International Journal of Chemical, Materials and Biomolecular Sciences
ISSN: 2415-6620
Vol:9, No:11, 2015

Effects of Roughness Elements on Heat Transfer during Natural Convection

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***Abstract*—**The present study focused on the investigation of the effects of roughness elements on heat transfer during natural convection in a rectangular cavity using numerical technique. Roughness elements were introduced on the bottom hot wall with a normalized amplitude (A*/H) of 0.1. Thermal and hydrodynamic behaviors were studied using computational method based on Lattice Boltzmann method (LBM). Numerical studies were performed for a laminar flow in the range of Rayleigh number (*Ra*) from 10^3 to 10^6 for a rectangular cavity of aspect ratio (L/H) 2.0 with a fluid of Prandtl number (*Pr*) 1.0. The presence of the sinusoidal roughness elements caused a minimum to maximum decrease in the heat transfer as 7% to 17% respectively compared to smooth enclosure. The results are presented for mean Nusselt number (*Nu*), isotherms and streamlines.

***Keywords*—**Natural convection, Rayleigh number, surface roughness, Nusselt number, Lattice Boltzmann Method.

I. Introduction

NATURAL convection induced by buoyancy force is a wide ranging heat transfer phenomenon from near to far environment [1]. Its importance is further pronounced due to its incorporation of passive safety systems in Small Modular Reactors (SMRs) and advanced generations of nuclear reactors, where it is being used for heat removal systems during normal and shutdown phases [2], [3]. It has a range of applications: heating and cooling of buildings, solar panels, electronic devices, growth of crystal during solidification of liquids, and in nuclear industry like nuclear reactor design and safety, reactor insulation, decay heat removal and cooling of radioactive waste containers etc.[4]

Buoyancy induced flows are studied in two types of enclosures smooth and partitioned. Partitioned enclosure heat transfer and transport phenomenon is quite different as compared to that of smooth. Whereas, in case of partitioned enclosure, heat transfer phenomena depends on nature and size of partitions. Some general examples of such cases are rooms, buildings with windows, solar energy collectors etc. [1]. Apart from smooth enclosures, extensive study has been performed on partial enclosures both computationally [4]-[6] and experimentally [1], [7], [8]. Bajorek [7] in his experimental study reported a decrease in the heat transfer about 10-21% due to the presence of adiabatic rectangular roughness elements both on bottom and top walls with side walls isothermal. Amin [9] described the effects of multiple rectangular roughness elements on heat transfer concluding that it causes a decrease and also increase in the rectangular enclosures. Das [10] studied the influence of sinusoidal bottom wall, its frequency and aspect ratio on the heat transfer using finite volume method in a rectangular enclosure.

Hasan et al. [11] performed numerical study of heat transfer and fluid flow due to the presence of wavy wall at uniform heat flux of a square enclosure and reported a decrease in the heat transfer. Shakerin et al. [8] in his study observed an increase in the heat transfer due to the presence of roughness element on vertical isothermal wall with adiabatic horizontal walls. Oztop [12] in a recent study reported that an isothermal partition on adiabatic horizontal wall with side walls as isothermal causes a reduction in heat transfer. Shaw et al. [5] observed that an adiabatic partition on adiabatic horizontal wall with side walls as isothermal also causes a decrease in heat transfer by using cubic spline method. Yucel et al. [4] mentioned some important studies related to partitioned enclosures.

The purpose of the present work is to study the effects of sinusoidal roughness elements on heat transfer in a rectangular enclosure. Sinusoidal roughness elements are located on hot wall instead of making hot wall as sinusoidal as was done in previous studies. The sinusoidal roughness elements are at same boundary condition as the corresponding wall. An arbitrary amplitude of 0.1 of 08 roughness elements was chosen for this study. The computational work has been performed by using state of the art computational method based on on Bhatnagar-Gross-Krook (BGK) model of Lattice Boltzmann Method. Simulations have been performed for a fluid of Pr number 1.0 for a laminar natural convection in range of Rayleigh Number (Ra) 10^3 to 10^6. The paper is organized as, in Section II, a brief introduction is provided for computational method, Section III presented geometry for present study and validation of computational code with previous studies, results along with related discussion are presented in Section IV, and finally, conclusion has been drawn based on the simulation results.

II. Description of Work

Lattice Boltzmann method has emerged as an alternative to traditional numerical methods based on finite volume and finite difference techniques [13]. LBM has a significant capability to simulate single and two phase flows and heat transfer phenomenon, steady as well as transient, buoyancy induced flows, condensation and evaporation in complex geometries [14]. This method based on kinetic theory of gas, was first introduced in 1988 by McNamara et al. to overcome problems associated with cellular gas automata [15]. Since

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that, it has been increasingly used for flow simulation based on particle distribution function. Unlike traditional numerical methods, LBM solves dynamics of hypothetical particle based on Boltzmann equation. LBM has many advantages like its ability to simulate complex and porous geometries which does not need special treatments, ease of algorithm implementation, and no need to solve Laplace equation at every iteration [15]. Although LBM has proved to be a promising tool for computational study of fluid and heat transfer phenomenon but still has some limitations because of its numerical instability at highly turbulent flows, but studies are in progress to overcome and enhance LBM stability for these flows. A detailed study regarding all models for LBM to improve its stability for turbulent flows has been explained in the [16].

Computational studies to investigate the effects of sinusoidal roughness elements on natural convection were performed by using computational algorithm based on single relaxation time Bhatnagr-Gross and Krook (BGK) model of LBM. Two-dimensional simulations for a geometry shown in Fig. 1 with corresponding boundary conditions were carried out in a laminar flow region for a Newtonian fluid of Pr number 1.0. The range of Ra number was investigated from 10^3 to 10^6. The dimensionless number called average Nusselt number (Nu), Prandtl number (Pr) and Rayleigh number (Ra) were calculated by using following relations. The average Nu was calculated along vertical walls and through entire domain of the fluid.

$$Nu_{av} = 1 + \frac{< U.\theta > H}{k\Delta\theta} \tag{1}$$

$$Nu_{av} = \frac{H}{L}\int_0^L \left(\frac{\partial\theta}{\partial x}\right) dy \tag{2}$$

$$Ra_L = \frac{g\beta\Delta T H^3}{\upsilon\alpha} \tag{3}$$

$$P_r = \frac{\upsilon}{\alpha} \tag{4}$$

Lattice Boltzmann method which was originated from lattice gas automata has a proven ability to be an alternative to traditional numerical schemes based on finite volume, finite difference, and finite elements methods [16], [17]. Main advantages of LBM are; easy to make algorithm, easy to treat complex boundaries, local computing, and no solution of Laplace equation at every time step [14]. LBM was first introduced in 1980s, and has demonstrated significant performance to simulate single and multiphase flow, condensation and evaporation, and buoyancy induced flows with complex geometries. The fundamental equation which simulates Navier-Stokes and energy equations based on hypothetical particle given by Boltzmann equation (6);

$$f_i(x + e_i\Delta t, t + \Delta t) = f_i(x,t) - \frac{\left(f_i(x,t) - f_i^{eq}(x,t)\right)}{\tau} + \Delta t F_{ext} \tag{5}$$

'F_{ext}' is external force and 'τ' is relaxation time. In the present study, nine velocity (D2Q9) and five temperature (D2Q5) directions were considered in two-dimensional study of the natural convection flow and heat transfer. Boussinesq approximations were used to incorporate the effects of buoyancy in external force term. All other properties were considered constant. The values of relaxation factors for Navier-Stokes and energy equations are calculated using kinematic viscosity and thermal diffusivity.

Proper boundary conditions are essential to have accurate numerical results and to achieve fast convergence. For velocities on wall, bounce back boundary conditions were implemented on all walls and solid nodes present in the cavity. The isothermal boundary conditions were implemented on hot and cold walls respectively as illustrated by Sukop and Thorne [14]. The horizontal walls were considered adiabatic or insulated.

Relaxation times 'τ' for momentum and energy equations are given by following equations respectively.

$$\boldsymbol{\tau_f} = \left(\boldsymbol{3\upsilon} + \tfrac{1}{2}\right) \tag{6}$$

$$\boldsymbol{\tau_{energy}} = \left(\boldsymbol{3\alpha} + \tfrac{1}{2}\right) \tag{7}$$

'α' is thermal diffusivity and 'υ' is kinematic viscosity of fluid. [14] A detailed study and respective recent developments in the LBM models are explained in reference [14]-[16].

III. Benchmarking

This part is divided into two parts. In the first part, grid independence study was carried out for rectangular cavity as shown in Fig. 2. In the second part, code validation is performed by comparing present results with previous studies. In the present study, the Boussinesq approximations are used in order to solve Navier-Stokes equation with buoyancy as external force. Density of fluid is assumed as constant in continuity equation whereas, it varies with temperature in momentum equation [14],[15].

$$\rho - \rho_\infty = \rho_\infty \beta \Delta T \tag{8}$$

and force term introduced in velocity term is calculated as

$$V = V + g\beta\tau_f\left(T - T_{ref}\right) \tag{9}$$

T_{ref} is mean temperature of cold and hot walls.

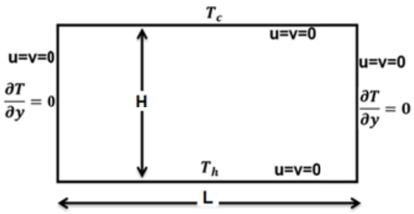

Fig. 1 Smooth rectangular cavity with aspect ratio 2.0

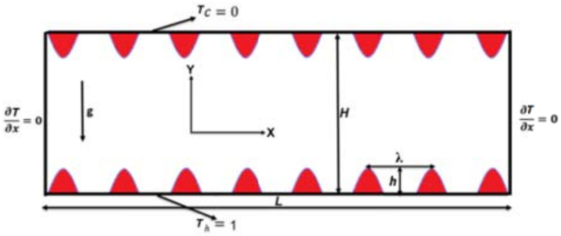

Fig. 2 Rough rectangular cavity with aspect ratio 2.0 in the presence of the roughness elements

In order to verify grid independence, simulations were run for *Ra* number of 10^4 on three different grid sizes as shown in the Table I. The comparison of results with [19] shows that an increase in the number of grid points enhances accuracy of results. A comparison shows a maximum percentage error for fine grid as 0.06%. This comparison proved that results of code are mesh independent and hence for better accuracy fine grid is utilized in further studies.

TABLE I
MESH INDEPENDENCE RESULTS

Ra	H	Present	Hollande	% Err
	100	2.3707	2.3911	0.8532
10^4	200	2.3835	2.3911	0.3178
	300	2.3896	2.3911	0.0627

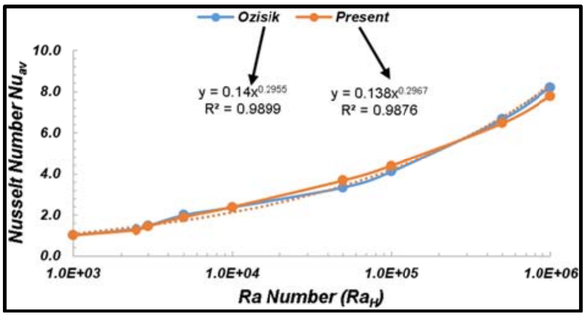

Fig. 3 Validation of present results of average Nu with previous studies of Ozisik [18]

In Fig. , average *Nu* values are plotted against [18]. Present results are in good agreement up to value of Ra equal to $5x10^5$ and deviate slightly as is clear from power law relation of both results. Also, the slight deviation at very high Ra number is in accordance with previous studies reported by [20], [21], as LBM has lesser heat transfer. Another validation was also performed with a square cavity of aspect ratio (H/L) 1.0 with side walls as isothermal and horizontal walls as adiabatic. The results are compared with benchmark solution of [22]. The results shown in Table II are in excellent agreement with previous studies.

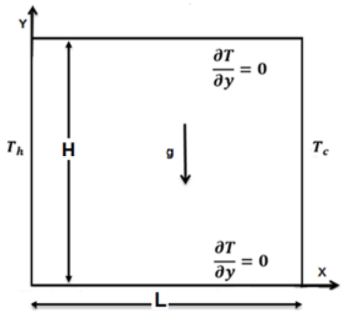

Fig. 4 Square cavity with side wall heating

TABLE II
VALIDATION OF PRESENT RESULTS OF AVERAGE NU WITH [22]

Ra	Present	Davis	% Error
10^3	1.1128	1.1180	0.47
10^4	2.2381	2.2430	0.22
10^5	4.5198	4.5190	0.02
10^6	8.8021	8.8000	0.02

A comparison study was performed for grid independence and for a range of Ra number with previous benchmark solution. Based on the comparison above, numerical results of present computational code can be considered as mesh independent and reliable within a good accuracy.

Simulations for rectangular cavity with the roughness present on the hot wall were performed up to Ra number 10^6. All simulations were carried out using Intel (R) core (TM) i7-4820k CPU@3.7GHz with 32GB RAM.

IV. RESULTS AND DISCUSSION

Buoyancy induced natural convection in smooth and partial enclosures heated from side and bottom has been studied for long in the past, because of its diverse applications. Besides enclosures heated from side, enclosures heated at bottom are considered inherently unstable [1], [23]. In the present study, natural convection flow of Newtonian fluid in the rectangular geometry is assumed as steady, two dimensional, and laminar. The range of simulation for Ra is from 10^3 to 10^6. The Boussinesq approximations are used to apply external force with negligible viscous dissipation [15]. Numerical simulations were performed for smooth as well as rough enclosure in order to make a comparison (results for smooth cavity is not presented because of space restrictions). Boundary conditions are illustrated in Figs. 1 and 2. Flow behavior inside the rough enclosure is presented in the form of isotherms and streamlines, and a quantitative comparison is made for heat transfer in form of Nu for both enclosures. Amount of heat transferred from smooth and rough hot wall in both cases is calculated using (1).

Some previous studies related to rough rectangular cavities are worth mentioning before present results are discussed.

International Journal of Chemical, Materials and Biomolecular Sciences
ISSN: 2415-6620
Vol:9, No:11, 2015

Amin [6], [23] performed numerical experiments of such a cavity with rectangular roughness elements on hot wall with different boundary conditions and aspect ratio. He observed that up to an amplitude of roughness element of 0.125, heat transfer is reduced and onset of convection is delayed. A comparison of average Nu of both cavities shows that amount of heat transfer in rough cavity is always smaller than smooth as shown in Fig. 5. The decrease in Nu is smaller in the range of Ra 2.5×10^3 to 10^4 but it increases almost linearly after that. So, there is a direct relationship of decrease in heat transfer to Ra number with amplitude of roughness element of 0.1. The main difference between present and [23] is shape of roughness elements and period, but results for average heat transfer are not too different. The reason behind this decrease may be due to decrease in amount of the fluid, because of incorporation of roughness elements. Other reason for degradation of heat transfer in cavity may be due to deceleration of fluid caused by presence of roughness elements [23].

Also, at lower Ra up to 10^4, amount of heat transfer decrement is smaller than for Ra $>10^4$. This shows that roughness may be delaying the onset of convection in the enclosure as compare to smooth. Therefore, heat transfer is dominated by the conduction for up to larger Ra values and hence cause degradation in heat transfer [23].

The natural convection phenomenon and flow behavior have been studied and explained extensively for smooth enclosures [17]. In case of side wall heating in a rectangular cavity with insulated bottom and top walls, fluid flows upward along hot wall and downward along cold wall leaving a central region known as 'core'. But in case of bottom heating and insulated vertical walls, behavior of fluid is different in the central region, which plays an important role as Ra increases. Fluid flows upward in the central region and downward alongside adiabatic walls. This become clearer not only through streamlines but by isothermal behavior of fluid as Ra increases shown in Figs. 6-11. Heat transfer phenomenon is dominated by pure conduction at least up to Ra ~ 2000, and Nu remains unity. This was also experimentally verified by [24]. As Ra increases, isotherms stratification increases and convection starts shrinking towards hot and cold walls [17]. Moreover, flows remain bi-cellular through whole range of Ra numbers during smooth as well as rough cavity up to 10^6.

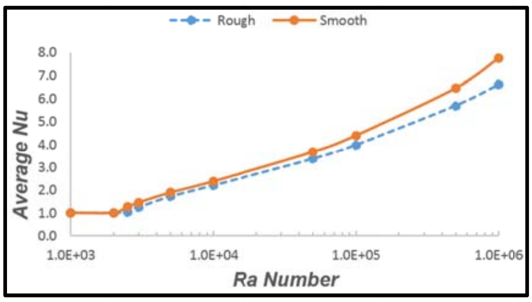

Fig. 5 Average values of Nu for smooth and rough cavity

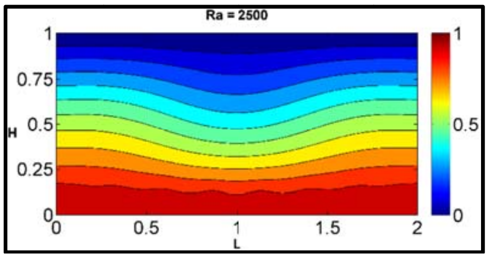

Fig. 6 Isotherms for rough rectangular cavity at Ra - 2500

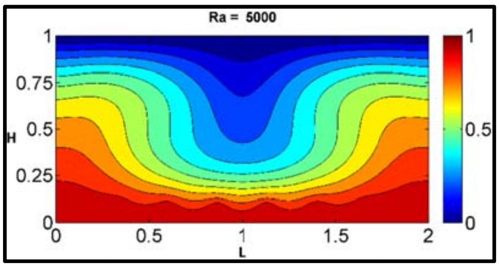

Fig. 7 Isotherms for rough rectangular cavity at Ra - 5000

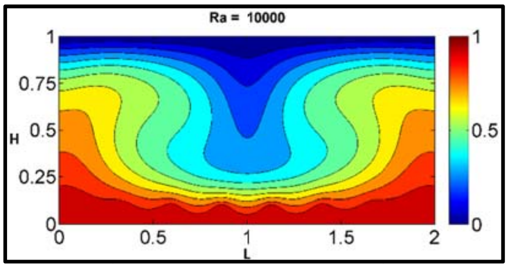

Fig. 8 Isotherms for rough rectangular cavity at Ra - 10000

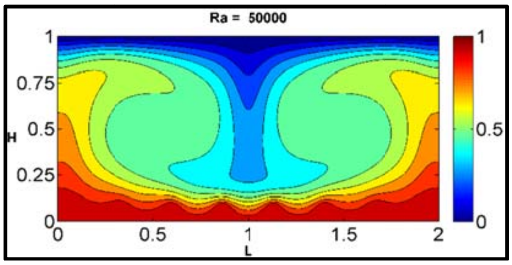

Fig. 9 Isotherms for rough rectangular cavity at Ra - 50000

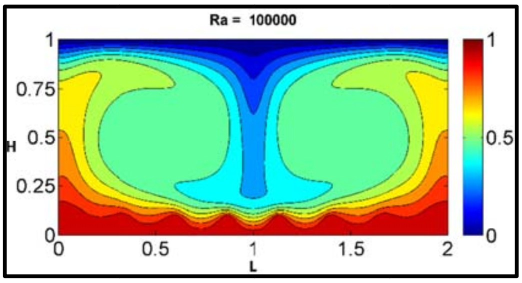

Fig. 10 Isotherms for rough rectangular cavity at Ra - 100000

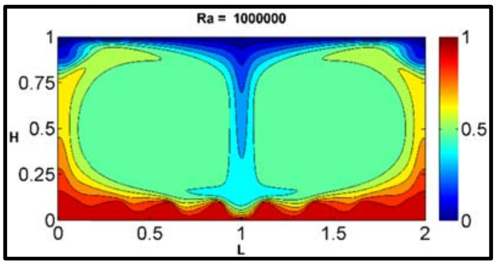

Fig. 11 Isotherms for rough rectangular cavity at Ra - 1000000

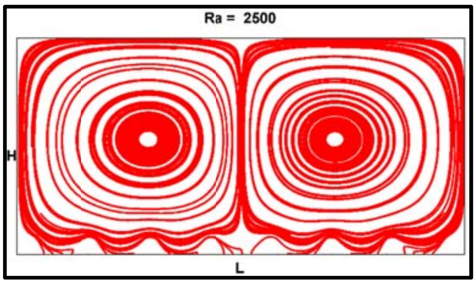

Fig. 12 Streamlines for rough rectangular cavity at Ra - 2500

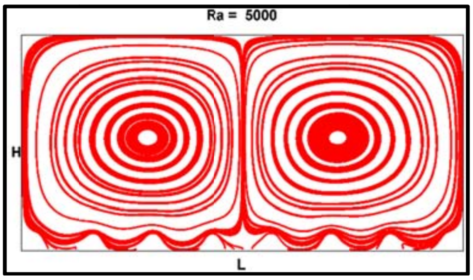

Fig. 13 Streamlines for rough rectangular cavity at Ra - 5000

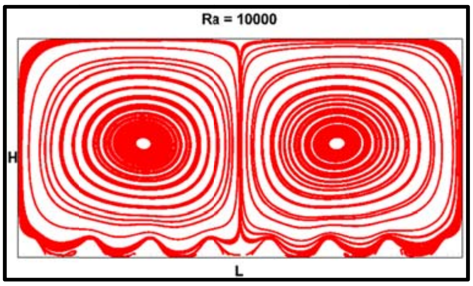

Fig. 14 Streamlines for rough rectangular cavity at Ra - 10000

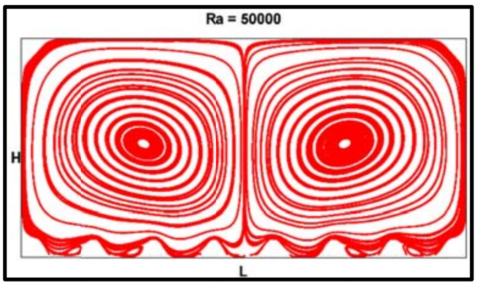

Fig. 15 Streamlines for rough rectangular cavity at Ra - 50000

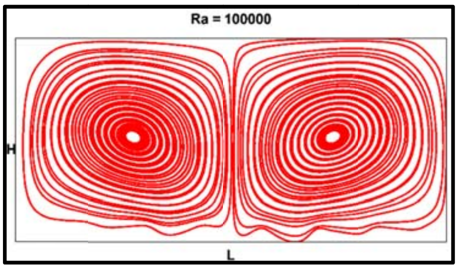

Fig. 16 Streamlines for rough rectangular cavity at Ra - 100000

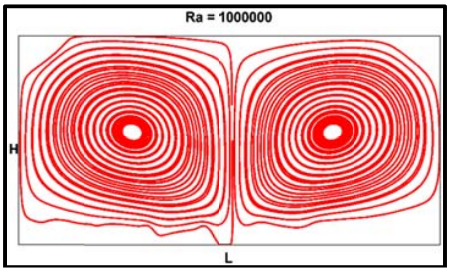

Fig. 17 Streamlines for rough rectangular cavity at Ra - 1000000

V. CONCLUSION

Numerical study was carried out for a rough rectangular cavity of aspect ratio 2.0 in the presence of roughness elements heated at the bottom. The roughness was introduced in the form of sinusoidal shape elements of amplitude 0.1. Simulations were performed for Newtonian fluid of *Pr* equal to 1.0 for a range of *Ra* 10^3 to 10^6. Behavior of streamlines and isotherms of rough cavity was not too different as compared to smooth cavity. The amount of average heat transfer was degraded in the presence of the sinusoidal roughness elements. A maximum to minimum decrease in *Nu* was observed to be 17% to 7% respectively.

NOMENCLATURE

lu	Lattice unit
ρ	Density
β	Thermal coefficient
T_w	Wall or surface temperature
T_{ref}	Reference or Mean temperature
Δt	Time Step
υ	Kinematic viscosity
α	Thermal diffusivity
f	Density Distribution function

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