Study of the Electromagnetic Resonances of a Cavity with an Aperture Using Numerical Method and Equivalent Circuit Method

Ming-Chu Yin, Ping-An Du

Abstract—The shielding ability of a shielding cavity with an aperture will be greatly degraded at resonance frequencies, and the resonance modes and frequencies are affected by aperture resonances and aperture-cavity coupling, which are closely related with aperture sizes. The equivalent circuit method and numerical method of Transmission Line Matrix (TLM) are used to analyze the effects of aperture resonances and aperture-cavity coupling on the electromagnetic resonances of a cavity with an aperture in this paper. Both analytical and numerical results show that the resonance modes of a shielding cavity with an aperture consist of cavity resonance modes and aperture resonance modes, and the resonance frequencies will shift with the change of the aperture sizes because of the aperture resonances and aperture-cavity coupling. Variation rules of electromagnetic resonances with aperture sizes for a cavity with an aperture are given, which will be useful for design of shielding cavities.

Keywords—Aperture-cavity coupling, equivalent circuit method, resonances, shielding equipment.

I. INTRODUCTION

SHIELDING cavities are frequently used to reduce the emissions or improve the immunities of their internal electronic equipments. For purpose of ventilation, control, and communication, shielding cavities always contain apertures in their panels. Electromagnetic resonances may occur when a cavity with apertures irradiated by external electromagnetic waves, and at these resonance frequencies, the electromagnetic field responses within the cavity are extremely high, which will make the shielding capability of the cavity greatly degraded. Therefore, it is necessary to research the electromagnetic resonances of a cavity with apertures.

The resonances of a cavity with apertures are affected by many design parameters, such as the characteristics of the EM interferences, the sizes, shapes, and distributions of apertures, and cavity sizes. Many numerical methods such as finite-difference time-domain (FDTD) method [1], method of

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moments (MoM) [2], and transmission line matrix (TLM) method [3], [4] have been used for the analyses of the shielding effectiveness (SE) and resonances. The effects of the polarization and incident directions of the incident wave, pacing and shapes of the aperture on SE are analyzed in [3], and the relationship between the characteristics of aperture and SE is analyzed in [4]. Besides, lots of analytical methods such as [5]-[10] have also been represented. In [5], [6], the shielding problem is solved by equivalent circuit methods, which can be used for prediction of the SE of a cavity with apertures. Aperture resonances and coupling are discussed in [7], [8]. References [9], [10] consider that the electromagnetic resonances of a cavity with apertures contain cavity resonances, aperture resonances, and aperture-cavity resonances. However, most of those researches focus on the SE or resonances themselves, although some of them discuss the effects of some design parameters on resonances, the change trends of resonances with aperture sizes are not included. Therefore, in this paper, the effects of aperture sizes on resonances of a cavity with an aperture are researched based on both the equivalent circuit method and the TLM method, and some conclusions are given, which are helpful for shielding cavity design.

The rest of this paper is organized as follows: In Section II, the equivalent circuit method, the TLM method, and formulas for the calculation of resonances are introduced. In Section III, some calculation examples are made and variation rules of resonances with aperture sizes are given. Finally, some conclusions are drawn in Section IV.

II. THEORY

The discussed problem in this paper is the resonances of a metallic shielding cavity with an aperture irradiated by external electromagnetic interference. The shape of cavity can be rectangular and cylindrical, and shape of aperture can be rectangular, ellipse, and so on.

A. Equivalent Circuit Method

An equivalent circuit model for the prediction of the SE of a rectangular cavity with a rectangular aperture irradiated by a normal incident plane wave with its E-field polarizing in the y direction is presented by [5], and its geometry model is shown in Fig. 1. The internal sizes of the cavity are $a \times b \times d$ and the thickness is t. The sizes of the aperture are $w \times l$. The monitor point P locates at the center axis of the front panel of the cavity, and its z-coordinate is p.
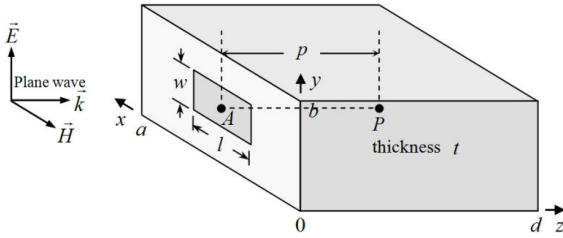
Fig. 1 The geometry model of a rectangular cavity contains a rectangular aperture

Fig. 2 shows the equivalent circuit model for the shielding problem shown in Fig. 1. In this model, the plane wave is represented by voltage $V_0 = \|\vec{E}\|$ and impedance $Z_0 \approx 377\Omega$, the aperture is represented by a length of coplanar strip line shorted at each end with characteristic impedance Z_{0s} and propagation constant k_0, and the cavity is represented by a rectangular waveguide shorted at the end with characteristic impedance Z_g and propagation constant k_g.
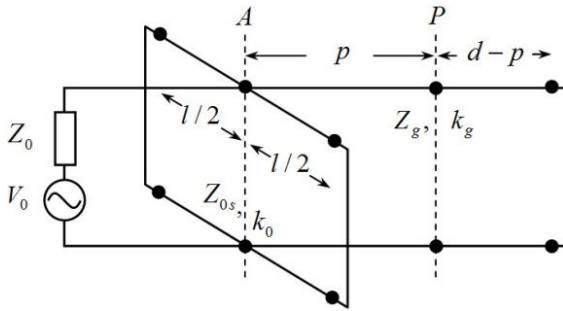

Fig. 2 The equivalent circuit for the shielding problem

The aperture effective width w_e given in [11] is

$$w_e = w - \frac{5t}{4\pi}(1 + \ln\frac{4\pi w}{t}) \tag{1}$$

If $w_e < b/\sqrt{2}$, the aperture characteristic impedance Z_{0s} can be expressed as

$$Z_{0s} = 120\pi^2 \left[\ln\left(2\frac{1 + \sqrt[4]{1-(w_e/b)^2}}{1 - \sqrt[4]{1-(w_e/b)^2}}\right)\right]^{-1} \tag{2}$$

Then the aperture impedance can be given by

$$Z_{ap} = \frac{1}{2}\frac{l}{a}jZ_{0s}\tan\frac{k_0 l}{2} \tag{3}$$

The equivalent voltage V_1 and source impedance Z_1 at the aperture can be obtained by using Thevenin's theorem, which are

$$V_1 = V_0 Z_{ap}/(Z_{ap} + Z_0) \tag{4}$$

$$Z_1 = Z_0 Z_{ap}/(Z_{ap} + Z_0) \tag{5}$$

The equivalent voltage V_2, source impedance Z_2, and load impedance Z_3 at point P can now be expressed as

$$V_2 = \frac{V_1}{\cos k_g p + j(Z_1/Z_2)\sin k_g p} \tag{6}$$

$$Z_2 = \frac{Z_1 + jZ_g \tan k_g p}{1 + j(Z_1/Z_g)\tan k_g p} \tag{7}$$

$$Z_3 = jZ_g \tan k_g(d - p) \tag{8}$$

where Z_g is the characteristic impedance of the cavity, for TE modes

$$Z_g = Z_g^{TE} = Z_0/\sqrt{1-(m\lambda/2a)^2 - (n\lambda/2b)^2} \tag{9}$$

For TM modes

$$Z_g = Z_g^{TM} = Z_0\sqrt{1-(m\lambda/2a)^2 - (n\lambda/2b)^2} \tag{10}$$

and the propagation constant of the cavity is

$$k_g = k_0\sqrt{1-(m\lambda/2a)^2 - (n\lambda/2b)^2} \tag{11}$$

where $k_0 = 2\pi/\lambda$ is the wave number of free space, m and n are mode numbers for directions of a and b, respectively.

Then, the voltage response at P is given by

$$V_{Rp} = V_2 Z_3/(Z_2 + Z_3) \tag{12}$$

In order to predict the higher order modes of propagation, [12] extends the voltage response according to the waveguide theory. In terms of [13], [14], the E-field of the TE wave propagating in a rectangular waveguide can be expressed as

$$E_x = \frac{j\omega\mu}{k_c^2}\frac{n\pi}{b}H_m \cos(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})e^{-j\beta z} \tag{13}$$

$$E_y = -\frac{j\omega\mu}{k_c^2}\frac{m\pi}{a}H_m \sin(\frac{m\pi x}{a})\cos(\frac{n\pi y}{b})e^{-j\beta z} \tag{14}$$

$$E_z = 0 \tag{15}$$

And the E-field of the TM wave propagating in a rectangular waveguide can be expressed as

$$E_x = -\frac{j\beta}{k_c^2}\frac{m\pi}{a}E_m\cos(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})e^{-j\beta z} \qquad (16)$$

$$E_y = -\frac{j\beta}{k_c^2}\frac{n\pi}{b}E_m\sin(\frac{m\pi x}{a})\cos(\frac{n\pi y}{b})e^{-j\beta z} \qquad (17)$$

$$E_z = E_m\sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})e^{-j\beta z} \qquad (18)$$

where k_c is the cutoff wave number and β is the phase constant of the waveguide, μ and ε are the permeability and permittivity of the medium within the waveguide, respectively, H_m and E_m are constants determined by excitation source.

The voltage response in the equivalent circuit should have the same form of the E-field of the wave propagating in the waveguide given by (13)-(18) according to the relationship between voltage and E-field. Therefore, the voltage response V_{Rp} given by (12) is actually the y component of the TE mode voltage response, which can be represented by V_{pymn}^{TE}, then its final form can be expressed as

$$V_{pymn}^{TE} = \begin{cases} 0, & m = 0 \\ \dfrac{V_1\sin(k_g(d-p))}{\sin(k_gd) - j(Z_1/Z_g^{TE})\cos(k_gd)}, & m \neq 0 \end{cases} \qquad (19)$$

Therefore, the x component of the TE mode voltage response can be given by

$$V_{pxmn}^{TE} = \begin{cases} 0, & m = 0 \\ -\dfrac{an}{bm}V_{pymn}^{TE}, & m \neq 0 \end{cases} \qquad (20)$$

And the z component of the TE mode voltage response is

$$V_{pzmn}^{TE} = 0 \qquad (21)$$

For TM mode, in order to avoid some mutations which cannot occur in the actual problem, Z_g given by (9) is used to calculate the x and y components of the TM mode voltage response, and Z_g given by (10) is used to calculate the z component of the TM mode voltage response. Therefore, the y component of the TM mode voltage response is given by

$$V_{pymn}^{TM} = \begin{cases} 0, & m = 0 \ or \ n = 0 \\ \dfrac{V_1\sin(k_g(d-p))}{\sin(k_gd) - j(Z_1/Z_g^{TE})\cos(k_gd)}, & m,n \neq 0 \end{cases} \qquad (22)$$

The x component of the TM mode voltage response is given by

$$V_{pxmn}^{TM} = \begin{cases} 0, & n = 0 \\ \dfrac{bm}{an}V_{pymn}^{TM}, & n \neq 0 \end{cases} \qquad (23)$$

And the z component of the TM mode voltage response is given by

$$V_{pzmn}^{TM} = \begin{cases} 0, & m \ or \ n = 0 \\ -\dfrac{k_c^2}{\beta}\dfrac{b}{n\pi}\dfrac{V_1\cos(k_g(d-p))}{\sin(k_gd) - j(Z_1/Z_g^{TM})\cos(k_gd)}, & m,n \neq 0 \end{cases} \qquad (24)$$

The total voltage components at point P, which are the summations of those mode voltage response components given by (19)-(24), can be given by

$$V_{px} = \sum_{m,n}(V_{pxmn}^{TE} + V_{pxmn}^{TM}) \qquad (25)$$

$$V_{py} = \sum_{m,n}(V_{pymn}^{TE} + V_{pymn}^{TM}) \qquad (26)$$

$$V_{pz} = \sum_{m,n}(V_{pzmn}^{TE} + V_{pzmn}^{TM}) \qquad (27)$$

Therefore, the total voltage response at point P is given by

$$V_p = \sqrt{\left|V_{px}\right|^2 + \left|V_{py}\right|^2 + \left|V_{pz}\right|^2} \qquad (28)$$

At point P, in the absence of the cavity, the load impedance is Z_0, the voltage response is $V_p' = V_0/2$, therefore, the SE can now be given by

$$SE_p = -20\log_{10}\left|V_p/V_p'\right| = -20\log_{10}\left|2V_p/V_0\right| \qquad (29)$$

B. TLM Method

TLM method is a differential form numerical method base on both the Huygens' Principle of light propagation model and the equivalent transmission line theory in the time domain. Using the comparability of the electromagnetic field and the voltage and current pulse and the analog of the matrix form of Maxwell's equations and the transmission line equation, the value of the electric and magnetic fields in TLM method are obtained by calculating the voltages of shunt-wound nodes and currents of series-wound nodes in the scattering matrix [15]. Further information and application of TLM method can be found in many literatures, such as [15] and [16].

Lots of commercial softwares based on TLM method have been developed, such as CST MICROSTRIPES (CST-MS) studio, a 3-D electromagnetic simulation tool based on TLM method, which is adopted in our studies.

C. Resonances

1. Aperture Inherent Resonances

A rectangular aperture in a conducting plane can be

considered as a length of coplanar strip transmission line shorted at each end [5], therefore, in terms of the transmission line theory, aperture resonances occur when the length of aperture is odd times of the half-wavelength of the incident wave. Actually, for an arbitrary shape aperture, its resonance frequency satisfies the relationship given by [7], which is

$$f_{a\_re} = (2k+1)\frac{c}{2l_{max}} \qquad (30)$$

where l_{max} is the maximum size of the aperture in the direction perpendicular to the incident E-field, c is the speed of light, and k denotes natural number.

2. Cavity Inherent Resonances

An empty cavity can be seen as a resonator. For a rectangular cavity, its resonance frequency given by [13] and [14] is expressed as

$$f_{c\_re} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{h}{d}\right)^2} \qquad (31)$$

where a, b and d are sizes of the cavity, m, n and h are mode numbers for directions of a, b and d, respectively.

For a cylindrical cavity, its resonance frequency for TEmnh modes given by [13], [14] is expressed as

$$f_{c\_re} = \frac{c}{2}\sqrt{\left(\frac{q_{mn}}{\pi r}\right)^2 + \left(\frac{h}{l}\right)^2} \qquad (32)$$

And its resonance frequency for TMmnh modes given by [13], [14] is expressed as

$$f_{c\_re} = \frac{c}{2}\sqrt{\left(\frac{q'_{mn}}{\pi r}\right)^2 + \left(\frac{h}{l}\right)^2} \qquad (33)$$

where q_{mn}, q'_{mn} are zero roots of Bessel function and Bessel function's derivative, respectively, r and l are radius and axial length of the cavity, respectively, and h is the mode number for direction of l.

III. CALCULATION RESULTS AND ANALYSES

In this section, some calculation examples are studied by equivalent circuit method coded by Matlab and TLM method coded by CST MICROSTRIPES (CST-MS). In all cases, the excitation EM interference is a normal incident plane wave polarizing in the y direction, its frequency range is 0-2 GHz, the thickness of the cavity is 1 mm, and the monitor point P locates at the central axis of the front panel of the cavity, and its z-coordinates is 200 mm. The material of the cavity is set to be perfect conductor in the circuit method and aluminum in TLM method.

Fig. 3 shows the configurations of the cases 1-3. In these three cases, the sizes of the cavity are 300 mm × 120 mm × 260 mm, and the width of the aperture is 10 mm. The only difference among them is the length of the aperture, which is 30 mm in case 1, 150 mm in case 2, and 280 mm in case 3. Some special theoretical resonance frequencies calculated by (30) and (31) for cases 1-3 are listed in Table I, where TEmnh represents the resonance modes of the cavity, and Aij represents the resonance modes of the aperture, in which the first index i denotes the case number and the second index j denotes the order of the resonance modes. For example, A31 represents the first order resonance mode of the aperture in case 3.

Fig. 4 shows the SE results of case 1 calculated by equivalent circuit method and TLM method, where the sizes of the aperture are 30 mm × 10 mm, and the resonance modes and their theoretical resonance frequencies are also marked. Because the maximum size of the aperture in case 1 is 30 mm, the first theoretical resonance frequency of the aperture is 5 GHz according to (30), and the upper limit of the frequency range in our study is 2 GHz, which is much less than 5 GHz, therefore, there is no resonances occur in aperture. It can be seen from Fig. 4 that only the cavity resonance modes of TE101, TE102, TE301, TE103, and TE302 appear, and the resonance frequencies calculated by circuit method and TLM method are in good agreement with those theoretical values calculated by (31).
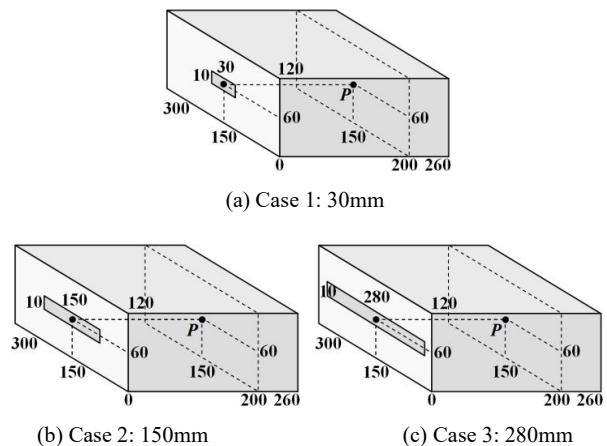

(a) Case 1: 30mm

(b) Case 2: 150mm

(c) Case 3: 280mm

Fig. 3 The configurations of cases 1-3

TABLE I
SOME SPECIAL THEORETICAL RESONANCE FREQUENCIES FOR CASES 1-3

Resonance Modes	Resonance Frequencies (GHz)	Resonance Modes	Resonance Frequencies (GHz)
A31	0.536	TE301	1.607
TE101	0.763	A32	1.607
A21	1	TE103	1.802
TE102	1.258	TE302	1.892

Fig. 5 shows the SE results of case 2 calculated by equivalent circuit method and TLM method, where the sizes of the aperture are 150 mm × 10 mm, and the resonance modes and their theoretical resonance frequencies are also marked. Both

the SE results of circuit method and TLM method show that the aperture resonance mode of A21 appears, that because the maximum size of the aperture in case 2 is 150 mm, the first theoretical resonance frequency of the aperture is 1 GHz according to (30), which is included in the frequency range in our study. Besides, the resonance frequencies nearby 1 GHz (the theoretical resonance frequency of the aperture) deviate from their theoretical values in the direction away from 1 GHz. For example, the frequency of TE101 calculated by TLM method is 0.73 GHz, which deviates from its theoretical value 0.763 GHz calculated by (31) in the direction of the left of 1 GHz, on the contrary, the frequency of TE102 calculated by TLM method is 1.295 GHz, which deviates from its theoretical value 1.258 GHz calculated by (31) in the direction of the right of 1 GHz.

Fig. 6 shows the SE results of case 3 calculated by equivalent circuit method and TLM method, where the sizes of the aperture are 280 mm × 10 mm, and the resonance modes and their theoretical resonance frequencies are also marked. Both the SE results of circuit method and TLM method show that the aperture resonance mode of A31 appears in case 3, and because the frequency of A31 is 0.536 GHz according to (30), which is less than the frequency of TE101 (0.763 GHz), A31 is the first order mode of case 3. And because the theoretical frequencies of TE301 and A32 are the same, both of them are 1.607 GHz according to (30) and (31), they overlap in the SE results curves. Besides, similar to case 2, the resonance frequencies nearby the 0.536 GHz (the theoretical resonance frequency of A31) and 1.607 GHz (the theoretical resonance frequency of A32), such as TE101 and TE103, deviate from their theoretical values.
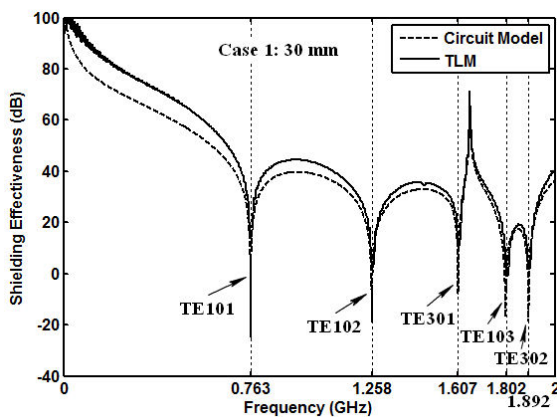

Fig. 4 The SE results for case 1

Fig. 7 shows the configurations of cases 4-6. In these three cases, all parameters except the aperture shape and sizes are the same as those in cases 1-3. Apertures in cases 4-6 are ellipse and their sizes are plotted in Fig. 7. And Fig. 8 shows the SE results of cases 4-6 calculated by TLM method. Because the sizes of cavity and the maximum sizes of apertures are the same as those in cases 1-3, the variation rules of resonance modes and frequencies with aperture sizes are the same as those in cases 1-3.
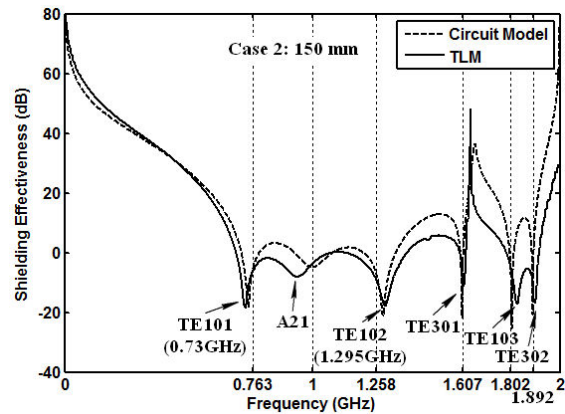

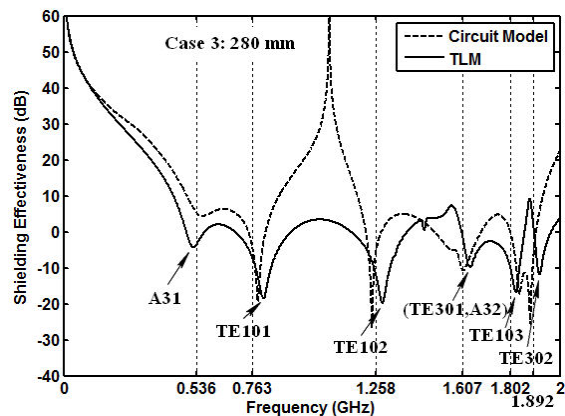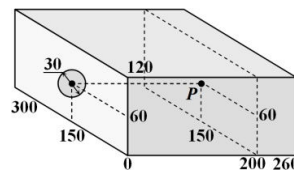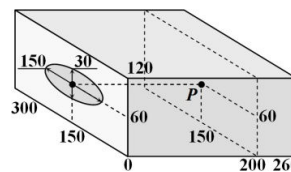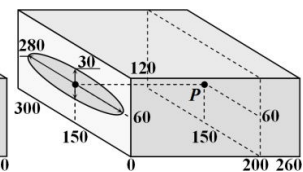Fig. 5 The SE results for case 2

Fig. 6 The SE results for case 3

(a) Case 4: 30mm

(b) Case 5: 150mm

(c) Case 6: 280mm

Fig. 7 The configurations of cases 4-6

Fig. 9 shows the configurations of cases 7-9. In these three cases, cavity is cylindrical, whose radius is 120 mm and axial length is 260 mm, and aperture is rectangular, whose width is 10 mm. The only difference among them is the aperture length, which is 30 mm in case 7, 120 mm in case 8, and 200 mm in case 9. The corresponding SE results calculated by TLM method are shown in Fig. 10. It can be seen that when aperture

resonances occur, the aperture resonance modes will appear, and the resonance frequencies nearby the theoretical aperture resonance frequencies deviate from their theoretical values.
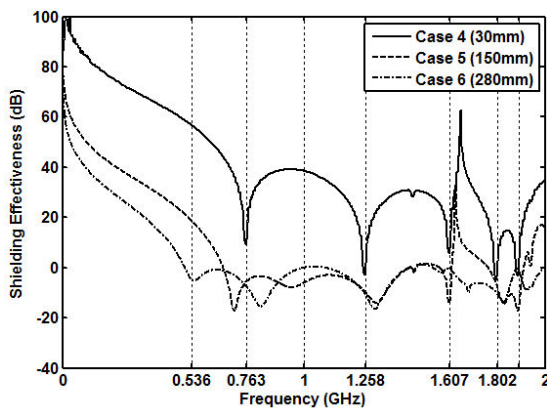

Fig. 8 The SE results for cases 4-6

(a) Case 7: 30mm

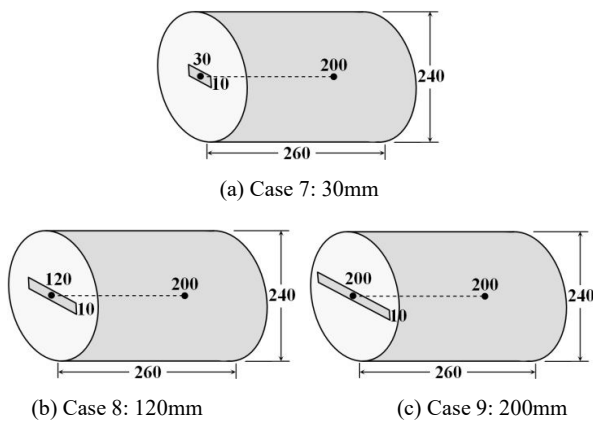(b) Case 8: 120mm   (c) Case 9: 200mm
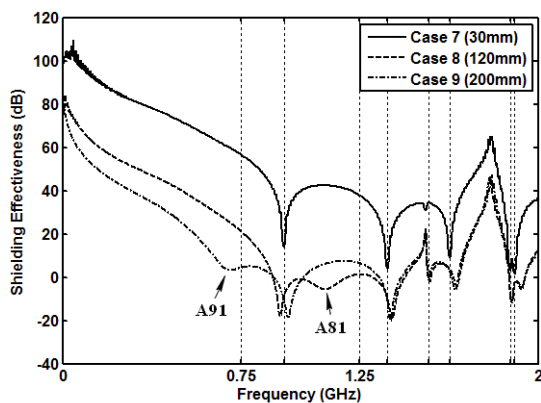
Fig. 9 The configurations of cases 7-9

Fig. 10 The SE results for cases 7-9

Based on the analyses given above, some laws of resonances can be concluded as follows:

1) The resonance modes of a cavity with an aperture only contain cavity resonance modes and the resonance frequencies are in good agreement with their theoretical values at those frequencies far from those frequencies where aperture resonances occur.

2) The resonance modes of a cavity with an aperture contain both the resonances modes of the cavity and aperture when aperture resonances occur, and the resonance frequencies nearby the frequencies where aperture resonances occur will deviate from their theoretical values.

IV. CONCLUSIONS

We have discussed the effects of the aperture resonances and aperture-cavity coupling on the resonances of a cavity with an aperture irradiated by an external electromagnetic wave based on both the equivalent circuit method and numerical method of TLM. A variety of aperture sizes, rectangular and round apertures, and rectangular and cylindrical cavities have been included in our calculation examples. Both the analytical and numerical results show that the resonance modes of a cavity with an aperture include aperture resonance modes and cavity resonance modes, and the resonance frequencies nearby those frequencies, where aperture resonances occur will deviate from their theoretical values because of the aperture-cavity coupling, and the closer they get to the theoretical aperture resonance frequency. The greater distance they deviate, which means that the resonance modes and frequencies of a cavity with an aperture are influenced by the aperture sizes and frequencies of incident EM waves. The study in this paper will be of use to designers of shielding cavities.

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