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Approximating Maximum Speed on Road from Curvature Information of Bezier Curve

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Abstract**—**Bezier curves have useful properties for path generation problem, for instance, it can generate the reference trajectory for vehicles to satisfy the path constraints. Both algorithms join cubic Bezier curve segment smoothly to generate the path. Some of the useful properties of Bezier are curvature. In mathematics, curvature is the amount by which a geometric object deviates from being flat, or straight in the case of a line. Another extrinsic example of curvature is a circle, where the curvature is equal to the reciprocal of its radius at any point on the circle. The smaller the radius, the higher the curvature thus the vehicle needs to bend sharply. In this study, we use Bezier curve to fit highway-like curve. We use different approach to find the best approximation for the curve so that it will resembles highway-like curve. We compute curvature value by analytical differentiation of the Bezier Curve. We will then compute the maximum speed for driving using the curvature information obtained. Our research works on some assumptions; first, the Bezier curve estimates the real shape of the curve which can be verified visually. Even though, fitting process of Bezier curve does not interpolate exactly on the curve of interest, we believe that the estimation of speed are acceptable. We verified our result with the manual calculation of the curvature from the map.

Keywords**—**Speed estimation, path constraints, reference trajectory, Bezier curve.

I. Introduction

TRANSPORTATION plays an important role in our daily activity. After revolution of transportation, commons form of transportation includes planes, trains, automobiles and other two-wheel devices such as bikes or motorcycles. Transportation is also crucial for economic purposes where raw materials, fuels, food, and manufactured goods are located where they are wanted. Through transportation, goods are moved from where there are surpluses to where there are shortages.

Modes of transportation are divided into several parts includes air, rail, road, water, cable, pipeline and space and basically it operates using three ways land, water and air. There are certain factors that we need to consider when we choose transportation such as speed, comfort, cost, flexibility, regularity, and safety. When we consider speed of transport, air transport is the quickest mode of transport but it costliest of them all.

Trains for example are exposed to the risk of derailment resulting from severe vibration induced by centrifugal forces from different kind of configuration [8]. A train entering curve from a straight line, from curve to another curve with different radius and leaving a curve to enter a straight line are types of sudden changes that can cause discomfort to passenger. Figs. 1 and 2 show the transition curves for 2 different radiuses which happened in rail configuration. Therefore, a transition curve from one form to another is needed. In order to construct route design for highway and railway, it is important to have joining curve between two different arc lengths that we called transition curve [1].

Transition curve may ease up the transition, prevent accident caused by sharp changes in direction and reduces shaking of vehicles in operation. It also makes the transition smoother, more stable, and safer.

Road design may be done heuristically but one should consider the topological factors which limit the flexibility of design. Examples of topological factor are hill, valley and mountain. These obstacles make the vehicles or transportation less efficient, as transition curves may not be available due to these factors. Therefore, maximum speeds are assigned in specific location to prevent accident. Similar to railways, road driver need to reduce their speed of vehicle at specific location.

Recently driverless car had been developed. Automated Driving (AD) offers an excellent solution due to its capability to optimize traffic flows, thus decreasing traffic jams and accidents [4]. It may also reduce fuel consumption resulting in the reduction of carbon dioxide and other noxious emissions [3]. In addition, a significant increase of road safety and comfort is to be expected due to avoidance of human driving errors. Reference [3] predicts the scope, identify actions and timeframes of the successive future developments of smart systems technologies, linking them to milestones defined for the progress of automated driving [4].

Maximum speed estimation will allow an automated car to speed under the threshold for safety procedure. In this paper, we fit Bezier curve on a road map and compute the curvature of this curve as this information is needed for the maximum speed estimation.

In curve and surface design, it is often desirable to have planar transition curve, where transition curve between two circles [9] can combine to form C-shape curve and S-shape curve, circle to straight lines or vice-versa and circle to circle with different radius. This transition curve is useful for highway and railway route design. It is very important for all motion planning of autonomous vehicles because it relate to the curvature and continuity. Curvature tends to have an

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inverse correlation with speed of vehicle [2]. Other than that, consistency and continuity of track alignment design are essential in improving the travel speed and efficiency of transportation.

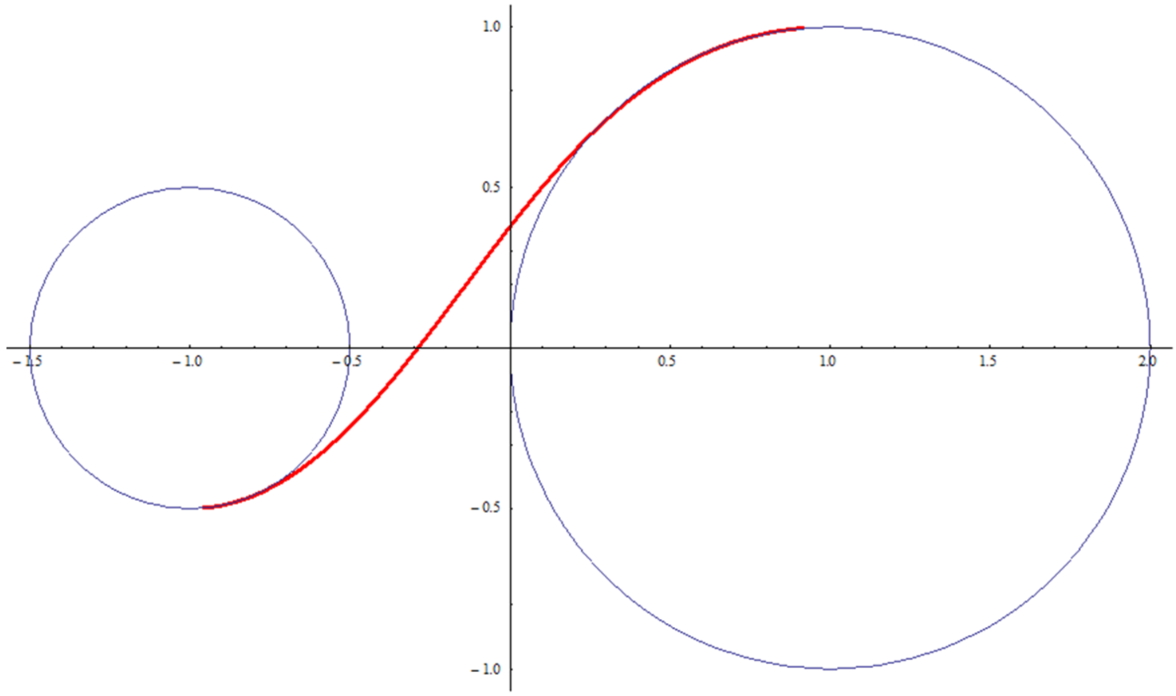

Fig. 1 S-shape transition curve for two different radiuses.

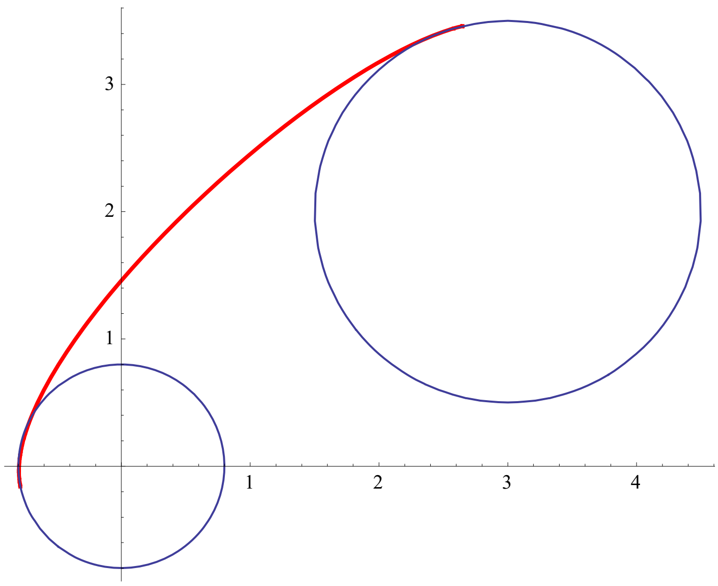

Fig. 2 C-shape transition curve for two different radiuses.

If a vehicle were to follow the road precisely when a tangent is followed immediately by a curve, the driver would have to turn the steering wheel instantaneously to match the changing direction [10]. Obviously, this is not possible and some slower rate of turning the steering wheel would be followed.

Curvature of circular curve remains the same but for transition curve it will change because of its characteristics that the radius is changing across its length. The absolute value of the reciprocal of curvature equals to radius of curvature [5]. If the radius of the circular curve is sufficiently large, the transition path followed by the vehicle is easily contained within the width of the lane in which the vehicle is travelling. As the circular radius decrease, however, it becomes necessary to modify the shape by elevating the road to accommodate comfort driving of the vehicle [10].

II. Classical Theory

A. Radial Forces

Transition curve lengths must be designed so that they minimized passenger discomfort and maximize safety. A vehicle travelling at constant speed along a curve is subjected to a centrifugal force expressed as:

$$F = \frac{mv^2}{R} \tag{1}$$

where; $m =$ mass of the vehicle (kg), $v =$ speed of the vehicle (m/s), $R =$ radius of the curvature (m).

Smaller value of R, the higher value of force F, acting on the vehicle and the faster the speed the force is also higher. To ensure the comfort of the occupants of the vehicle, the force should be allowed to increase gradually in a linear fashion from zero to the full value experienced on the circular arc. Therefore, it can be expressed as:

$$F \propto \frac{1}{R} \quad \text{and} \quad F \propto L \tag{2}$$

and can combined so that $L/R = K$ where K is a constant. Two curves can be used to provide the transitional section of curve. These are the clothoid and the cubic parabola. The clothoid is a true spiral and thus matches the requirement that RL should be a constant, K. The cubic parabola is not a true spiral because RL is not a constant. It can, however, be used over a certain range of circular curvature and has the advantage of being less complex than the clothoid [10].

B. Continuity

There have several types of continuity which is parametric continuity and geometric continuity. C are refer to parametric continuity where G are geometric continuity. $C^0$and G^0 were refer to point continuity, where point continuity means two curve are connected at their end points.

$C^1$and G^1were continuity of tangency across two curves. In order for two curves to be C^1 or G^1 they also need to be connected at their end point and are therefore C^0 and G^0as well. Tangency continuity means the tangent at the beginning of one curve is parallel to the tangent of the other curve and both share their origin point. In addition C^1 requires the tangents to be of the same length as well.

C^2 and G^2 were continuity of curvature across two connected curves. All previous conditions have to be fulfilled in order for C^2 and/or G^2 to be possible. Continuity of curvature is important for reflection lines for high quality surfaces. Automotive class surfaces usually attempt to achieve even higher continuity levels of G^3/C^3 but for architecture this level of surface quality is usually not necessary as it also requires high quality production methods usually too expensive for large scale constructs.

C. Radial Speed and Design Speed

Radial forces act on a vehicle as it travel around a curve and this is why transition curves necessary. A vehicle of mass m travelling at constant speed, v along a curve of radius, r is subjected to radial force F such that $F = mv^2/R$, which is when $R = \infty, F = 0$.

Roads are designed according to a speed design which is constant for a given stretch of roadway. Thus a vehicle must be able to travels comfortably and safety at the length of a given stretch of road at the design speed regardless of bends. The mass of vehicle is also assumed to be constant.

Road speed design procedure reflects the association between curvature and speed. The basic principal of horizontal curve design is derived from application of the kinematics equation. Centrifugal acceleration (CA) is total lateral acceleration, applied through pavement superelevation and tire-pavement friction on vehicle negotiating a circular curve due to vehicle movement:

$$CA = \frac{V^2}{R} = (e + f)g \tag{3}$$

where; V is vehicle speed, R is curve radius, e is pavement superelevation rate, f is the tire-pavement side friction factor, and g is acceleration of gravity [7]. Application of (3) to S.I. units leads to the following expression:

$$V_d^2 = 127 * R * (e + f) \tag{4}$$

where; V_d is the design speed in kilometres per hour(kph), R is the curve radius in meters(m), e is the pavement superelevation rate(m/m), f is the tire-pavement side friction coefficient. Then, the equivalent formula is:

$$V_d^2 = \frac{85660*(e+f)}{DC} \tag{5}$$

where; DC is the degree of curve, in degrees per 100-ft (30.5m) arc, V_d is the design speed.

III. Speed Estimation

A. Heuristic Speed Estimation

For the discussion of this paper we have chosen road from Relau to Balik Pulau in Penang Island, Malaysia shown in Fig. 3, as it has many sharp turns area. Firstly, we mark 10 point, marked as red point, along that road. For each red point we draw the biggest possible circle that is bounded by the road. From this circle, we obtain the radius therefore the curvature value to be plugged into (4). Fig. 4 shows the road map with 10 circles drawn for curvature estimation coloured with alternate blue and red.

Fig. 3 Roadmap from Relau to Balik Pulau marked by red point

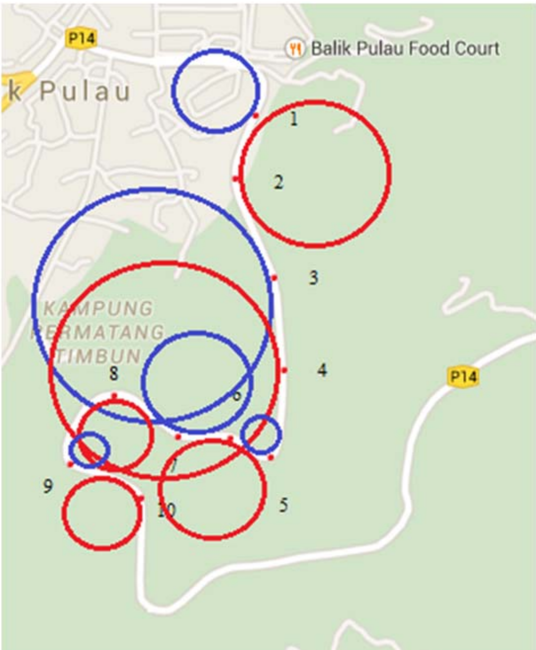

Fig. 4 Blue and Red Circle are drawn and touched red point

By constructing the biggest possible circle which touches the corner of the road, the radius can be used to compute the curvature hence calculating the maximum safe speed. By using US standard [7], to find normal velocity with e and f are constant variable, we can have velocity design speed of highway like in Table I. Real time observation has been made by driving along that road. We identify average velocity along that 10th red spot was 40km/h and speed limit was place at second red spot that we only allowed driving at a speed of 60km/h.

TABLE I
DESIGN VELOCITY WITH THEIR RESPECTIVE RADIUS AND COORDINATE

Point	Coordinates, (x, y)	Radius (m), r	Design Velocity (km/h), V_d
1	(150, 420)	62380	36.58
2	(139, 382)	90352	52.98
3	(162, 324)	177105	103.86
4	(167, 270)	135528	79.48
5	(160, 220)	19510	11.44
6	(136, 231)	60617	35.28
7	(104, 231)	56693	33.25
8	(66, 255)	46337	27.17
9	(41, 216)	20781	12.19
10	(83, 194)	36067	21.15

B. Maximum Speed Based on Bezier Curve

For the next method, we use cubic Bezier curve definition. Since cubic Bezier curve can only have 4 point, we divide 10th red spot into 3 different parts. Every part of the curve we calculate separately.

$$z(t) = (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t) t^2 P_2 + t^3 P_3 \quad (6)$$

For every consecutive P0, P1, P2 and P3 point we replace with 4 coordinate points from 10 red point. On each of the consecutive P0, P1, P2 and P3 we can have new definition of cubic Bezier curve. From there we can calculate curvature of that point from cubic Bezier definition. By application of (7), we use to calculate curvature for each of the new definition of cubic Bezier curve.

$$\kappa = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}} \quad (7)$$

TABLE II
AVERAGE VELOCITY FOR THE FIRST CONSECUTIVE 4 POINT OF 1ST PIECEWISE CURVE OF CUBIC BEZIER CURVE

Time, t	Curvature, κ	Velocity Design, V_d
0.0	0.00341907	80.83
0.1	0.00242514	113.96
0.2	0.00161627	170.99
0.3	0.00103317	267.50
0.4	0.00062047	445.42
0.5	0.000316811	872.36
0.6	0.0000755603	3657.63
0.7	0.000136719	2021.46
0.8	0.000345409	800.13
0.9	0.000571928	483.23
1	0.000835646	330.73

By using re-adjust control points, we can generate approximation 10 points of B-spline curve of degree three approximately lie on the original control point. This method was an alternative manual method to find approximation without using piecewise cubic Bezier. The lower the degree, the closer B-spline curve follow the polyline.

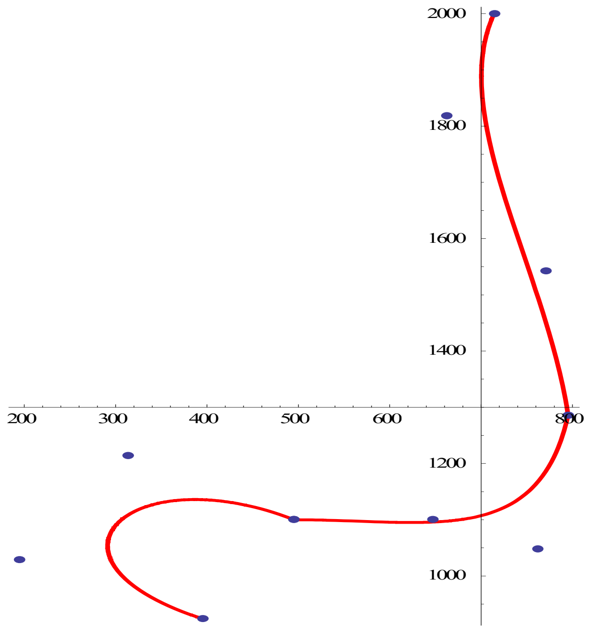

Fig. 5 Graph being stitched-up together at 4th and 7th point without curvature continuity

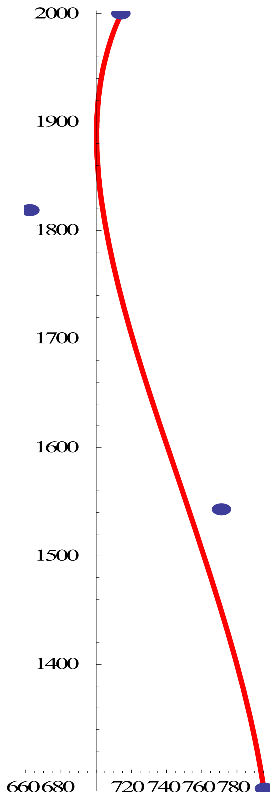

Fig. 6 1st piecewise curve of cubic Bezier curve

TABLE III
AVERAGE VELOCITY FOR THE SECOND CONSECUTIVE 4 POINT OF 2ND PIECEWISE CURVE OF CUBIC BEZIER CURVE

Time, t	Curvature, κ	Velocity Design, V_d
0.0	0.0013892	198.94
0.1	0.00259406	106.54
0.2	0.00474796	58.21
0.3	0.0076126	36.30
0.4	0.00906271	30.50
0.5	0.00740876	37.30
0.6	0.00449674	61.46
0.7	0.00212101	130.30
0.8	0.000495718	557.52
0.9	0.000643006	429.81
1	0.00150391	183.77

TABLE IV
AVERAGE VELOCITY FOR THE THIRD CONSECUTIVE 4 POINT OF 3RD PIECEWISE CURVE OF CUBIC BEZIER CURVE

Time, t	Curvature, κ	Velocity Design, V_d
0.0	0.00321057	86.08
0.1	0.00470423	58.75
0.2	0.00673412	41.04
0.3	0.00920858	30.01
0.4	0.0116632	23.70
0.5	0.013093	21.11
0.6	0.0124356	22.22
0.7	0.00989018	27.94
0.8	0.00688858	40.12
0.9	0.00448731	61.59
1	0.00287396	96.16

TABLE V
TIME, CURVATURE AND VELOCITY 9TH DEGREE OF BEZIER CURVE

Time, t	Curvature, κ	Velocity Design, V_d
0.0	0.00455876	60.62
0.1	0.00056292	490.96
0.2	0.000348771	792.42
0.3	0.00120516	229.32
0.4	0.0034957	79.06
0.5	0.00587868	47.01
0.6	0.00248057	111.41
0.7	0.000952666	290.10
0.8	0.00793351	34.84
0.9	0.0149203	18.52
1	0.00383195	72.12

TABLE VI
TIME, CURVATURE AND VELOCITY OF 3RD DEGREE OF BEZIER CURVE USING RE-ADJUST METHOD

Time, t	Coordinate, (x, y)	Curvature, κ	Velocity Design, V_d
0.0	(150, 419)	0.00667426	41.41
0.1	(145, 379)	0.000917857	301.11
0.2	(152, 334)	0.000410039	674.01
0.3	(155, 293)	0.00136311	202.75
0.4	(150, 262)	0.00416671	66.33
0.5	(135, 245)	0.00640685	43.14
0.6	(113, 239)	0.00194827	141.86
0.7	(88, 238)	0.00144054	191.85
0.8	(65, 232)	0.00776668	35.58
0.9	(55, 215)	0.0129415	21.36
1	(82, 195)	0.00376704	73.37

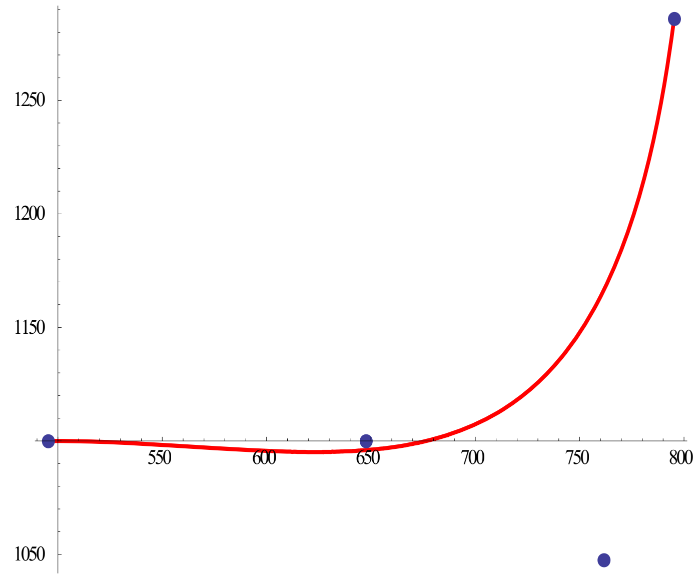

Fig. 7 2nd piecewise curve of cubic Bezier curve

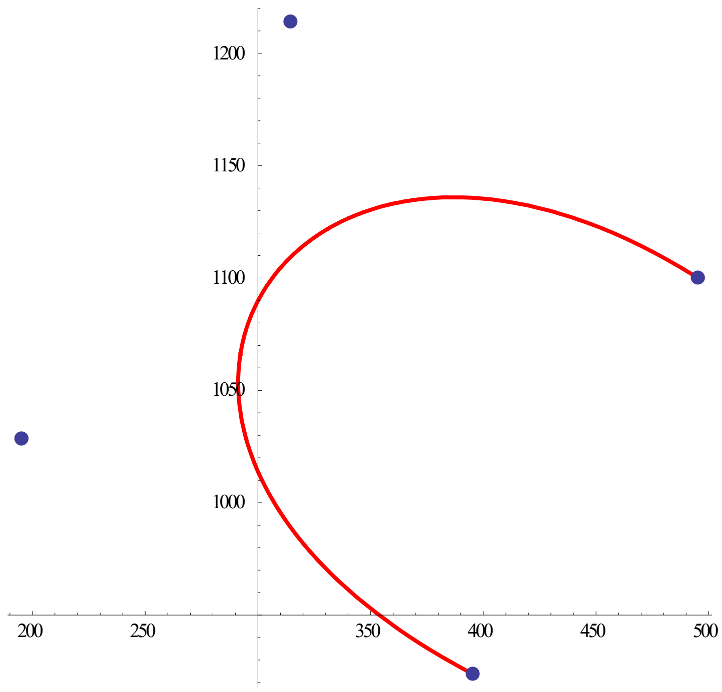

Fig. 8 3rd piecewise curve of cubic Bezier curve

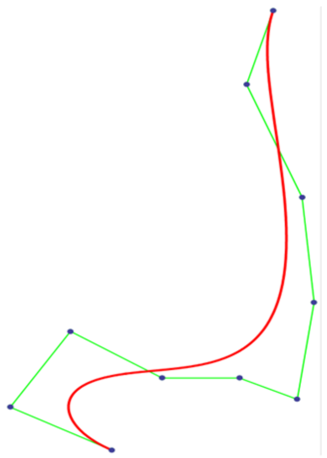

Fig. 9 10 point with 9th degree of Bezier curve

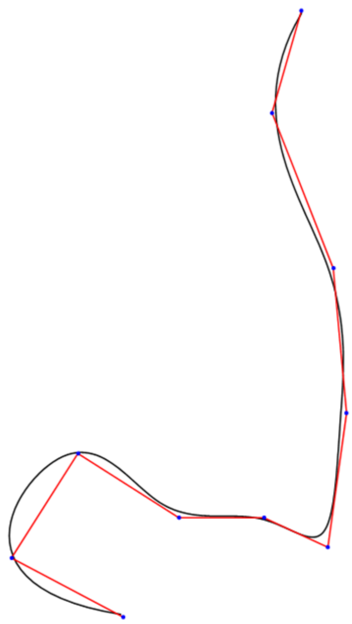

Fig. 10 3rd degree curve with re-adjust control point

TABLE VII
COMPARISON OF TIME, COORDINATE AND VELOCITY OF HEURISTICALLY WITH MAXIMUM SPEED BASED ON BEZIER CURVE INFORMATION

Velocity Design by (4), V_d	Selected Coordinate, (x, y)	Time, t	Coordinate on the curve which is closed to (x, y) by (6), $(\bar{x}, \bar{y})$	Velocity Design, $V_{\bar{d}}$
36.58	(150, 420)	0.0	(150, 419)	41.41
52.98	(139, 382)	0.1	(145, 379)	301.11
103.86	(162, 324)	0.2	(152, 334)	674.01
79.48	(167, 270)	0.3	(155, 293)	202.75
11.44	(160, 220)	0.4	(150, 262)	66.33
35.28	(136, 231)	0.5	(135, 245)	43.14
33.25	(104, 231)	0.6	(113, 239)	141.86
-	-	0.7	(88, 238)	191.85
27.17	(66, 255)	0.8	(65, 232)	35.58
12.19	(41, 216)	0.9	(55, 215)	21.36
21.15	(83, 194)	1	(82, 195)	73.37

IV. CONCLUSION

Bezier curve have a potential to estimates the maximum driving speed allowed for a safety driving on road. Due to the facts that the curve does not interpolate the points, thus small changes in coordinates will affect the curvature of the curve. Therefore, it creates bigger difference toward speed estimation on a curve. The contribution of this paper is to summarize curvature information of Bezier curve for the purpose of approximating maximum speed on a road and utilizing verification between heuristic method and Bezier method in order to estimate the maximum allowed speed.

As shown in Fig. 5, 4th and 7th points are the last point of a curve and initial point of another curve, stitched up together. When curve segments are joined, the starting point of the second curve segment coincides with the ending point of the first curve segment [6]. From there we can observe that there will be redundant calculation on 4th and 7th point since that it will be the last point for the 1st part of the curve and be last point for the consecutive part of the curve. Therefore, graph will be over-plotting on last and initial point of calculation. Other than that in Figs. 6-8, we can see that initial point and last point does not have any information before and after the calculation. If using Bezier curve or B-spline curve of degree 10, the curve still does not lie on control point because it follows convex hull properties. Fig. 10 show us that we need lower degree of the curve in order the curve will closely follow the polyline or control point. Here we can see that we need lower degree of the curve, but at the same time we need to have the curve that lies on the control point that have smooth shape. Thus, tangency and geometric continuity will play their roles.

Some improvement can be made by increasing the number of red point along the curve. Past studies shown that when increase the number of control points of the curve it will refine the shape of the curve become smoother thus we can calculate the speed limit more accurate and precise. Another improvement also can be made by increasing or elevating the degree of freedom of the curve, such that at the moment we using cubic Bezier but we need to divide the curve into several part and also we use 10 degree of freedom on the single curve without being divide the curve into part by part.

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