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# Soliton Interaction in Multi-Core Optical Fiber: Application to WDM System

S. Arun Prakash, V. Malathi, M. S. Mani Rajan

***Abstract*****—**The analytical bright two soliton solution of the 3-coupled nonlinear Schrödinger equations with variable coefficients in birefringent optical fiber is obtained by Darboux transformation method. To the design of ultra-speed optical devices, Soliton interaction and control in birefringence fiber is investigated. Lax pair is constructed for *N* coupled NLS system through AKNS method. Using two-soliton solution, we demonstrate different interaction behaviors of solitons in birefringent fiber depending on the choice of control parameters. Our results shows that interactions of optical solitons have some specific applications such as construction of logic gates, optical computing, soliton switching, and soliton amplification in wavelength division multiplexing (WDM) system.

***Keywords*****—**Optical soliton, soliton interaction, soliton switching, WDM.

## I. Introduction

IN the context of optical fiber communications and nonlinear optics, nonlinear Schrödinger equations have been employed to describe the optical pulse propagation in nonlinear optical fiber. Since the theoretical prediction [1] and experimental observation [2], optical solitons has potential applications in the optical communication. Optical solitons ascend when the linear dispersion and nonlinear effects are exactly balanced. Optical soliton propagation in single-mode fibers is governed by the nonlinear Schrödinger (NLS) equation, which involves the GVD and self-phase modulation (SPM) [3]. However, single-mode optical fibers are not really "single-mode" but are actually bimodal because of the birefringence induced by various imperfections randomly distributed along the fiber [4]. In the picosecond regime, the governing model for the vector solitons propagation in the birefringent fibers is the coupled nonlinear Schrödinger (CNLS) system [5].

Actual observations suggest that the core medium in a real fiber is not homogeneous due to some nonuniformity factors such as the variation in the lattice parameters of the fiber and fluctuation of the fiber geometry [6]. One of the most exciting phenomena associated with solitons is their collisions. It is well known that if the solitons interact like particles cross each other unaffectedly only by a phase shift, then the collision is elastic. In addition, the physical quantities such as amplitudes and velocities are conserved. In particular, optical solitons, which exhibit the fascinating characteristics of self-guided beams, are an attractive and active area of research, because of their potential applications in many areas, including all optical switching, optical data storage, design of logic gates, and processing. To enhance the communication quality of high-bit rate and long-distance optical communication systems, soliton interaction based on the NLS-typed equations have been worked out [7], [8]. Recently, soliton collision in CNLS system has been investigated by [9]. Very recently, soliton interaction in CNLS equation with higher order effects has been performed in [10].

## II. Model

To investigate this case we consider 3-CNLSE with variable coefficients as:

$$
\begin{aligned}
& i q_{1z} + \frac{\beta(z)}{2} q_{1tt} + \gamma(z)(|q_1|^2 + |q_2|^2 + |q_3|)q_1 + i\delta(z) q_1 = 0 \\
& i q_{2z} + \frac{\beta(z)}{2} q_{2tt} + \gamma(z)(|q_1|^2 + |q_2|^2 + |q_3|)q_2 + i\delta(z) q_2 = 0 \\
& i q_{3z} + \frac{\beta(z)}{2} q_{3tt} + \gamma(z)(|q_1|^2 + |q_2|^2 + |q_3|)q_3 + i\delta(z) q_3 = 0
\end{aligned}
\tag{1}
$$

where $q_1$, $q_2$ and $q_3$ are slowly varying envelopes of three optical modes, the variables $z$ and $t$, respectively, correspond to the propagation distance and time. $\beta(z), \gamma(z), \delta(z)$ are the variable coefficients which are associated with group velocity dispersion (GVD), nonlinearity and fiber loss/ gain, respectively. In practical applications, the model is of primary interest not only for the dispersion and for nonlinear management of a soliton in inhomogeneous systems, but also to examine the soliton propagation in wavelength division multiplexing (WDM) system.

## III. Lax Pair

In this section, with symbolic computation, we will construct the Lax pair of (1) via the Ablowitz–Kaup–Newell–Segur scheme (AKNS) [11]. From the Lax pair, multi soliton solutions can be generated effectively by means of Darboux transformation.

$$\psi_t = U\psi; \qquad \psi_z = V\psi;$$
$$\psi = (\varphi_1, \varphi_2, \varphi_3)^T$$

$$U = iJ + P\ , \ V = 2i\lambda^2 J + \lambda P + Q$$

S. Arun Prakash and V.Malathy are with the Electrical Department, Anna University, Madurai Region, Ramanathapuram 623513, Tamilnadu, India (phone: 9790328649, +91 9894068686; e-mail: arunprakashmadurai@ gmail.com vmeee@autmdu.ac.in).

M. S. Mani Rajan is with the Physics Department, Anna University, Ramanathapuram 623513, Tamilnadu, India (phone: +91 9940740238; e-mail: senthilmanirajanofc@ gmail.com).

$$J=\begin{pmatrix}-1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix},\quad P=\sqrt{\frac{\gamma}{2\beta}}\begin{pmatrix}0&q_1&q_2&q_3\\-q_1^*&0&0&0\\-q_2^*&0&0&0\\-q_3^*&0&0&0\end{pmatrix} \tag{2}$$

$$Q=\frac{i}{2}\begin{pmatrix}\gamma(z)\sum_{P=1}^{3}|q_P|^2 & \sqrt{2\gamma(z)\beta(z)}\,q_{1t} & \sqrt{2\gamma(z)\beta(z)}\,q_{2t} & \sqrt{2\gamma(z)\beta(z)}\,q_{3t}\\ \sqrt{2\gamma(z)\beta(z)}\,q_{1t}^* & -\gamma(z)|q_1|^2 & -\gamma(z)q_2q_1^* & -\gamma(z)q_3q_1^*\\ \sqrt{2\gamma(z)\beta(z)}\,q_{2t}^* & -\gamma(z)q_1q_2^* & -\gamma(z)|q_2|^2 & -\gamma(z)q_3q_2^*\\ \sqrt{2\gamma(z)\beta(z)}\,q_{3t}^* & -\gamma(z)q_1q_3^* & -\gamma(z)q_2q_3^* & -\gamma(z)|q_3|^2\end{pmatrix}$$

In the above set of equations (2), * indicates complex conjugate, superscript $T$ indicates transposed matrix and the subscripts $z$ and $t$ denote derivatives with respect to $z$ and $t$. From the compatibility condition, (1) can be derived. The compatibility condition is given;

$$U_z-V_t+[U,V]=0 \tag{3}$$

## IV. Darboux Transformation

It is well known that the Darboux transformation method [12] is a straightforward and useful tool for generating the soliton solutions by symbolic computation.

$$\psi=D\psi=(\lambda I_{4X4}-S)\psi$$

where $D$ is called Darboux matrix.

$$S=H\,\Lambda\,H^{-1} \tag{4}$$

where $H$ is a non-singular matrix

$$U_1=\lambda\,J+P_1$$

$$P_1=P+J\,S-S\,J$$

To construct $S$ matrix, we consider $H$ and $\Lambda$ as,

$$H=\begin{pmatrix}\phi_1&-\phi_2^*&-\phi_3^*&-\phi_4^*\\\phi_2&\phi_1^*&0&0\\\phi_3&0&\phi_1^*&0\\\phi_4&0&0&\phi_1^*\end{pmatrix}$$

$$\Lambda=\begin{pmatrix}-i\lambda&0&0&0\\0&-i\lambda^*&0&0\\0&0&-i\lambda^*&0\\0&0&0&-i\lambda^*\end{pmatrix}$$

$$S=\begin{pmatrix}-i\lambda^*-\frac{i(\lambda-\lambda^*)\phi_1\phi_1^*}{\Delta} & -\frac{i(\lambda-\lambda^*)\phi_1\phi_2^*}{\Delta} & -\frac{i(\lambda-\lambda^*)\phi_1\phi_3^*}{\Delta} & -\frac{i(\lambda-\lambda^*)\phi_1\phi_4^*}{\Delta}\\ -\frac{i(\lambda-\lambda^*)\phi_2\phi_1^*}{\Delta} & -i\lambda^*-\frac{i(\lambda-\lambda^*)\phi_2\phi_2^*}{\Delta} & -\frac{i(\lambda-\lambda^*)\phi_2\phi_3^*}{\Delta} & -\frac{i(\lambda-\lambda^*)\phi_2\phi_4^*}{\Delta}\\ -\frac{i(\lambda-\lambda^*)\phi_3\phi_1^*}{\Delta} & -\frac{i(\lambda-\lambda^*)\phi_3\phi_2^*}{\Delta} & -i\lambda^*-\frac{i(\lambda-\lambda^*)\phi_3\phi_3^*}{\Delta} & -\frac{i(\lambda-\lambda^*)\phi_3\phi_4^*}{\Delta}\\ -\frac{i(\lambda-\lambda^*)\phi_4\phi_1^*}{\Delta} & -\frac{i(\lambda-\lambda^*)\phi_4\phi_2^*}{\Delta} & -\frac{i(\lambda-\lambda^*)\phi_4\phi_3^*}{\Delta} & -i\lambda^*-\frac{i(\lambda-\lambda^*)\phi_4\phi_4^*}{\Delta}\end{pmatrix}$$

$$\Delta=|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2$$

$$q_2(1)=\sqrt{\frac{\gamma(z)}{\beta(z)}}(-\sqrt{2}\xi_1\,c_1\exp(-2i\,A)Sech(2B_1)-2\sqrt{2}\,c_2\,T) \tag{5}$$

$$q_2(2)=\sqrt{\frac{\gamma(z)}{\beta(z)}}(-\sqrt{2}\xi_1\,c_1\exp(-2i\,A)Sech(2B_1)-2\sqrt{2}\,c_2\,T)$$

$$q_2(3)=\sqrt{\frac{\gamma(z)}{\beta(z)}}(-\sqrt{2}\xi_1\,c_1\exp(-2i\,A)Sech(2B_1)-2\sqrt{2}\,c_2\,T)$$

$$T=\frac{U}{V}$$

$$V=a_1\,2\,Cosh(2\,B_2)-2\,a_3\,Sech(2\,B_1)[2\,Cosh(2\,y)+2\,Cos(2\,x)]+ \\ 2\,\xi_1^2\,Sech^2(2\,B_1)\,2\,Cosh(2\,B_1)[2\,Cosh(2\,y)+2\cos(2\,x)]$$

$$U=a_1\exp(-2i\,A_2)+ \\ i\,a_2\,Sech(2B_1)[\exp(-2i\,A_1)\,2\,Sinh(2B_2)-\exp(-2i\,A_2)\,2Sinh(2B_1)]- \\ a_3Sech(2B_1)[\exp(-2i\,A_1)\,2\,Cosh(2B_2)+\exp(-2i\,A_2)\,2Cosh(2B_1)]+ \\ \xi_1^2\,Sech^2(2\,B_1)\exp(-2iA_1)[2\,Cosh(2y)+2Cos(2x)]$$

$$a_1=(\eta_2-\eta_1)^2+(\xi_2+\xi_1)^2$$

$$a_2=\xi_1\,(\eta_2-\eta_1)$$

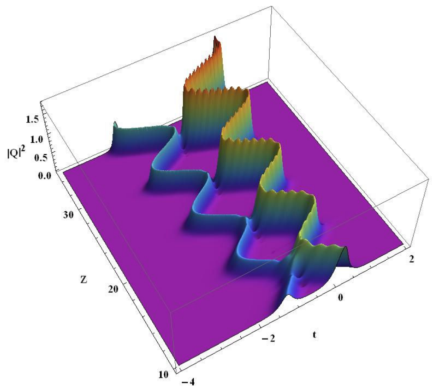
$$a_3=\xi_1\,(\xi_2+\xi_1)$$

Soliton interaction can be investigated by manipulating the control parameters related with soliton dynamics. Here, using multi soliton solution, soliton interaction to be investigated for a special case. On the *other* hand, to investigate the soliton solution of (1), we can properly select the different forms of variable coefficients such as group velocity dispersion and nonlinearity due to the influence of the variable coefficient functions on the soliton dynamics.

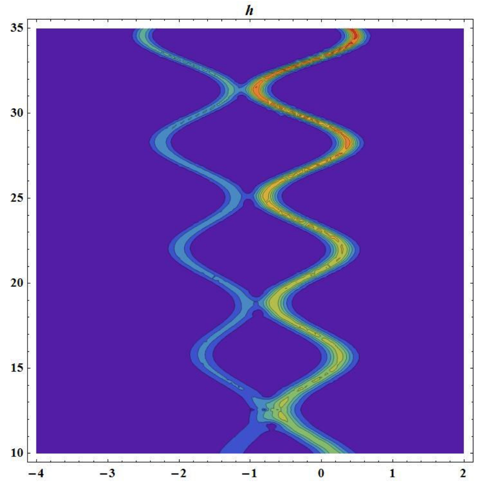
## V. Result and Discussion

It is very meaningful to know the effects of each operation on solitons, for the knowledge is very helpful for soliton management in experiment. To investigate this case, let us consider a fiber, whose dispersion and nonlinearity varies periodically along its length as given in [13], [14].

$$D(z)=\cos(\sigma\,z) \\ R(z)=\cos(\sigma\,z) \tag{6}$$

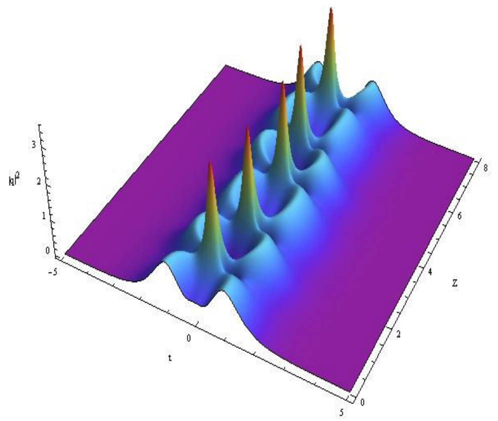



(a)

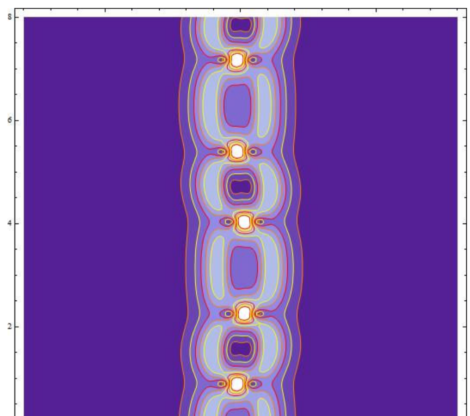



**(b)**

Fig. 1 (a) Intensity profile of two solitons solutions. The parameters are $\eta_1$= -0.4, $\eta$2=-0.4, $\xi_1$=-0.25, $\xi_2$=0.4, σ=0.05 (b) Corresponding contour plot




(a)

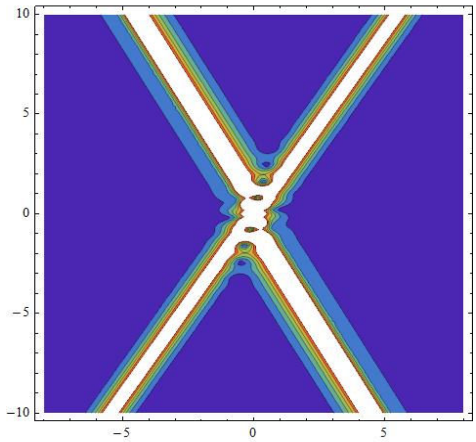


(b)

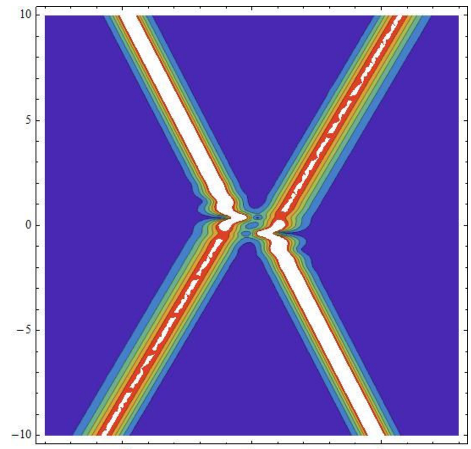
Fig. 2 (a) Intensity profile of two solitons solutions. The parameters are $\eta_1$= -0.03, $\eta_2$=-0.04, $\xi_1$=-1, $\xi_2$=1.5 (b) Corresponding contour plot

For the choice of (6), we observed that the snake like motion of the two solitons during the course of propagation without any interaction which is essential for optical communication system. This means that solitons are transmit stably and propagate along the optical fibers with a constant separation among them as depicted in Fig. 1 (a). These properties are significant for increasing the bit rate in optical fiber communication.

During the propagation, amplitude and width are invariant. On the contrary, when the amplitude and velocity parameters are manipulated properly, the solitons interact with each other periodically as shown in Fig. 2 (a). This means that the two solitons attract and repel each other alternately. Such a special structure is known as the bounded states of two solitons. It can be noted that two solitons collide elastically without perturbation, that is to say, after collision the two soliton maintain their original shapes and velocities with only a phase shift at the moment of collision. This is also one of the important properties of solitons.




(a)




(b)

Fig. 3 (a) Contour plot of two soliton solutions. The parameters are $\eta_1$= -0.08, $\eta_2$=0.09, $\xi_1$=-1, $\xi_2$=-1.5. (b) Contour plot of two soliton solutions. The parameters are $\eta_1$= -0.08, $\eta_2$=0.07, $\xi_1$=-0.09, $\xi_2$=1.4

If we set $\beta=\gamma=1$, then $\delta$ becomes zero in (1). For this choice, (1) reduced to 3-coupled nonlinear Schrödinger equations with constant coefficients.

By comparing Figs. 3 (a) and (b), we may conclude that stirring of phase shift in solitons after the collision can be controlled via manipulating the values of control parameters. From Fig. 3 (a), one can conclude that out of two solitons, one soliton will undergo the phase shift after the collision while another one is propagate without phase shift. On the other hand, both solitons are experienced with phase shifts after the interaction as shown in Fig. 3 (b). In fact, these interactions with phase shifting behaviors can be advisably controlled by taking the different values of physical parameters.

## VI. Conclusion

In order to understand the soliton propagation in WDM system of optical fiber communication, *3*-coupled nonlinear Schrödinger equation which governs the optical soliton pulse propagation in multi-core fibers is investigated. Using constructed Lax pair, we elucidate the generalized *3*-coupled nonlinear Schrödinger equation analytically through Darboux transformation method. Analytical investigations have been made to understand the dynamics of solitons under the variation of nonlinear parameter and modulation of dispersion effect. Some graphical illustrations in Figs. 1–3 may provide useful insight into the description of optical solitons propagation in the inhomogeneous multi-core fibers with a specific dispersion profile and various interaction scenarios for homogeneous fiber. It has also been obtained that the combined effects of controlling both the group velocity dispersion and the nonlinearity distribution can restrict interactions between the neighboring solitons. Finally, we demonstrate the various interaction dynamics of bright solitons for constant coefficients. The fascinating collision properties with completely or partially switching of energy between the solitons open possibilities for future applications in the design of logical gates, fiber directional couplers, quantum information processors and soliton switching devices.

## References


[1] A. Hasegawa, F. Tappert, Appl. Phys. Lett. 23 (1973)142– 144.
[2] L.F. Mollenauer, R.H. Stolen, J.P. Gordon, Phys. Rev. Lett. 45 (1980)1095–1098.
[3] G.P. Agrawal, *Nonlinear Fiber Optics*; Academic Press: New York, 1995.
[4] I. P. Kaminow, IEEE J. Quantum Electron. 17 (1981) 15.
[5] C. R. Menyuk, Opt. Lett. 12 (1987) 614.
[6] F. Abdullaeev, Theory of Solitons in Inhomogeneous Media, Wiley, New York, 1994
[7] R. Ganapathy, K. Porsezian, A. Hasegawa, V.N. Serkin, IEEE J. Quantum Electron. 44 (2008) 383–390.
[8] Y.S. Kivshar, G.P. Agrawal, Optical Solitons: From Fibers to Photonic Crystals. Academic Press, San Diego (2003)
[9] M. Wang, W.R. Shan, X. Lü, Y.S. Xue, Z.Q. Lin, B. Tian, Appl. Math. Comput. 219 (2013) 11258–11264.
[10] D.S. Wang, S. Yin, Y. Tian, Y. Liu, Appl. Math. Comput. 229 (2014) 296–309.
[11] M.J. Ablowitz, D.J. Kaup, A.C. Newell, H. Segur, Phys. Rev. Lett. 1973, 31, 125–127.
[12] V. Matveev, M. Salle, Darboux Transformation and Solitons, Springer, Berlin, 1991.
[13] M.S. Mani Rajan, A. Mahalingam, A. Uthayakumar, K. Porsezian, Commun. Nonlinear Sci. Numer. Simul. 18 (2013) 1410–1432.
[14] M.S. Mani Rajan, A. Mahalingam, J. Math. Phys. 54 (2013) 043514.


**S. Arun Prakash** received his Bachelor of Engineering in Electrical and Electronics from Kamaraj College of Engineering Virudhunagar and Master of Engineering in Power Systems from Velammal College of Engineering, Chennai. Tamilnadu, India.

He is a member of Institution of Engineers (India) and Life Member of the Indian Society for Technical Education. Currently working as Assistant Professor in the Department of Electrical and Electronics Engineering, Anna University, University College of Engineering Ramanathapuram, Tamilnadu, India. His area of research is intelligent techniques application to power system protection.

**V. Malathi** is working as professor in the department of Electrical and Electronics Engineering in Anna University Regional Centre, Madurai. She completed her Bachelor degree in College of Engineering Guindy and her Masters in Thiyagaraja College of Engg, Madurai. She completed her Ph.D in Anna University Chennai, her areas of interest are intelligent techniques, and its applications, Smart Grid, FPGA based power system and Automation.

She is a member of Institution of Engineers (India) and Life Member of the Indian Society for Technical Education.

She is working as Professor in Department of Electrical and Electronics Engineering, Anna University, Regional Centre Madurai, Tamilnadu, India. Her area of research is intelligent techniques application to power system protection.

**M. S. Mani Rajan** received his B.Sc and M.Sc degree in Physics from Madurai Kamaraj University, Madurai, Tamilnadu, India. He received M.Phil from Bharathidasan University, Tiruchirappalli, India. He received Ph.D in from Anna University, Chennai. He published over 12 papers in the field of soliton based optical fiber communication systems in highly reputed international journals and he published more than 20 papers in various international conferences.

He is a Life member of Indian Physics Association (IPA), Member in Indian Laser Association (ILA) and Photonics society of India (PSI).

He is currently working as Assistant Professor in the Department of Physics, Anna University, University College of Engineering Ramanathapuram, Tamilnadu, India. His area of research is soliton propagation in various nonlinear media, photonic crystal fiber and erbium doped fiber. His most significant contribution has been made in the area of pulse compression and amplification of optical solitons via nonlinear tunneling. He proposed many novel methods for generation of ultrashort pulses and optical solitons for different applications.