-n_3} & m_1\le m\le m_3,\\ & n_2\le n\le n_3, y_1\le y\le y_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_{P|Q,R}(p,q,r)=\begin{cases}\dfrac{p-p_1}{p_2-p_1} & p_1\le p\le p_2,\\ & q_1\le q\le q_3, r_1\le r\le r_3\\ \dfrac{p-p_3}{p_2-p_3} & p_2\le p\le p_3,\\ & q_1\le q\le q_3, r_1\le r\le r_3,\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_{Q|P,R}(p,q,r)=\begin{cases}\dfrac{q-q_1}{q_2-q_1} & p_1\le p\le p_3,\\ & q_1\le q\le q_2, r_1\le r\le r_3,\\ \dfrac{q-q_3}{q_2-q_3} & p_1\le p\le p_3,\\ & q_2\le q\le q_3, r_1\le r\le r_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_{R|P,Q}(p,q,r)=\begin{cases}\dfrac{r-r_1}{r_2-r_1} & p_1\le p\le p_3,\\ & q_1\le q\le q_3, r_1\le r\le r_2\\ \dfrac{r-r_3}{r_2-r_3} & p_1\le p\le p_3,\\ & q_1\le q\le q_3, r_2\le r\le r_3\\ 0 & \text{otherwise.}\end{cases}$$

The membership surfaces of column vectors of the imprecise matrix will be as:

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:9, No:12, 2015

$$\mu_C(x,z,m) = \begin{cases} \frac{(x-x_1)(z-z_1)(m-m_1)}{(x_2-x_1)(z_2-z_1)(m_2-m_1)} & x_1 \le x \le x_2, \\ & z_1 \le z \le z_2, m_1 \le m \le m_2 \\ \frac{(x-x_1)(z-z_3)(m-m_1)}{(x_2-x_1)(z_2-z_3)(m_2-m_1)} & x_1 \le x \le x_2, \\ & z_2 \le z \le z_3, m_1 \le m \le m_2 \\ \frac{(x-x_3)(z-z_1)(m-m_1)}{(x_2-x_3)(z_2-z_1)(m_2-m_1)} & x_2 \le x \le x_3, \\ & z_1 \le z \le z_2, m_1 \le m \le m_2 \\ \frac{(x-x_3)(z-z_3)(m-m_1)}{(x_2-x_3)(z_2-z_3)(m_2-m_1)} & x_2 \le x \le x_3, \\ & z_2 \le z \le z_3, m_1 \le m \le m_2 \\ \frac{(x-x_1)(z-z_1)(m-m_3)}{(x_2-x_1)(z_2-z_1)(m_2-m_3)} & x_1 \le x \le x_2, \\ & z_1 \le z \le z_2, m_2 \le m \le m_3 \\ \frac{(x-x_1)(z-z_3)(m-m_3)}{(x_2-x_1)(z_2-z_3)(m_2-m_3)} & x_1 \le x \le x_2, \\ & z_2 \le z \le z_3, m_2 \le m \le m_3 \\ \frac{(x-x_3)(z-z_1)(m-m_3)}{(x_2-x_3)(z_2-z_1)(m_2-m_3)} & x_2 \le x \le x_3, \\ & z_1 \le z \le z_2, m_2 \le m \le m_3 \\ \frac{(x-x_3)(z-z_3)(m-m_3)}{(x_2-x_3)(z_2-z_3)(m_2-m_3)} & x_2 \le x \le x_3, \\ & z_2 \le z \le z_3, m_2 \le m \le m_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(y,w,n) = \begin{cases} \frac{(y-y_1)(w-w_1)(n-n_1)}{(y_2-y_1)(w_2-w_1)(n_2-n_1)} & y_1 \le y \le y_2, \\ & w_1 \le w \le w_2, n_1 \le n \le n_2 \\ \frac{(y-y_1)(w-w_3)(n-n_1)}{(y_2-y_1)(w_2-w_3)(n_2-n_1)} & y_1 \le y \le y_2, \\ & w_2 \le w \le w_3, n_1 \le n \le n_2 \\ \frac{(y-y_3)(w-w_1)(n-n_1)}{(y_2-y_3)(w_2-w_1)(n_2-n_1)} & y_2 \le y \le y_3, \\ & w_1 \le w \le w_2, n_1 \le n \le n_2 \\ \frac{(y-y_3)(w-w_3)(n-n_1)}{(y_2-y_3)(w_2-w_3)(n_2-n_1)} & y_2 \le y \le y_3, \\ & w_2 \le w \le w_3, n_1 \le n \le n_2 \\ \frac{(y-y_1)(w-w_1)(n-n_3)}{(y_2-y_1)(w_2-w_1)(n_2-n_3)} & y_1 \le y \le y_2, \\ & w_1 \le w \le w_2, n_2 \le n \le n_3 \\ \frac{(y-y_1)(w-w_3)(n-n_3)}{(y_2-y_1)(w_2-w_3)(n_2-n_3)} & y_1 \le y \le y_2, \\ & w_2 \le w \le w_3, n_2 \le n \le n_3 \\ \frac{(y-y_3)(w-w_1)(n-n_3)}{(y_2-y_3)(w_2-w_1)(n_2-n_3)} & y_2 \le y \le y_3, \\ & w_1 \le w \le w_2, n_2 \le n \le n_3 \\ \frac{(y-y_3)(w-w_3)(n-n_3)}{(y_2-y_3)(w_2-w_3)(n_2-n_3)} & y_2 \le y \le y_3, \\ & w_2 \le w \le w_3, n_2 \le n \le n_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(p,q,r) = \begin{cases} \frac{(p-p_1)(q-q_1)(r-r_1)}{(p_2-p_1)(q_2-q_1)(r_2-r_1)} & p_1 \le p \le p_2, \\ & q_1 \le q \le q_2, r_1 \le r \le r_2 \\ \frac{(p-p_1)(q-q_3)(r-r_1)}{(p_2-p_1)(q_2-q_3)(r_2-r_1)} & p_1 \le p \le p_2, \\ & q_2 \le q \le q_3, r_1 \le r \le r_2 \\ \frac{(p-p_3)(q-q_1)(r-r_1)}{(p_2-p_3)(q_2-q_1)(r_2-r_1)} & p_2 \le p \le p_3, \\ & q_1 \le q \le q_2, r_1 \le r \le r_2 \\ \frac{(p-p_3)(q-q_3)(r-r_1)}{(p_2-p_3)(q_2-q_3)(r_2-r_1)} & p_2 \le p \le p_3, \\ & q_2 \le q \le q_3, r_1 \le r \le r_2 \\ \frac{(p-p_1)(q-q_1)(r-r_3)}{(p_2-p_1)(q_2-q_1)(r_2-r_3)} & p_1 \le p \le p_2, \\ & q_1 \le q \le q_2, r_2 \le r \le r_3 \\ \frac{(p-p_1)(q-q_3)(r-r_3)}{(p_2-p_1)(q_2-q_3)(r_2-r_3)} & p_1 \le p \le p_2, \\ & q_2 \le q \le q_3, r_2 \le r \le r_3 \\ \frac{(p-p_3)(q-q_1)(r-r_3)}{(p_2-p_3)(q_2-q_1)(r_2-r_3)} & p_2 \le p \le p_3, \\ & q_1 \le q \le q_2, r_2 \le r \le r_3 \\ \frac{(p-p_3)(q-q_3)(r-r_3)}{(p_2-p_3)(q_2-q_3)(r_2-r_3)} & p_2 \le p \le p_3, \\ & q_2 \le q \le q_3, r_2 \le r \le r_3 \\ 0 & \text{otherwise.} \end{cases}$$

III. Arithmetic Operations of Imprecise Matrix

Consider two matrices

$$A = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix}$$

and

$$B = \begin{pmatrix} P & Q \\ R & S \end{pmatrix}$$

where the row membership surfaces of A and B are as:

$$\mu_{A_R}(x,y) = \begin{cases} \frac{(x-x_1)(y-y_1)}{(x_2-x_1)(y_2-y_1)} & x_1 \le x \le x_2, y_1 \le y \le y_2 \\ \frac{(x-x_1)(y-y_3)}{(x_2-x_1)(y_2-y_3)} & x_1 \le x \le x_2, y_2 \le y \le y_3 \\ \frac{(x-x_3)(y-y_1)}{(x_2-x_3)(y_2-y_1)} & x_2 \le x \le x_3, y_1 \le y \le y_2 \\ \frac{(x-x_3)(y-y_3)}{(x_2-x_3)(y_2-y_3)} & x_2 \le x \le x_3, y_2 \le y \le y_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{A_R}(z,w) = \begin{cases} \frac{(z-z_1)(w-w_1)}{(z_2-z_1)(w_2-w_1)} & z_1 \le z \le z_2, w_1 \le w \le w_2 \\ \frac{(z-z_1)(w-w_3)}{(z_2-z_1)(w_2-w_3)} & z_1 \le z \le z_2, w_2 \le w \le w_3 \\ \frac{(z-z_3)(w-w_1)}{(z_2-z_3)(w_2-w_1)} & z_2 \le z \le z_3, w_1 \le w \le w_2 \\ \frac{(z-z_3)(w-w_3)}{(z_2-z_3)(w_2-w_3)} & z_2 \le z \le z_3, w_2 \le w \le w_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{B_R}(p,q) = \begin{cases} \frac{(p-p_1)(q-q_1)}{(p_2-p_1)(q_2-q_1)} & p_1 \le p \le p_2, q_1 \le q \le q_2 \\ \frac{(p-p_1)(q-q_3)}{(p_2-p_1)(q_2-q_3)} & p_1 \le p \le p_2, q_2 \le q \le q_3 \\ \frac{(p-p_3)(q-q_1)}{(p_2-p_3)(q_2-q_1)} & p_2 \le p \le p_3, q_1 \le q \le q_2 \\ \frac{(p-p_3)(q-q_3)}{(p_2-p_3)(q_2-q_3)} & p_2 \le p \le p_3, q_2 \le q \le q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{B_R}(r,s) = \begin{cases} \frac{(r-r_1)(s-s_1)}{(r_2-r_1)(s_2-s_1)} & r_1 \le r \le r_2, s_1 \le s \le s_2 \\ \frac{(r-r_1)(s-s_3)}{(r_2-r_1)(s_2-s_3)} & r_1 \le r \le r_2, s_2 \le s \le s_3 \\ \frac{(r-r_3)(s-s_1)}{(r_2-r_3)(s_2-s_1)} & r_2 \le r \le r_3, s_1 \le s \le s_2 \\ \frac{(r-r_3)(s-s_3)}{(r_2-r_3)(s_2-s_3)} & r_2 \le r \le r_3, s_2 \le s \le s_3 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of A and B are as:

$$\mu_{A_C}(x,z) = \begin{cases} \frac{(x-x_1)(z-z_1)}{(x_2-x_1)(z_2-z_1)} & x_1 \le x \le x_2, z_1 \le z \le z_2 \\ \frac{(x-x_1)(z-z_3)}{(x_2-x_1)(z_2-z_3)} & x_1 \le x \le x_2, z_2 \le z \le z_3 \\ \frac{(x-x_3)(z-z_1)}{(x_2-x_3)(z_2-z_1)} & x_2 \le x \le x_3, z_1 \le z \le z_2 \\ \frac{(x-x_3)(z-z_3)}{(x_2-x_3)(z_2-z_3)} & x_2 \le x \le x_3, z_2 \le z \le z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{A_C}(y,w) = \begin{cases} \frac{(y-y_1)(w-w_1)}{(y_2-y_1)(w_2-w_1)} & y_1 \le y \le y_2, w_1 \le w \le w_2 \\ \frac{(y-y_1)(w-w_3)}{(y_2-y_1)(w_2-w_3)} & y_1 \le y \le y_2, w_2 \le w \le w_3 \\ \frac{(y-y_3)(w-w_1)}{(y_2-y_3)(w_2-w_1)} & y_2 \le y \le y_3, w_1 \le w \le w_2 \\ \frac{(y-y_3)(w-w_3)}{(y_2-y_3)(w_2-w_3)} & y_2 \le y \le y_3, w_2 \le w \le w_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{B_C}(p,r) = \begin{cases} \frac{(p-p_1)(r-r_1)}{(p_2-p_1)(r_2-r_1)} & p_1 \le p \le p_2, r_1 \le r \le r_2 \\ \frac{(p-p_1)(r-r_3)}{(p_2-p_1)(r_2-r_3)} & p_1 \le p \le p_2, r_2 \le r \le r_3 \\ \frac{(p-p_3)(r-r_1)}{(p_2-p_3)(r_2-r_1)} & p_2 \le p \le p_3, r_1 \le r \le r_2 \\ \frac{(p-p_3)(r-r_3)}{(p_2-p_3)(r_2-r_3)} & p_2 \le p \le p_3, r_2 \le r \le r_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{B_C}(q,s)=\begin{cases}\frac{(q-q_1)(s-s_1)}{(q_2-q_1)(s_2-s_1)} & q_1\le q\le q_2, s_1\le s\le s_2\\ \frac{(q-q_1)(s-s_3)}{(q_2-q_1)(s_2-s_3)} & q_1\le q\le q_2, s_2\le s\le s_3\\ \frac{(q-q_3)(s-s_1)}{(q_2-q_3)(s_2-s_1)} & q_2\le q\le q_3, s_1\le s\le s_2\\ \frac{(q-q_3)(s-s_3)}{(q_2-q_3)(s_2-s_3)} & q_2\le q\le q_3, s_2\le s\le s_3\\ 0 & \text{otherwise.}\end{cases}$$

Here, for simplicity and without any loss of generality it is assumed that $X=[x_1,x_2,x_3]$, $Y=[y_1,y_2,y_3]$, $Z=[z_1,z_2,z_3]$, $W=[w_1,w_2,w_3]$ and $P=[p_1,p_2,p_3]$, $Q=[q_1,q_2,q_3]$, $R=[r_1,r_2,r_3]$, $S=[s_1,s_2,s_3]$ are triangular imprecise numbers.

A. *Addition of Imprecise Matrix*

Addition of two imprecise matrices A and B will be as:

$$\begin{aligned}A+B &= \begin{pmatrix} X & Y\\ Z & W\end{pmatrix}+\begin{pmatrix} P & Q\\ R & S\end{pmatrix}\\ &= \begin{pmatrix} L & M\\ N & K\end{pmatrix}\end{aligned}$$

where $L=X+P=[x_1+p_1,x_2+p_2,x_3+p_3]$, $M=Y+Q=[y_1+q_1,y_2+q_2,y_3+q_3]$, $N=Z+R=[z_1+r_1,z_2+r_2,z_3+r_3]$ and $K=W+S=[w_1+s_1,w_2+s_2,w_3+s_3]$. Now, the row membership surfaces of the matrix

$$C=\begin{pmatrix} L & M\\ N & K\end{pmatrix}$$

will be as follows

$$\mu_{C_R}(l,m)=$$

$$\begin{cases}\frac{\{(x+p)-(x_1+p_1)\}\{(y+q)-(y_1+q_1)\}}{\{(x_2+p_2)-(x_1+p_1)\}\{(y_2+q_2)-(y_1+q_1)\}} & x_1+p_1\le x+p\le x_2+p_2,\ y_1+q_1\le y+q\le y_2+q_2\\ \frac{\{(x+p)-(x_1+p_1)\}\{(y+q)-(y_3+q_3)\}}{\{(x_2+p_2)-(x_1+p_1)\}\{(y_2+q_2)-(y_3+q_3)\}} & x_1+p_1\le x+p\le x_2+p_2,\ y_2+q_2\le y+q\le y_3+q_3\\ \frac{\{(x+p)-(x_3+p_3)\}\{(y+q)-(y_1+q_1)\}}{\{(x_2+p_2)-(x_3+p_3)\}\{(y_2+q_2)-(y_1+q_1)\}} & x_2+p_2\le x+p\le x_3+p_3,\ y_1+q_1\le y+q\le y_2+q_2\\ \frac{\{(x+p)-(x_3+p_3)\}\{(y+q)-(y_3+q_3)\}}{\{(x_2+p_2)-(x_3+p_3)\}\{(y_2+q_2)-(y_3+q_3)\}} & x_2+p_2\le x+p\le x_3+p_3,\ y_2+q_2\le y+q\le y_3+q_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_{C_R}(n,k)=$$

$$\begin{cases}\frac{\{(z+r)-(z_1+r_1)\}\{(w+s)-(w_1+s_1)\}}{\{(z_2+r_2)-(z_1+r_1)\}\{(w_2+s_2)-(w_1+s_1)\}} & z_1+r_1\le z+r\le z_2+r_2,\ w_1+s_1\le w+s\le w_2+s_2\\ \frac{\{(z+r)-(z_1+r_1)\}\{(w+s)-(w_3+s_3)\}}{\{(z_2+r_2)-(z_1+r_1)\}\{(w_2+s_2)-(w_3+s_3)\}} & z_1+r_1\le z+r\le z_2+r_2,\ w_2+s_2\le w+s\le w_3+s_3\\ \frac{\{(z+r)-(z_3+r_3)\}\{(w+s)-(w_1+s_1)\}}{\{(z_2+r_2)-(z_3+r_3)\}\{(w_2+s_2)-(w_1+s_1)\}} & z_2+r_2\le z+r\le z_3+r_3,\ w_1+s_1\le w+s\le w_2+s_2\\ \frac{\{(z+r)-(z_3+r_3)\}\{(w+s)-(w_3+s_3)\}}{\{(z_2+r_2)-(z_3+r_3)\}\{(w_2+s_2)-(w_3+s_3)\}} & z_2+r_2\le z+r\le z_3+r_3,\ w_2+s_2\le w+s\le w_3+s_3\\ 0 & \text{otherwise.}\end{cases}$$

Similarly, the column membership surfaces of the matrix C will be as:

$$\mu_{C_C}(l,n)=$$

$$\begin{cases}\frac{\{(x+p)-(x_1+p_1)\}\{(z+r)-(z_1+r_1)\}}{\{(x_2+p_2)-(x_1+p_1)\}\{(z_2+r_2)-(z_1+r_1)\}} & x_1+p_1\le x+p\le x_2+p_2,\ z_1+r_1\le z+r\le z_2+r_2\\ \frac{\{(x+p)-(x_1+p_1)\}\{(z+r)-(z_3+r_3)\}}{\{(x_2+p_2)-(x_1+p_1)\}\{(z_2+r_2)-(z_3+r_3)\}} & x_1+p_1\le x+p\le x_2+p_2,\ z_2+r_2\le z+r\le z_3+r_3\\ \frac{\{(x+p)-(x_3+p_3)\}\{(z+r)-(z_1+r_1)\}}{\{(x_2+p_2)-(x_3+p_3)\}\{(z_2+r_2)-(z_1+r_1)\}} & x_2+p_2\le x+p\le x_3+p_3,\ z_1+r_1\le z+r\le z_2+r_2\\ \frac{\{(x+p)-(x_3+p_3)\}\{(z+r)-(z_3+r_3)\}}{\{(x_2+p_2)-(x_3+p_3)\}\{(z_2+r_2)-(z_3+r_3)\}} & x_2+p_2\le x+p\le x_3+p_3,\ z_2+r_2\le z+r\le z_3+r_3\\ 0 & \text{otherwise.}\end{cases}$$

$$\mu_{C_C}(m,k)=$$

$$\begin{cases}\frac{\{(y+q)-(y_1+q_1)\}\{(w+s)-(w_1+s_1)\}}{\{(y_2+q_2)-(y_1+q_1)\}\{(w_2+s_2)-(w_1+s_1)\}} & y_1+q_1\le y+q\le y_2+q_2,\ w_1+s_1\le w+s\le w_2+s_2\\ \frac{\{(y+q)-(y_1+q_1)\}\{(w+s)-(w_3+s_3)\}}{\{(y_2+q_2)-(y_1+q_1)\}\{(w_2+s_2)-(w_3+s_3)\}} & y_1+q_1\le y+q\le y_2+q_2,\ w_2+s_2\le w+s\le w_3+s_3\\ \frac{\{(y+q)-(y_3+q_3)\}\{(w+s)-(w_1+s_1)\}}{\{(y_2+q_2)-(y_3+q_3)\}\{(w_2+s_2)-(w_1+s_1)\}} & y_2+q_2\le y+q\le y_3+q_3,\ w_1+s_1\le w+s\le w_2+s_2\\ \frac{\{(y+q)-(y_3+q_3)\}\{(w+s)-(w_3+s_3)\}}{\{(y_2+q_2)-(y_3+q_3)\}\{(w_2+s_2)-(w_3+s_3)\}} & y_2+q_2\le y+q\le y_3+q_3,\ w_2+s_2\le w+s\le w_3+s_3\\ 0 & \text{otherwise.}\end{cases}$$

B. *Subtraction of Imprecise Matrix*

Subtraction of two imprecise matrices A and B will be

$$\begin{aligned}A-B &= \begin{pmatrix} X & Y\\ Z & W\end{pmatrix}-\begin{pmatrix} P & Q\\ R & S\end{pmatrix}\\ &= \begin{pmatrix} L & M\\ N & K\end{pmatrix}\end{aligned}$$

where $L=X-P=[x_1-p_3,x_2-p_2,x_3-p_1]$, $M=Y-Q=[y_1-q_3,y_2-q_2,y_3-q_1]$, $N=Z-R=[z_1-r_3,z_2-r_2,z_3-r_1]$ and $K=W-S=[w_1-s_3,w_2-s_2,w_3-s_1]$ respectively. Now, the row membership surfaces of the matrix

$$C=\begin{pmatrix} L & M\\ N & K\end{pmatrix}$$

will be as follows

$$\mu_{C_R}(l,m)=$$

$$\begin{cases}\frac{\{(x-p)-(x_1-p_3)\}\{(y-q)-(y_1-q_3)\}}{\{(x_2-p_2)-(x_1-p_3)\}\{(y_2-q_2)-(y_1-q_3)\}} & x_1-p_3\le x-p\le x_2-p_2,\ y_1-q_3\le y-q\le y_2-q_2\\ \frac{\{(x-p)-(x_1-p_3)\}\{(y-q)-(y_3-q_1)\}}{\{(x_2-p_2)-(x_1-p_3)\}\{(y_2-q_2)-(y_3-q_1)\}} & x_1-p_3\le x-p\le x_2-p_2,\ y_2-q_2\le y-q\le y_3-q_1\\ \frac{\{(x-p)-(x_3-p_1)\}\{(y-q)-(y_1-q_3)\}}{\{(x_2-p_2)-(x_3-p_1)\}\{(y_2-q_2)-(y_1-q_3)\}} & x_2-p_2\le x-p\le x_3-p_1,\ y_1-q_3\le y-q\le y_2-q_2\\ \frac{\{(x-p)-(x_3-p_1)\}\{(y-q)-(y_3-q_1)\}}{\{(x_2-p_2)-(x_3-p_1)\}\{(y_2-q_2)-(y_3-q_1)\}} & x_2-p_2\le x-p\le x_3-p_1,\ y_2-q_2\le y-q\le y_3-q_1\\ 0 & \text{otherwise.}\end{cases}$$

and

$$\mu_{C_R}(n,k) = \begin{cases} \dfrac{\{(z-r)-(z_1-r_3)\}\{(w-s)-(w_1-s_3)\}}{\{(z_2-r_2)-(z_1-r_3)\}\{(w_2-s_2)-(w_1-s_3)\}} & z_1-r_3 \le z-r \le z_2-r_2, \\ & w_1-s_3 \le w-s \le w_2-s_2 \\ \dfrac{\{(z-r)-(z_1-r_3)\}\{(w-s)-(w_3-s_1)\}}{\{(z_2-r_2)-(z_1-r_3)\}\{(w_2-s_2)-(w_3-s_1)\}} & z_1-r_3 \le z-r \le z_2-r_2, \\ & w_2-s_2 \le w-s \le w_3-s_1 \\ \dfrac{\{(z-r)-(z_3-r_1)\}\{(w-s)-(w_1-s_3)\}}{\{(z_2-r_2)-(z_3-r_1)\}\{(w_2-s_2)-(w_1-s_3)\}} & z_2-r_2 \le z-r \le z_3-r_1, \\ & w_1-s_3 \le w-s \le w_2-s_2 \\ \dfrac{\{(z-r)-(z_3-r_1)\}\{(w-s)-(w_3-s_1)\}}{\{(z_2-r_2)-(z_3-r_1)\}\{(w_2-s_2)-(w_3-s_1)\}} & z_2-r_2 \le z-r \le z_3-r_1, \\ & w_2-s_2 \le w-s \le w_3-s_1 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of the matrix C will be as:

$$\mu_{C_C}(l,n) = \begin{cases} \dfrac{\{(x-p)-(x_1-p_3)\}\{(z-r)-(z_1-r_3)\}}{\{(x_2-p_2)-(x_1-p_3)\}\{(z_2-r_2)-(z_1-r_3)\}} & x_1-p_3 \le x-p \le x_2-p_2, \\ & z_1-r_3 \le z-r \le z_2-r_2 \\ \dfrac{\{(x-p)-(x_1-p_3)\}\{(z-r)-(z_3-r_1)\}}{\{(x_2-p_2)-(x_1-p_3)\}\{(z_2-r_2)-(z_3-r_1)\}} & x_1-p_3 \le x-p \le x_2-p_2, \\ & z_2-r_2 \le z-r \le z_3-r_1 \\ \dfrac{\{(x-p)-(x_3-p_1)\}\{(z-r)-(z_1-r_3)\}}{\{(x_2-p_2)-(x_3-p_1)\}\{(z_2-r_2)-(z_1-r_3)\}} & x_2-p_2 \le x-p \le x_3-p_1, \\ & z_1-r_3 \le z-r \le z_2-r_2 \\ \dfrac{\{(x-p)-(x_3-p_1)\}\{(z-r)-(z_3-r_1)\}}{\{(x_2-p_2)-(x_3-p_1)\}\{(z_2-r_2)-(z_3-r_1)\}} & x_2-p_2 \le x-p \le x_3-p_1, \\ & z_2-r_2 \le z-r \le z_3-r_1 \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\mu_{C_C}(m,k) = \begin{cases} \dfrac{\{(y-q)-(y_1-q_3)\}\{(w-s)-(w_1-s_3)\}}{\{(y_2-q_2)-(y_1-q_3)\}\{(w_2-s_2)-(w_1-s_3)\}} & y_1-q_3 \le y-q \le y_2-q_2, \\ & w_1-s_3 \le w-s \le w_2-s_2 \\ \dfrac{\{(y-q)-(y_1-q_3)\}\{(w-s)-(w_3-s_1)\}}{\{(y_2-q_2)-(y_1-q_3)\}\{(w_2-s_2)-(w_3-s_1)\}} & y_1-q_3 \le y-q \le y_2-q_2, \\ & w_2-s_2 \le w-s \le w_3-s_1 \\ \dfrac{\{(y-q)-(y_3-q_1)\}\{(w-s)-(w_1-s_3)\}}{\{(y_2-q_2)-(y_3-q_1)\}\{(w_2-s_2)-(w_1-s_3)\}} & y_2-q_2 \le y-q \le y_3-q_1, \\ & w_1-s_3 \le w-s \le w_2-s_2 \\ \dfrac{\{(y-q)-(y_3-q_1)\}\{(w-s)-(w_3-s_1)\}}{\{(y_2-q_2)-(y_3-q_1)\}\{(w_2-s_2)-(w_3-s_1)\}} & y_2-q_2 \le y-q \le y_3-q_1, \\ & w_2-s_2 \le w-s \le w_3-s_1 \\ 0 & \text{otherwise.} \end{cases}$$

C. Multiplication of imprecise matrix

Multiplication of two imprecise matrices A and B will be as:

$$\begin{aligned} A.B &= \begin{pmatrix} X & Y \\ Z & W \end{pmatrix} \cdot \begin{pmatrix} P & Q \\ R & S \end{pmatrix} \\ &= \begin{pmatrix} L & M \\ N & K \end{pmatrix} \end{aligned}$$

where

$$\begin{aligned} L &= XP + YR \\ &= [x_1p_1, x_2p_2, x_3p_3] \end{aligned}$$

$$\begin{aligned} M &= XQ + YS \\ &= [y_1q_1, y_2q_2, y_3q_3] \end{aligned}$$

$$\begin{aligned} N &= ZP + WR \\ &= [z_1r_1, z_2r_2, z_3r_3] \end{aligned}$$

$$\begin{aligned} K &= ZQ + WS \\ &= [w_1s_1, w_2s_2, w_3s_3] \end{aligned}$$

The membership functions of L, M, N and K are as:

$$\mu_{L|M}(l,m) = \begin{cases} \dfrac{1}{2(x_2-x_3)(p_2-p_3)}\{-(x_1p_2-2x_1p_1+x_2p_1) \\ \quad +\sqrt{(x_1p_2-2x_1p_1+x_2p_1)^2-4(x_1p_1-l)(x_2-x_1)(p_2-p_1)}\} & x_1p_1 \le l \le x_2p_2, \\ & y_1q_1 \le m \le y_3q_3 \\ \dfrac{1}{2(x_2-x_3)(p_2-p_3)}\{-(x_3p_2-2x_3p_3+x_2p_3) \\ \quad -\sqrt{(x_3p_2-2x_3p_3+x_2p_3)^2-4(x_3p_3-l)(x_2-x_3)(p_2-p_3)}\} & x_2p_2 \le l \le x_3p_3, \\ & y_1q_1 \le m \le y_3q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{M|L}(l,m) = \begin{cases} \dfrac{1}{2(y_2-y_3)(q_2-q_3)}\{-(y_1q_2-2y_1q_1+y_2q_1) \\ \quad +\sqrt{(y_1q_2-2y_1q_1+y_2q_1)^2-4(y_1q_1-m)(y_2-y_1)(q_2-q_1)}\} & y_1q_1 \le m \le y_2q_2, \\ & x_1p_1 \le l \le x_3p_3 \\ \dfrac{1}{2(y_2-y_3)(q_2-q_3)}\{-(y_3q_2-2y_3q_3+y_2q_3) \\ \quad -\sqrt{(y_3q_2-2y_3q_3+y_2q_3)^2-4(y_3q_3-m)(y_2-y_3)(q_2-q_3)}\} & y_2q_2 \le m \le y_3q_3, \\ & x_1p_1 \le l \le x_3p_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{N|K}(n,k) = \begin{cases} \dfrac{1}{2(z_2-z_3)(r_2-r_3)}\{-(z_1r_2-2z_1r_1+z_2r_1) \\ \quad +\sqrt{(z_1r_2-2z_1r_1+z_2r_1)^2-4(z_1r_1-n)(z_2-z_1)(r_2-r_1)}\} & z_1r_1 \le n \le z_2r_2, \\ & w_1s_1 \le k \le w_3s_3 \\ \dfrac{1}{2(z_2-z_3)(r_2-r_3)}\{-(z_3r_2-2z_3r_3+z_2r_3) \\ \quad -\sqrt{(z_3r_2-2z_3r_3+z_2r_3)^2-4(z_3r_3-n)(z_2-z_3)(r_2-r_3)}\} & z_2r_2 \le n \le z_3r_3, \\ & w_1s_1 \le k \le w_3s_3 \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\mu_{K|N}(n,k = \begin{cases} \dfrac{1}{2(w_2-w_3)(s_2-s_3)}\{-(w_1s_2-2w_1s_1+w_2s_1) \\ \quad +\sqrt{(w_1s_2-2w_1s_1+w_2s_1)^2-4(w_1s_1-k)(w_2-w_1)(s_2-s_1)}\} & w_1s_1 \le k \le w_2s_2, \\ & z_1r_1 \le n \le z_3r_3 \\ \dfrac{1}{2(w_2-w_3)(s_2-s_3)}\{-(w_3s_2-2w_3s_3+w_2s_3) \\ \quad -\sqrt{(w_3s_2-2w_3s_3+w_2s_3)^2-4(w_3s_3-k)(w_2-w_3)(s_2-s_3)}\} & w_2s_2 \le k \le w_3s_3, \\ & z_1r_1 \le n \le z_3r_3 \\ 0 & \text{otherwise.} \end{cases}$$

Now, the row and column membership surfaces of the matrix

$$\begin{aligned} C &= A.B \\ &= \begin{pmatrix} X & Y \\ Z & W \end{pmatrix} \cdot \begin{pmatrix} P & Q \\ R & S \end{pmatrix} \\ &= \begin{pmatrix} L & M \\ N & K \end{pmatrix} \end{aligned}$$

will be as shown in (1)-(4).

$$\mu_{C_R}(l,m)=\begin{cases}\frac{1}{4(x_2-x_3)(p_2-p_3)(y_2-y_3)(q_2-q_3)}\{-(x_1p_2-2x_1p_1+x_2p_1)\\ \quad+\sqrt{(x_1p_2-2x_1p_1+x_2p_1)^2-4(x_1p_1-l)(x_2-x_1)(p_2-p_1)}\}\{-(y_1q_2-2y_1q_1+y_2q_1)\\ \quad+\sqrt{(y_1q_2-2y_1q_1+y_2q_1)^2-4(y_1q_1-m)(y_2-y_1)(q_2-q_1)}\} & x_1p_1\le l\le x_2p_2, y_1q_1\le m\le y_2q_2\\ \frac{1}{4(x_2-x_3)(p_2-p_3)(y_2-y_3)(q_2-q_3)}\{-(x_1p_2-2x_1p_1+x_2p_1)\\ \quad+\sqrt{(x_1p_2-2x_1p_1+x_2p_1)^2-4(x_1p_1-l)(x_2-x_1)(p_2-p_1)}\}\{-(y_3q_2-2y_3q_3+y_2q_3)\\ \quad-\sqrt{(y_3q_2-2y_3q_3+y_2q_3)^2-4(y_3q_3-m)(y_2-y_3)(q_2-q_3)}\} & x_1p_1\le l\le x_2p_2; y_2q_2\le m\le y_3q_3\\ \frac{1}{4(x_2-x_3)(p_2-p_3)(y_2-y_3)(q_2-q_3)}\{-(x_3p_2-2x_3p_3+x_2p_3)\\ \quad-\sqrt{(x_3p_2-2x_3p_3+x_2p_3)^2-4(x_3p_3-l)(x_2-x_3)(p_2-p_3)}\}\{-(y_1q_2-2y_1q_1+y_2q_1)\\ \quad+\sqrt{(y_1q_2-2y_1q_1+y_2q_1)^2-4(y_1q_1-m)(y_2-y_1)(q_2-q_1)}\} & x_2p_2\le l\le x_3p_3; y_1q_1\le m\le y_2q_2\\ \frac{1}{4(x_2-x_3)(p_2-p_3)(y_2-y_3)(q_2-q_3)}\{-(x_3p_2-2x_3p_3+x_2p_3)\\ \quad-\sqrt{(x_3p_2-2x_3p_3+x_2p_3)^2-4(x_3p_3-l)(x_2-x_3)(p_2-p_3)}\}\{-(y_3q_2-2y_3q_3+y_2q_3)\\ \quad-\sqrt{(y_3q_2-2y_3q_3+y_2q_3)^2-4(y_3q_3-m)(y_2-y_3)(q_2-q_3)}\} & x_2p_2\le l\le x_3p_3; y_2q_2\le m\le y_3q_3\\ 0 & \text{otherwise.}\end{cases}\tag{1}$$

$$\mu_{C_R}(n,k)=\begin{cases}\frac{1}{4(z_2-z_3)(r_2-r_3)(w_2-w_3)(s_2-s_3)}\{-(z_1r_2-2z_1r_1+z_2r_1)\\ \quad+\sqrt{(z_1r_2-2z_1r_1+z_2r_1)^2-4(z_1r_1-n)(z_2-z_1)(r_2-r_1)}\}\{-(w_1s_2-2w_1s_1+w_2s_1)\\ \quad+\sqrt{(w_1s_2-2w_1s_1+w_2s_1)^2-4(w_1s_1-k)(w_2-w_1)(s_2-s_1)}\} & z_1r_1\le n\le z_2r_2; w_1s_1\le k\le w_2s_2\\ \frac{1}{4(z_2-z_3)(r_2-r_3)(w_2-w_3)(s_2-s_3)}\{-(z_1r_2-2z_1r_1+z_2r_1)\\ \quad+\sqrt{(z_1r_2-2z_1r_1+z_2r_1)^2-4(z_1r_1-n)(z_2-z_1)(r_2-r_1)}\}\{-(w_3s_2-2w_3s_3+w_2s_3)\\ \quad-\sqrt{(w_3s_2-2w_3s_3+w_2s_3)^2-4(w_3s_3-k)(w_2-w_3)(s_2-s_3)}\} & z_1r_1\le n\le z_2r_2; w_2s_2\le k\le w_3s_3\\ \frac{1}{4(z_2-z_3)(r_2-r_3)(w_2-w_3)(s_2-s_3)}\{-(z_3r_2-2z_3r_3+z_2r_3)\\ \quad-\sqrt{(z_3r_2-2z_3r_3+z_2r_3)^2-4(z_3r_3-n)(z_2-z_3)(r_2-r_3)}\}\{-(w_1s_2-2w_1s_1+w_2s_1)\\ \quad+\sqrt{(w_1s_2-2w_1s_1+w_2s_1)^2-4(w_1s_1-k)(w_2-w_1)(s_2-s_1)}\} & z_2r_2\le n\le z_3r_3; w_1s_1\le k\le w_2s_2\\ \frac{1}{4(z_2-z_3)(r_2-r_3)(w_2-w_3)(s_2-s_3)}\{-(z_3r_2-2z_3r_3+z_2r_3)\\ \quad-\sqrt{(z_3r_2-2z_3r_3+z_2r_3)^2-4(z_3r_3-n)(z_2-z_3)(r_2-r_3)}\}\{-(w_3s_2-2w_3s_3+w_2s_3)\\ \quad-\sqrt{(w_3s_2-2w_3s_3+w_2s_3)^2-4(w_3s_3-k)(w_2-w_3)(s_2-s_3)}\} & z_2r_2\le n\le z_3r_3; w_2s_2\le k\le w_3s_3\\ 0 & \text{otherwise.}\end{cases}\tag{2}$$

$$\mu_{C_C}(l,n)=\begin{cases}\frac{1}{4(x_2-x_3)(p_2-p_3)(z_2-z_3)(r_2-r_3)}\{-(x_1p_2-2x_1p_1+x_2p_1)\\ \quad+\sqrt{(x_1p_2-2x_1p_1+x_2p_1)^2-4(x_1p_1-l)(x_2-x_1)(p_2-p_1)}\}\{-(z_1r_2-2z_1r_1+z_2r_1)\\ \quad+\sqrt{(z_1r_2-2z_1r_1+z_2r_1)^2-4(z_1r_1-n)(z_2-z_1)(r_2-r_1)}\} & x_1p_1\le l\le x_2p_2; z_1r_1\le n\le z_2r_2\\ \frac{1}{4(x_2-x_3)(p_2-p_3)(z_2-z_3)(r_2-r_3)}\{-(x_1p_2-2x_1p_1+x_2p_1)\\ \quad+\sqrt{(x_1p_2-2x_1p_1+x_2p_1)^2-4(x_1p_1-l)(x_2-x_1)(p_2-p_1)}\}\{-(z_3r_2-2z_3r_3+z_2r_3)\\ \quad-\sqrt{(z_3r_2-2z_3r_3+z_2r_3)^2-4(z_3r_3-n)(z_2-z_3)(r_2-r_3)}\} & x_1p_1\le l\le x_2p_2; z_2r_2\le n\le z_3r_3\\ \frac{1}{4(x_2-x_3)(p_2-p_3)(z_2-z_3)(r_2-r_3)}\{-(x_3p_2-2x_3p_3+x_2p_3)\\ \quad-\sqrt{(x_3p_2-2x_3p_3+x_2p_3)^2-4(x_3p_3-l)(x_2-x_3)(p_2-p_3)}\}\{-(z_1r_2-2z_1r_1+z_2r_1)\\ \quad+\sqrt{(z_1r_2-2z_1r_1+z_2r_1)^2-4(z_1r_1-n)(z_2-z_1)(r_2-r_1)}\} & x_2p_2\le l\le x_3p_3; z_1r_1\le n\le z_2r_2\\ \frac{1}{4(x_2-x_3)(p_2-p_3)(z_2-z_3)(r_2-r_3)}\{-(x_3p_2-2x_3p_3+x_2p_3)\\ \quad-\sqrt{(x_3p_2-2x_3p_3+x_2p_3)^2-4(x_3p_3-l)(x_2-x_3)(p_2-p_3)}\}\{-(z_3r_2-2z_3r_3+z_2r_3)\\ \quad-\sqrt{(z_3r_2-2z_3r_3+z_2r_3)^2-4(z_3r_3-n)(z_2-z_3)(r_2-r_3)}\} & x_2p_2\le l\le x_3p_3; z_2r_2\le n\le z_3r_3\\ 0 & \text{otherwise.}\end{cases}\tag{3}$$

$$\mu_{C_R}(n,k)=\begin{cases}\frac{1}{4(y_2-y_3)(q_2-q_3)(w_2-w_3)(s_2-s_3)}\{-(y_1q_2-2y_1q_1+y_2q_1)\\ \quad+\sqrt{(y_1q_2-2y_1q_1+y_2q_1)^2-4(y_1q_1-m)(y_2-y_1)(q_2-q_1)}\}\{-(w_1s_2-2w_1s_1+w_2s_1)\\ \quad+\sqrt{(w_1s_2-2w_1s_1+w_2s_1)^2-4(w_1s_1-k)(w_2-w_1)(s_2-s_1)}\} & y_1q_1\le m\le y_2q_2; w_1s_1\le k\le w_2s_2\\ \frac{1}{4(y_2-y_3)(q_2-q_3)(w_2-w_3)(s_2-s_3)}\{-(y_1q_2-2y_1q_1+y_2q_1)\\ \quad+\sqrt{(y_1q_2-2y_1q_1+y_2q_1)^2-4(y_1q_1-m)(y_2-y_1)(q_2-q_1)}\}\{-(w_3s_2-2w_3s_3+w_2s_3)\\ \quad-\sqrt{(w_3s_2-2w_3s_3+w_2s_3)^2-4(w_3s_3-k)(w_2-w_3)(s_2-s_3)}\} & y_1q_1\le m\le y_2q_2; w_2s_2\le k\le w_3s_3\\ \frac{1}{4(y_2-y_3)(q_2-q_3)(w_2-w_3)(s_2-s_3)}\{-(y_3q_2-2y_3q_3+y_2q_3)\\ \quad-\sqrt{(y_3q_2-2y_3q_3+y_2q_3)^2-4(y_3q_3-m)(y_2-y_3)(q_2-q_3)}\}\{-(w_1s_2-2w_1s_1+w_2s_1)\\ \quad+\sqrt{(w_1s_2-2w_1s_1+w_2s_1)^2-4(w_1s_1-k)(w_2-w_1)(s_2-s_1)}\} & y_2q_2\le m\le y_3q_3; w_1s_1\le k\le w_2s_2\\ \frac{1}{4(y_2-y_3)(q_2-q_3)(w_2-w_3)(s_2-s_3)}\{-(y_3q_2-2y_3q_3+y_2q_3)\\ \quad-\sqrt{(y_3q_2-2y_3q_3+y_2q_3)^2-4(y_3q_3-m)(y_2-y_3)(q_2-q_3)}\}\{-(w_3s_2-2w_3s_3+w_2s_3)\\ \quad-\sqrt{(w_3s_2-2w_3s_3+w_2s_3)^2-4(w_3s_3-k)(w_2-w_3)(s_2-s_3)}\} & y_2q_2\le m\le y_3q_3; w_2s_2\le k\le w_3s_3\\ 0 & \text{otherwise.}\end{cases}\tag{4}$$

IV. Numerical Example

Consider two matrices

$$A = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix}$$

and

$$B = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}$$

Where $X_{11} = [1, 2, 3], X_{12} = [2, 3, 4], X_{21} = [3, 4, 5], X_{22} = [5, 6, 7]$ and $Y_{11} = [2, 3, 4], Y_{12} = [6, 7, 8], Y_{21} = [8, 9, 10], Y_{22} = [1, 2, 3]$ are imprecise numbers. For simplicity we have assumed that all are triangular imprecise numbers.

A. Addition of two imprecise matrices

Addition of two imprecise matrices A and B will be as follows

$$A + B = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} + \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

where $Z_{11} = [3, 5, 7]$, $Z_{12} = [8, 10, 12]$, $Z_{21} = [11, 13, 15]$ and $Z_{22} = [6, 8, 10]$ respectively. The membership functions of Z_{11}, Z_{12}, Z_{21} and Z_{22} for row vectors are as follows

$$\mu_{Z_{11}|Z_{12}}(z_{11}, z_{12}) = \begin{cases} \dfrac{z_{11}-3}{2} & 3 \le z_{11} \le 5, \\ & 8 \le z_{12} \le 12 \\ \dfrac{7-z_{11}}{2} & 5 \le z_{11} \le 7, \\ & 8 \le z_{12} \le 12 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{12}|Z_{11}}(z_{11}, z_{12}) = \begin{cases} \dfrac{z_{12}-8}{2} & 8 \le z_{12} \le 10, \\ & 3 \le z_{11} \le 7 \\ \dfrac{12-z_{12}}{2} & 10 \le z_{12} \le 12, \\ & 3 \le z_{11} \le 7 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{21}|Z_{22}}(Z_{21}, Z_{22}) = \begin{cases} \dfrac{z_{21}-11}{2} & 11 \le z_{21} \le 13, \\ & 6 \le z_{22} \le 10 \\ \dfrac{15-z_{21}}{2} & 13 \le z_{21} \le 15, \\ & 6 \le z_{22} \le 10 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{22}|Z_{21}}(Z_{21}, Z_{22}) = \begin{cases} \dfrac{z_{22}-6}{2} & 6 \le z_{22} \le 8, \\ & 11 \le z_{21} \le 15 \\ \dfrac{10-z_{22}}{2} & 8 \le z_{22} \le 10, \\ & 11 \le z_{21} \le 15 \\ 0 & \text{otherwise.} \end{cases}$$

Now, the row membership surfaces of the matrix

$$C = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

will be as follows

$$\mu_R(z_{11}, z_{12}) = \begin{cases} \dfrac{(z_{11}-3)(z_{12}-8)}{4} & 3 \le z_{11} \le 5, \\ & 8 \le z_{12} \le 10 \\ \dfrac{(z_{11}-3)(12-z_{12})}{4} & 3 \le z_{11} \le 5, \\ & 10 \le z_{12} \le 12 \\ \dfrac{(7-z_{11})(z_{12}-8)}{4} & 5 \le z_{11} \le 7, \\ & 8 \le z_{12} \le 10 \\ \dfrac{(7-z_{11})(12-z_{12})}{4} & 5 \le z_{11} \le 7, \\ & 10 \le z_{12} \le 12 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_R(z_{21}, z_{22}) \begin{cases} \dfrac{(z_{21}-11)(z_{22}-6)}{4} & 11 \le z_{21} \le 13, \\ & 6 \le z_{22} \le 8 \\ \dfrac{(z_{21}-11)(10-z_{22})}{4} & 11 \le z_{21} \le 13, \\ & 8 \le z_{22} \le 10 \\ \dfrac{(15-z_{21})(z_{22}-6)}{4} & 13 \le z_{21} \le 15, \\ & 6 \le z_{22} \le 8 \\ \dfrac{(15-z_{21})(10-z_{22})}{4} & 13 \le z_{21} \le 15, \\ & 8 \le z_{22} \le 10 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of the matrix C will be as follows

$$\mu_C(z_{11}, z_{21}) = \begin{cases} \dfrac{(z_{11}-3)(z_{21}-11)}{4} & 3 \le z_{11} \le 5, \\ & 11 \le z_{21} \le 13 \\ \dfrac{(z_{11}-3)(15-z_{21})}{4} & 3 \le z_{11} \le 5, \\ & 13 \le z_{21} \le 15 \\ \dfrac{(7-z_{11})(z_{21}-11)}{4} & 5 \le z_{11} \le 7, \\ & 11 \le z_{21} \le 13 \\ \dfrac{(7-z_{11})(15-z_{21})}{4} & 5 \le z_{11} \le 7, \\ & 13 \le z_{21} \le 15 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(z_{12}, z_{22}) = \begin{cases} \dfrac{(z_{12}-8)(z_{22}-6)}{4} & 8 \le z_{12} \le 10, \\ & 6 \le z_{22} \le 8 \\ \dfrac{(z_{12}-8)(10-z_{22})}{4} & 8 \le z_{12} \le 10, \\ & 8 \le z_{22} \le 10 \\ \dfrac{(12-z_{12})(z_{22}-6)}{4} & 10 \le z_{12} \le 12, \\ & 6 \le z_{22} \le 8 \\ \dfrac{(12-z_{12})(10-z_{22})}{4} & 10 \le z_{12} \le 12, \\ & 8 \le z_{22} \le 10 \\ 0 & \text{otherwise.} \end{cases}$$

B. Subtraction of Two Imprecise Matrices

Subtraction of two imprecise matrices A and B will be

$$A - B = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} - \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

where $Z_{11} = [-3, -1, 1]$, $Z_{12} = [-6, -4, -2]$, $Z_{21} = [-7, -5, -3]$ and $Z_{22} = [2, 4, 6]$ respectively. The membership functions of Z_{11}, Z_{12}, Z_{21} and Z_{22} for row vectors are as follows

$$\mu_{Z_{11}|Z_{12}}(z_{11}, z_{12}) = \begin{cases} \dfrac{z_{11}+3}{2} & -3 \le z_{11} \le -1, \\ & -6 \le z_{12} \le -2 \\ \dfrac{1-z_{11}}{2} & -1 \le z_{11} \le 1, \\ & -6 \le z_{12} \le -2 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{12}|Z_{11}}(z_{11},z_{12}) = \begin{cases} \dfrac{z_{12}+6}{2} & -6 \le z_{12} \le -4, \\ & -3 \le z_{11} \le 1 \\ \dfrac{-2-z_{12}}{2} & -4 \le z_{12} \le -2, \\ & -3 \le z_{11} \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{21}|Z_{22}}(z_{21},z_{22}) = \begin{cases} \dfrac{z_{21}+7}{2} & -7 \le z_{21} \le -5, \\ & 2 \le z_{22} \le 6 \\ \dfrac{-3-z_{21}}{2} & -5 \le z_{21} \le -3, \\ & 2 \le z_{22} \le 6 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{22}|Z_{21}}(z_{21},z_{22}) = \begin{cases} \dfrac{z_{22}-2}{2} & 2 \le z_{22} \le 4, \\ & -7 \le z_{21} \le -3 \\ \dfrac{6-z_{22}}{2} & 4 \le z_{22} \le 6, \\ & -7 \le z_{21} \le -3 \\ 0 & \text{otherwise.} \end{cases}$$

Now, the row membership surfaces of the matrix

$$C = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

will be as follows

$$\mu_R(z_{11},z_{12}) = \begin{cases} \dfrac{(z_{11}+3)(z_{12}+6)}{4} & -3 \le z_{11} \le -1, \\ & -6 \le z_{12} \le -4 \\ \dfrac{(z_{11}+3)(-2-z_{12})}{4} & -3 \le z_{11} \le -1, \\ & -4 \le z_{12} \le -2 \\ \dfrac{(1-z_{11})(z_{12}+6)}{4} & -1 \le z_{11} \le 1, \\ & -6 \le z_{12} \le -4 \\ \dfrac{(1-z_{11})(-2-z_{12})}{4} & -1 \le z_{11} \le 1, \\ & -4 \le z_{12} \le -2 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_R(z_{21},z_{22}) = \begin{cases} \dfrac{(z_{21}+7)(z_{22}-2)}{4} & -7 \le z_{21} \le -5, \\ & 2 \le z_{22} \le 4 \\ \dfrac{(z_{21}+7)(6-z_{22})}{4} & -7 \le z_{21} \le -5, \\ & 4 \le z_{22} \le 6 \\ \dfrac{(-3-z_{21})(z_{22}-2)}{4} & -5 \le z_{21} \le -3, \\ & 2 \le z_{22} \le 4 \\ \dfrac{(-3-z_{21})(6-z_{22})}{4} & -5 \le z_{21} \le -3, \\ & 4 \le z_{22} \le 6 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of the matrix C will be

$$\mu_C(z_{11},z_{21}) = \begin{cases} \dfrac{(z_{11}+3)(z_{21}+7)}{4} & -3 \le z_{11} \le -1, \\ & -7 \le z_{21} \le -5 \\ \dfrac{(z_{11}+3)(-3-z_{21})}{4} & -3 \le z_{11} \le -1, \\ & -5 \le z_{21} \le -3 \\ \dfrac{(1-z_{11})(z_{21}+7)}{4} & -1 \le z_{11} \le 1, \\ & -7 \le z_{21} \le -5 \\ \dfrac{(1-z_{11})(-3-z_{21})}{4} & -1 \le z_{11} \le 1, \\ & -5 \le z_{21} \le -3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(z_{12},z_{22}) = \begin{cases} \dfrac{(z_{12}+6)(z_{22}-2)}{4} & -6 \le z_{12} \le -4, \\ & 2 \le z_{22} \le 4 \\ \dfrac{(z_{12}+6)(6-z_{22})}{4} & -6 \le z_{12} \le -4, \\ & 4 \le z_{22} \le 6 \\ \dfrac{(-2-z_{12})(z_{22}-2)}{4} & -4 \le z_{12} \le -2, \\ & 2 \le z_{22} \le 4 \\ \dfrac{(-2-z_{12})(6-z_{22})}{4} & -4 \le z_{12} \le -2, \\ & 4 \le z_{22} \le 6 \\ 0 & \text{otherwise.} \end{cases}$$

C. Multiplication of Two Imprecise Matrices

Multiplication of two imprecise matrices A and B will be

$$A.B = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} \cdot \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

where $Z_{11} = [18, 33, 52]$, $Z_{12} = [8, 20, 36]$, $Z_{21} = [46, 66, 90]$ and $Z_{22} = [23, 40, 61]$ respectively. The membership functions of Z_{11}, Z_{12}, Z_{21} and Z_{22} for row vectors are as follows

$$\mu_{z_{11}|z_{12}}(z_{11},z_{12}) = \begin{cases} \dfrac{-13+\sqrt{25+8z_{11}}}{4} & 18 \le z_{11} \le 33, \\ & 8 \le z_{12} \le 36 \\ \dfrac{21-\sqrt{25+8z_{11}}}{4} & 33 \le z_{11} \le 52, \\ & 8 \le z_{12} \le 36 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{z_{12}|z_{11}}(z_{11},z_{12}) = \begin{cases} \dfrac{-5+\sqrt{9+2z_{12}}}{2} & 8 \le z_{12} \le 20, \\ & 18 \le z_{11} \le 52 \\ \dfrac{9-\sqrt{9+2z_{12}}}{2} & 20 \le z_{12} \le 36, \\ & 18 \le z_{11} \le 52 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{z_{21}|z_{22}}(z_{21},z_{22}) = \begin{cases} \dfrac{-9+\sqrt{2z_{21}-11}}{2} & 46 \le z_{21} \le 66, \\ & 23 \le z_{22} \le 61 \\ \dfrac{13-\sqrt{2z_{21}-11}}{2} & 66 \le z_{21} \le 90, \\ & 23 \le z_{22} \le 61 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{z_{22}|z_{21}}(z_{21},z_{22}) = \begin{cases} \dfrac{-15+\sqrt{41+8z_{22}}}{4} & 23 \le z_{22} \le 40, \\ & 46 \le z_{21} \le 90 \\ \dfrac{23-\sqrt{41+8z_{22}}}{4} & 40 \le z_{22} \le 61, \\ & 46 \le z_{21} \le 90 \\ 0 & \text{otherwise.} \end{cases}$$

Now, the row membership surfaces of the matrix

$$C = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

will be as follows

$$\mu_R(z_{11},z_{12}) =$$

$$\begin{cases} \dfrac{(-13+\sqrt{25+8z_{11}})(-5+\sqrt{9+2z_{12}})}{8} & 18 \le z_{11} \le 33, \\ & 8 \le z_{12} \le 20 \\ \dfrac{(-13+\sqrt{25+8z_{11}})(9-\sqrt{9+2z_{12}})}{8} & 18 \le z_{11} \le 33, \\ & 20 \le z_{12} \le 36 \\ \dfrac{(21-\sqrt{25+8z_{11}})(-5+\sqrt{9+2z_{12}})}{8} & 33 \le z_{11} \le 52, \\ & 8 \le z_{12} \le 20 \\ \dfrac{(21-\sqrt{25+8z_{11}})(9-\sqrt{9+2z_{12}})}{8} & 33 \le z_{11} \le 52, \\ & 20 \le z_{12} \le 36 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_R(z_{21}, z_{22})=\begin{cases} \frac{(-9+\sqrt{2z_{21}-11})(-15+\sqrt{41+8z_{22}})}{8} & 46 \le z_{21} \le 66, \; 23 \le z_{22} \le 40 \\ \frac{(-9+\sqrt{2z_{21}-11})(23-\sqrt{41+8z_{22}})}{8} & 46 \le z_{21} \le 66, \; 40 \le z_{22} \le 61 \\ \frac{(13-\sqrt{2z_{21}-11})(-15+\sqrt{41+8z_{22}})}{8} & 66 \le z_{21} \le 90, \; 23 \le z_{22} \le 40 \\ \frac{(13-\sqrt{2z_{21}-11})(23-\sqrt{41+8z_{22}})}{8} & 66 \le z_{21} \le 90, \; 40 \le z_{22} \le 61 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of the matrix C will be as follows

$$\mu_C(z_{11}, z_{21})=\begin{cases} \frac{(-13+\sqrt{25+8z_{11}})(-9+\sqrt{2z_{21}-11})}{8} & 18 \le z_{11} \le 33, \; 46 \le z_{21} \le 66 \\ \frac{(-13+\sqrt{25+8z_{11}})(13-\sqrt{2z_{21}-11})}{8} & 18 \le z_{11} \le 33, \; 66 \le z_{21} \le 90 \\ \frac{(21-\sqrt{25+8z_{11}})(-9+\sqrt{2z_{21}-11})}{8} & 33 \le z_{11} \le 52, \; 46 \le z_{21} \le 66 \\ \frac{(21-\sqrt{25+8z_{11}})(13-\sqrt{2z_{21}-11})}{8} & 33 \le z_{11} \le 52, \; 66 \le z_{21} \le 90 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(z_{12}, z_{22})=\begin{cases} \frac{(-5+\sqrt{9+2z_{12}})(-15+\sqrt{41+8z_{22}})}{8} & 8 \le z_{12} \le 20, \; 23 \le z_{22} \le 40 \\ \frac{(-5+\sqrt{9+2z_{12}})(23-\sqrt{41+8z_{22}})}{8} & 8 \le z_{12} \le 20, \; 40 \le z_{22} \le 61 \\ \frac{(9-\sqrt{9+2z_{12}})(-15+\sqrt{41+8z_{22}})}{8} & 20 \le z_{12} \le 36, \; 23 \le z_{22} \le 40 \\ \frac{(9-\sqrt{9+2z_{12}})(23-\sqrt{41+8z_{22}})}{8} & 20 \le z_{12} \le 36, \; 40 \le z_{22} \le 61 \\ 0 & \text{otherwise.} \end{cases}$$

V. Conclusion

In this article, we have shown the membership surface and arithmetic operations of imprecise matrix. We have shown here addition, subtraction and product of imprecise matrices. The numerical example of arithmetic operations are given only for 2×2 matrices. But using this method the arithmetic operations can be performed for $n \times n$ matrices too and it is possible to obtain the row and column membership surfaces of the matrix.

Acknowledgment

The author would like to thank Prof. Hemanta K. Baruah, Vice-Chancellor, Bodoland University, India for his valuable suggestion that greatly improved the manuscript.

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Dhruba Das completed his MSc in Statistics from Gauhati University, India in 2012. After completing his MSc, he has joined as a junior research fellow in the Department of Statistics, Gauhati University in 2013 for his PhD and currently he is pursuing his research under the guidance of Prof. Hemanta K. Baruah, Vice-Chancellor, Bodoland University, Kokrajhar, Assam, India.