Shear Strength of Reinforced Web Openings in Steel Beams

K. S. Sivakumaran, Bo Chen

Abstract—The floor beams of steel buildings, cold-formed steel floor joists in particular, often require large web openings, which may affect their shear capacities. A cost effective way to mitigate the detrimental effects of such openings is to weld/fasten reinforcements. A difficulty associated with an experimental investigation to establish suitable reinforcement schemes for openings in shear zone is that moment always coexists with the shear, and thus, it is impossible to create pure shear state in experiments, resulting in moment influenced results. However, Finite Element Method (FEM) based analysis can be conveniently used to investigate the pure shear behaviour of webs including webs with reinforced openings. This paper presents the details associated with the finite element analysis of thick/thin-plates (representing the web of hot-rolled steel beam, and the web of a cold-formed steel member) having a large reinforced opening. The study considered simply-supported rectangular plates subjected to in-plane shear loadings until failure (including post-buckling behaviour). The plate was modelled using geometrically non-linear quadrilateral shell elements, and non-linear stress-strain relationship based on experiments. Total Langrangian with large displacement/small strain formulation was used for such analyses. The model also considered the initial geometric imperfections. This study considered three reinforcement schemes, namely, flat, lip, and angle reinforcements. This paper discusses the modelling considerations and presents the results associated with the various reinforcement schemes under consideration.

Keywords—Cold-formed steel, finite element analysis, opening, reinforcement, shear resistance.

I. INTRODUCTION

L ARGE web openings are often cut-out in cold-formed steel floor joists of such buildings, usually at the construction site itself, in order to pass through pipes, wires, etc. A cost effective way to mitigate the detrimental effects of these impromptu web openings in the floor joists is to fasten reinforcements. An experimental investigation for shear problems always presents difficulties since moment always coexists with the shear. However, the finite element analysis method can be easily used to isolate the pure shear behaviour and investigate the shear strength of webs, including the webs with reinforced openings. This paper presents the details associated with the finite element analysis of thick/thin-web having a large reinforced opening, based on post-buckling behaviour of simply-supported rectangular plates with reinforced opening subjected to in-plane shear loadings until failure. Only a limited research exists on shear reinforcements

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on cold-formed steel webs. Pennock [1] carried out experimental studies on cold-formed steel joists with reinforced and unreinforced web openings subjected to bending, which considered both circular and square openings. The reinforcement scheme [1] considered was found to be inadequate for openings located, both, in high bending zones and in high shear zones. Acharya [2], [3] performed an experimental investigation on reinforcement schemes for cold-formed steel joists having large web openings. His studies considered both flexural and shear zones. Three reinforcement schemes were considered in his study for the shear reinforcement. Two of the reinforcement schemes were recommended by the AISI [4], which are (a) a steel plate having the same thickness, same size, and shape of the opening as the main joist and (b) a cold-formed steel joist section having the same thickness, same size and shape of the opening as the main joist. The third reinforcement scheme considered by [2], [3] consisted of four channel sections screw fastened around the opening. Acharya [2], [3] concluded that only the reinforcement scheme using the channel sections was adequate to restore the shear strength of cold-formed steel joists having web openings. Section II of this paper presents the finite element analysis models for web plates having a reinforced opening subjected to in-plane shear loadings. The plates were subjected to increasing shear loads until failure so as to capture the strength of such systems.

II. THE FINITE ELEMENT MODEL

In this section, a general finite element analysis model is proposed to investigate the behavior of plates with a reinforced square opening subjected to pure shear loads. The model consists of two components, namely, the main plate and the reinforcements. The main plate, representing the web of a cold-formed steel joist, is taken to have a length of 'a', a width of 'h' and a thickness of 't', resulting in an aspect ratio of (a/h) and a slenderness of (h/t). The plates considered in this investigation had a fixed aspect ratio (a/h=3) and varying slenderness ratios (h/t). Though the aspect ratio can influence the shear strength of plates, analytical studies [5] indicated that only a marginal change exists in plates having a/h >3, thus an aspect ratio of a/h = 3 was used to represent the cold-formed steel joists whose aspect ratio may be substantially higher than 3. The parametric study considered h/t = 50, 100, 150 and 200, representing thick to thin webs. The main plate is assumed to have a centrally located square opening of side dimensions 'dc'. This investigation considered dc/h = 0.6 representing 60% web opening, which is rather large. The

remaining web width for installation of reinforcement is thus (h-dc).

Fig. 1 shows the three reinforcement schemes, namely, flat reinforcement, lip reinforcement, and angle reinforcement, under consideration. Accordingly, all four edges of the square opening were reinforced with equal size reinforcements along all four edges. The width of the reinforcement was taken as 'hr'. Considering the flat reinforcement and assuming that the reinforcement is attached between the opening edge and web edge, the width of the reinforcement $h_r = (h-dc)/2$. Thus, the flat reinforcement consists of flat strips of metal of width hr and t_r along all four edges of the opening. In the lip reinforcement arrangement, all edges of the square opening were considered to be reinforced with lip plate of width h_r and t_r. The third reinforcement scheme under consideration consists of an angle, having equal size legs, fastened along all edges of the square opening. As shown in Fig. 1, one leg is fastened to the web, somewhat similar to the flat reinforcement, thus the other leg acts like the lip reinforcement. The width of each leg however, was taken as $h_r/2$, thus the total width of all three reinforcement schemes is hr. The reinforcements are assumed to be fully attached to the main plate with no additional constraints. Thus, though screws are widely used in construction practice, here, no screw will be modeled. In the parametric study presented in Section III, the thickness of the reinforcements t_r was taken to be multiple of main plate thickness ($t_r = n.t$).

A. The Finite Elements

The finite element models of the plate and reinforcement schemes were constructed and analyzed using the finite element analysis software ADINA [6]. In the current study the quadrilateral four-node shell elements for the non-linear analysis, which are capable of representing both flat and curved surfaces, were used. Each node of the shell element has six degrees of freedoms, namely three displacements and three rotations. This four-node element is capable of simulating both the membrane and the flexural behaviors of plates. The default 2 by 2 integration point arrangement for the 4-node element is used in the r-s element mid-plane. In the through-thickness direction 't' the Newton-Cotes rule for integration points is preferred rather than the default Gauss quadrature rule because, instead of having integration points only within the thickness of plates, the Newton-Cotes rule also has the integration points lying at both the top and bottom surfaces. This allows one to capture the gradually yielding response of the plate starting from the boundaries. Though the program default integration point number for Newton-Cotes rule is 3, in order to improve the accuracy of the model, a through thickness integration of 7 was used in the analyses. The quadrilateral four-node shell elements and the 2 x 2 x 7 integration scheme were used for both the main plate and the reinforcements, since both of them are plated elements.

B. The Mesh Quality

The mesh configurations for the three reinforcement schemes, which are not shown in this paper, are based on a convergence study [5]. Accordingly, 24-division mesh configuration is adequate for plates with an aspect ratio of 3 (a/b=3), resulting in 3456 elements for the main plate with an opening. The flat portions of the reinforcements in flat reinforcement and angle reinforcement also used similar mesh configuration. The lip reinforcement, however, contained 24 by 24 divisions, which gave a ratio of the longest element edge to the shortest element edge of 3. However, height of the lips associated with angle reinforcement was divided into 20 elements resulting in 24 by 20 divisions, which gave a ratio of the longest element edge to the shortest element edge of 5.
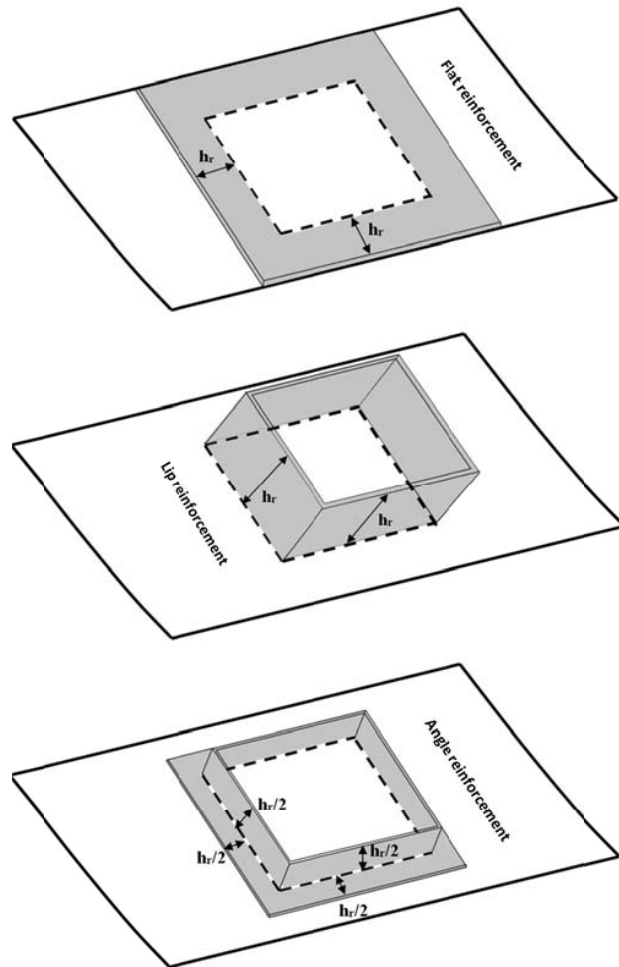

Fig. 1 The reinforcement schemes

C. Initial Geometric Imperfections and the Residual Stresses

The models included geometrical initial imperfections; however, possible residual stresses were ignored in this study, primarily based on [7] which concluded that the flexural residual stress has negligible or no effect on the ultimate strength of cold-formed steel sections. The main plate contained a double sine function imperfection and the amplitude of the imperfection was taken as $w_0 = 0.003h$, where h is the width of the main plate. The flat surfaces and

the flat edges of the lips of the reinforcements contained compatible imperfections.

D. The Material Model

Sivakumaran and Abdel-Rahman [8] has shown that, instead of a well-defined yield point, cold-formed steel exhibits a gradual yielding behavior followed by a certain level of strain hardening. It has also been shown that the yield strength and ultimate strength differ between the corner area and the flat area of a cold-formed steel section, primarily because of the cold-working at the corners. For analysis purpose, Sivakumaran and Abdel-Rahman [8] proposed an idealized multi-linear stress-strain relationship for cold-formed steel material at the corner zones and at the flat zones. Fig. 2 shows these proposed stress-strain relationships, where 'Fy' is the yield stress of steel, which depends on the steel grade selected in the analysis. In this investigation, the commonly used 350MPa yield strength was used as the value of 'Fy'. This idealized multi-linear stress-strain relationship for cold-formed steel was used for, both, the main plate and the reinforcement plate. Since this study focuses only on the flat plates, the stress-strain relationship associated with the flat area was used for the main plate, as well as the stiffeners. Plastic-multi-linear isothermal plasticity material models were chosen for the analysis, in order to simulate the non-linear material stress-strain relationship. It assumes the material to be elastic–plastic with strain hardening, following the isotropic hardening rule. The von Mises yield criterion is adopted as the yielding criterion for steel.
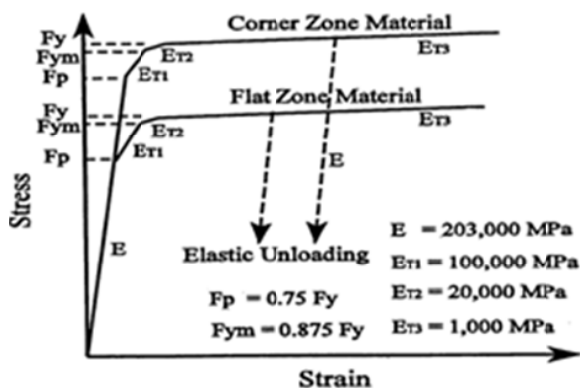

Fig. 2 Idealized material model [8]

E. Boundary Conditions and the Loading Conditions

The main plate was assumed to be simply supported along the four edges and was subjected to uniformly distributed shear loads applied along all four edges. The edges of the opening were left to move freely. The flat portions of the reinforcements were assumed to be fully attached to the main plate, represented by the same elements and the same finite element nodes.

F. The Analysis Technique

The analysis technique must capture the pre-buckling, post-buckling and the ultimate load level behavior of the models under consideration, as ultimate strength of plates with reinforced opening is of interest. Though ADINA [6] features included the automatic-time-stepping (ATS) method and the load-displacement-control (LDC) method, here the ATS was used which requires prescription of a load and time steps. If no convergence can be obtained through the user-defined load steps, the ADINA would automatically subdivide the time steps until the convergence is achieved.

III. ANALYSIS RESULTS

The plates with reinforced opening under consideration were assumed to have a length of a=300 mm, and a width of h=100 mm, which gave an aspect ratio of a/h=3. The amplitude of the initial geometric imperfection used was 0.3 mm. Furthermore, this study considered a 60% square opening with dc=60 mm. Therefore, the total width of the reinforcement plate h_r was taken to be a fixed value of 20mm, which is the one half of the remaining width of the plate above and below the opening. For the angle-reinforcement configuration, the reinforcement plate was assumed to be bent into angles with the width of each leg of the angle equals to (1/2). h_r =10 mm. As indicated the slenderness ratios of the plates considered in this investigation are h/t = 50, 100, 150 and 200, which covers from thick plates to the thinnest plates allowed in the AISI code [4]. In this study, considering the plate with different h/t one by one, reinforcements with increasing thickness were applied on plates to investigate the influence of the thickness of the reinforcements on the behavior and the ultimate shear capacity of such plates. Thus, for plates with each 'h/t' value, the reinforcement thickness to the main plate thickness ratio (t_r/t) increases from zero until the increase in the ultimate shear strength of plates is less than 1.0%. Generally, (t_r/t) was increased at an incremental step of one for each (h/t) value. However, intermediate steps may also be applied when needed. The multi-linear material model [8] with Fy = 350 MPa and ν = 0.3 was used for the material property of the plate as well as the reinforcements.

The behaviors of the plates considered in this section are illustrated through the shear stress versus the average shear strain diagrams. The average shear strain shown in the horizontal axis was obtained by dividing the y-displacement of the lower right corner of the plate by the length of the plate. Fig. 3 shows the applied shear stress versus the average shear strain diagrams for both solid plates and plates with reinforced openings. The behavior of the solid plate is shown in these diagrams, in order to be able to compare the behavior of the reinforced plates with that of solid plates. These figures also include the behavior of plates with unreinforced openings, which corresponds to the case of (t_r/t) = 0. Essentially, thickness of the reinforcement t_r=0 mm indicates that there was no reinforcements. Fig. 3 shows the shear stress-strain diagrams for plates with flat-reinforcements, for plates with lip-reinforcements, and for plates with angle-reinforcements. For illustration purposes only the lowest and the highest slenderness values considered in this investigation (i.e. h/t=50 and h/t=200) are shown in this figure. Each figure shows the shear stress-strain relationships as the size of the

reinforcement (t_r/t) increases. It may be noted that only few selected 't_r/t' values are plotted in Fig. 3. It can also be observed from Fig. 3 that solid plates are generally stiffer than plates with both unreinforced (i.e. $t_r/t = 0$) and reinforced openings. As anticipated, openings in plates tend to decrease

the stiffness of plates. Increasing reinforcement thickness t_r increases the stiffness as well as the strength of the system. Obviously, the ultimate strengths of plates with reinforced opening can be extracted from these graphs.
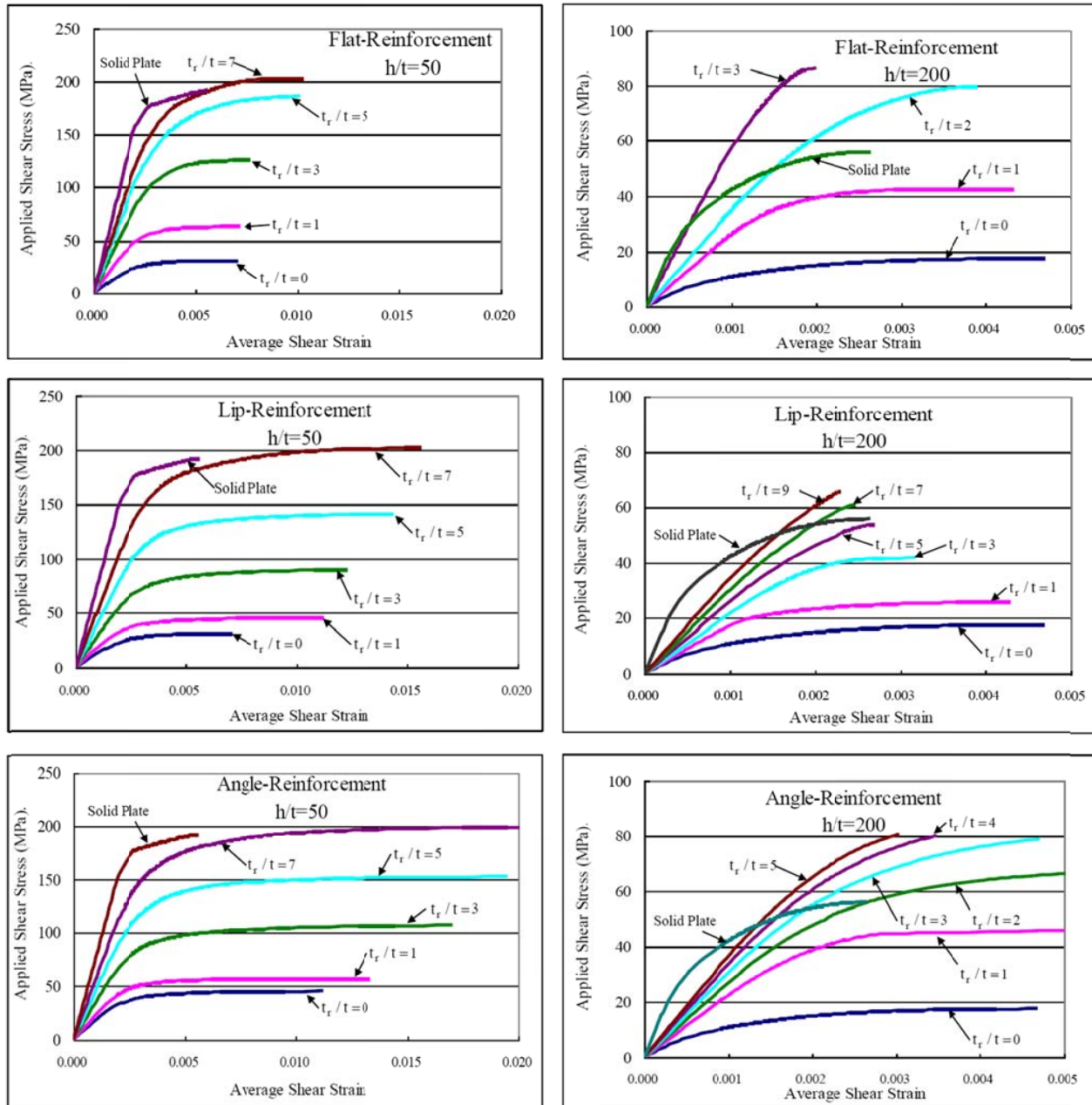

Fig. 3 Applied shear stress versus average shear strain

Table I shows the ultimate shear capacities of plates with reinforced openings obtained from the finite element analysis corresponding to h/t= 50, 100, 150, and 200. The table also shows the strengths of corresponding solid plate, ultimate shear strength of plate with unreinforced opening (row 1, t_r/t =0) and the strengths for the three reinforcement schemes

under consideration. The first column indicates the thickness of the reinforcement (t_r/t). As stated before, the 't_r/t' values were generally increased at an incremental step of one, however, in some analyses additional half steps were made in order to obtain enough data points for the later analysis. The

analyses were carried until the percentage increase in the ultimate shear strength of plates is less than 1.0%, essentially, no further increase in shear strength. At this point, the reinforcement has recaptured the loss in strength due to opening and any increase in reinforcement thickness increases the strength of opening region only, that the regions outside the opening becomes the weak-link that the member begins to fail in regions outside the opening. Fig. 4 shows the deformed shapes (magnified by 10%) of plates at the failure load levels. The figures are for plates with h/t=200 (thin plate), however, Fig. 4 (A) shows the plate with unreinforced opening, and Figs. 4 (B)-(D) are plates with adequate flat, lip and angle reinforcements, respectively, to restore the shear strength reduced by the opening. It can be seen from Fig. 4 that before reinforcements are applied, plates with 60% openings (dc/h=0.6) fail at the four corners of the openings. With adequate reinforcements, plates fail in diagonal shear buckling failure outside of the opening region. The normalized ultimate strength values may facilitate interpretation of these results. Fig. 5 shows the plots of the ultimate shear strength of plates with reinforced openings normalized by the ultimate shear strength of the corresponding solid plates versus 't_r/t' for plates with flat, lip, and the angle reinforcement schemes. For illustration purposes, only the lowest and the highest slenderness values considered in this investigation (i.e. h/t=50 and h/t=200) are shown in this figure. Note that the normalized strength of greater than 1.0 indicates that the reinforced plate has recaptured the original strength. By observing the normalized shear strengths, it can be seen from Fig. 5 that all three reinforcement schemes are capable of restoring the shear strength of plates which was compromised due to the presence of centrally located square opening. When an adequate amount of reinforcements is applied, the ultimate shear strength of a plate with a square opening can even increase beyond its original shear strength (solid plate and no openings). For example, when h/t=200, Fig. 5 shows that the ultimate shear strength of a plate with flat-reinforcement around the opening can be as high as about 1.5 times the original shear strength of a solid plate. This can be attributed to the fact that the reinforcement actually divides the plate into three panels. Two panels are on either side of the opening, and the third region is the reinforced opening. When enough reinforcements are provided the plate fails in the regions outside of the opening. Thus, the failure load of the plate is governed by the outside square panels. Chen [5] has shown in her studies that the ultimate shear strength of a plate increases with decreasing aspect ratio (a/h). Proper reinforcement scheme for an opening produces an effect similar to reduction of effective a/h of the plate, which results in an increase in the ultimate shear strength relative to the original solid plate. Furthermore, the reinforced opening edges may provide rotational restraints to the outer panels, thus, one edge of these outer panels is not simply supported, and thus experiences higher load than that of a simply supported plate. For example, when h/t=200, with an adequate amount of reinforcement ($t_r/t=3.5$), the ultimate shear strength $\tau_{ul(reinf.)}$ was 84.7MPa. The shear strength of a simply supported solid plate with

h/t=200 and a/h=1 would be $\tau_{ul(a/h = 1)}$ =81.50MPa. Thus, when enough reinforcement is applied, the ultimate shear strength of a plate with a/h=3 and reinforced opening is comparable to the ultimate shear strength of a solid plate with a/h=1.
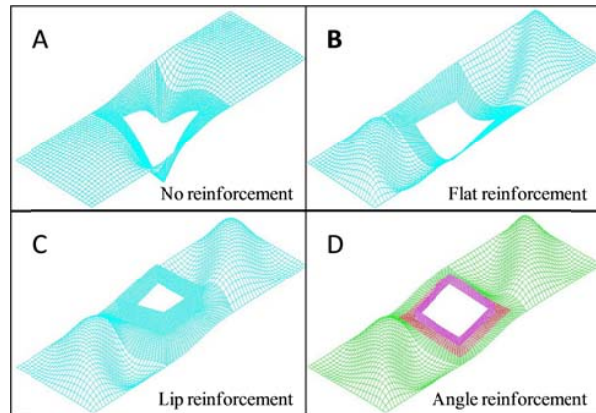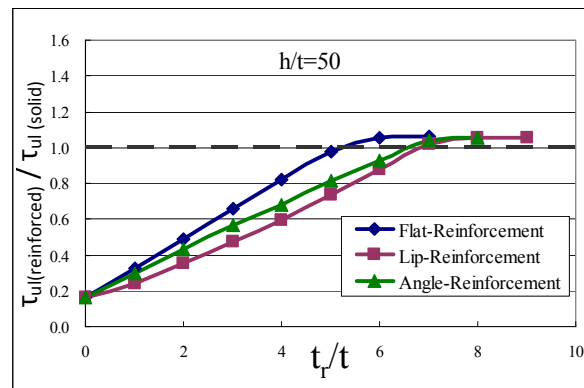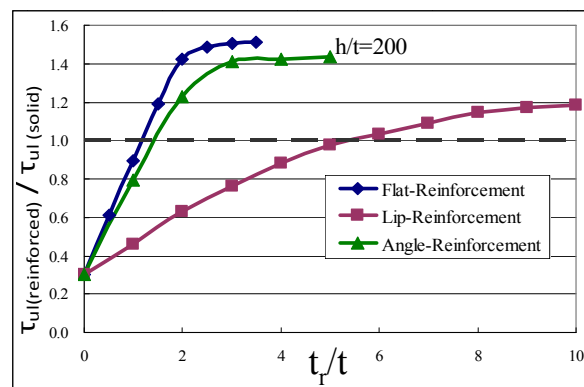

Fig. 4 Deformed shapes of plates containing sufficient opening reinforcement

(a)

(b)

Fig. 5 The impact of reinforcement thickness on the ultimate shear strength

TABLE I
ULTIMATE SHEAR STRENGTH OF REINFORCED WEB OPENINGS

t_r/t	h/t=50 (Ultimate Shear Strength of Solid Plate =191.6 MPa)			h/t=100 (Ultimate Shear Strength of Solid Plate =113.1 MPa)		
	Flat Reinforcement	Lip Reinforcement	Angle Reinforcement	Flat Reinforcement	Lip Reinforcement	Angle Reinforcement
0,0	31,3	31,3	31,3	23,6	23,6	23,6
1,0	62,9	45,5	56,8	62,9	39,1	53,9
1,5						
2,0	94,0	67,6	83,1	100,0	52,6	75,7
2,5						
3,0	125,5	90,3	108,0	128,8	68,8	96,3
3,5						
4,0	157,1	114,1	129,8	151,0	85,5	114,3
5,0	186,5	141,5	155,3	152,4	102,9	130,8
6,0	202,7	168,3	178,1		121,3	144,2
7,0	202,8	195,1	199,6		135,1	145,8
8,0		201,8	201,8		140,1	146,6
9,0		202,1	202,4		140,6	

t_r/t	h/t=150 (Ultimate Shear Strength of Solid Plate =74.5 MPa)			h/t=200 (Ultimate Shear Strength of Solid Plate =56.0 MPa)		
	Flat Reinforcement	Lip Reinforcement	Angle Reinforcement	Flat Reinforcement	Lip Reinforcement	Angle Reinforcement
0,0	19,3	19,3	19,3	17,0	17,0	17,0
1,0	58,7	32,0	51,2	50,1	25,6	44,6
1,5	77,8			66,6		
2,0	94,3	42,5	75,7	79,8	35,3	68,7
2,5	107,0			83,1		
3,0	108,1	53,9	96,3	84,1	42,8	78,8
3,5	108,9			84,7		
4,0		66,4	101,2		49,3	79,8
5,0		78,1	102,6		54,8	80,3
6,0		87,3	103,3		57,8	
7,0		91,1			61,1	
8,0		94,3			64,0	
9,0		95,0			65,7	

A. Effects of Reinforcement Thickness

Table I shows that for all three reinforcement schemes, as the thickness of the reinforcement (t_r/t) increases, the ultimate shear strength of the reinforced plate increases. From Fig. 5, it can be seen that, at the initial portion of the diagrams, the ultimate shear strength of plates with reinforced openings increases approximately linearly with increasing 't_r/t', but the increase in the ultimate shear strength reduces for higher values of 't_r/t'. Essentially, there is no strength gain beyond an optimal reinforcement thickness. From Fig. 5, it can be noted that it is easier to restore the shear capacity of slender plates by any one of the three reinforcement schemes. For example, for a plate with h/t=50 a flat-reinforcement of t_r/t=6 is needed to restore the shear capacity of the plate with opening to the shear capacity of a solid plate. However, for a plate with h/t=200 a flat-reinforcement of t_r/t=1.5 is enough to restore the shear capacity of that plate with opening to the shear capacity of a solid plate. Similar observations can be made with respect to the lip-reinforcement and the angle-reinforcement schemes.

B. Effects of Reinforcement Configuration

The total width h_r of the reinforcement, which was a fixed value of 20mm, and the length of the reinforcement, which was approximately equal to the perimeter of the opening, were the same for all three reinforcement configurations. The only variable is the thickness of the reinforcement represented by the ratio t_r/t, where t_r is the thickness of the reinforcement and t is the thickness of the plate. Thus, the effectiveness of a reinforcement scheme can be established by the comparing the required t_r/t values those would restore the shear strength of plates to their original strength (i.e. shear strength of solid plates). Fig. 5 compares the ultimate shear strength of plates having different reinforcement configurations for increasing thickness of the reinforcements (t_r/t). It was observed that for plates under consideration with 'h/t' values 50, 100, 150 and 200, the flat-reinforcement scheme required the least 't_r/t' value in order to restore the ultimate shear strength of a plate with an opening to that of a solid plate. For example, Fig. 5 corresponding to h/t=200 indicates that the flat-reinforcement scheme, the angle-reinforcement scheme, and the lip-reinforcement scheme can be used to restore the shear strength of plates with openings to that of the shear strength of solid plates using $t_r/t \approx 1.1$, $t_r/t \approx 1.3$, and $t_r/t \approx 4.3$, respectively. Similar trends were noticed for plates having h/t=50, 100 and 150. Thus, it was concluded that the flat-reinforcement scheme is the most effective scheme to reinforce a square opening in a web of a steel beam as compared to the other two reinforcement schemes. The lip-reinforcement is the least effective reinforcement configuration to restore the shear capacity of plates with square opening, and whereas the angle-reinforcement configuration falls between the flat and the lip-reinforcement configurations.

IV. CONCLUDING REMARKS

This paper considered the ultimate shear strength of plates with reinforced openings. The plates analyzed in this investigation has an aspect ratio of a/h=3, and a centrally located 60% square opening (dc/h=0.6). The four slenderness ratios (h/t) considered in this study were h/t=50, 100, 150 and 200, which cover from thick plate to the thinnest plate spectrum allowed in the AISI code [4]. Three reinforcement schemes, namely the flat-reinforcement, the lip-reinforcement and the angle-reinforcement, were applied on the plates in order to compare and evaluate the effectiveness of these three reinforcement schemes. It was shown from the study that with an adequate amount of reinforcement material, all three reinforcement schemes are capable of restoring the ultimate shear strength of a plate with a square opening to that of a solid plate. However, the flat-reinforcement is the most effective reinforcement scheme as compared to the other two reinforcement schemes.

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