

# Time-Dependent Behavior of Reinforced Concrete Beams under Sustained and Repeated Loading

Sultan Daud, John P. Forth, Nikolaos Nikitas

**Abstract**—The current study aims to highlight the loading characteristics impact on the time evolution (focusing particularly on long term effects) of the deformation of realized reinforced concrete beams. Namely the tension stiffening code provisions (i.e. within Eurocode 2) are reviewed with a clear intention to reassess their operational value and predicting capacity. In what follows the experimental programme adopted along with some preliminary findings and numerical modeling attempts are presented.

For a range of long slender reinforced concrete simply supported beams (4200 mm) constant static sustained and repeated cyclic loadings were applied mapping the time evolution of deformation. All experiments were carried out at the Heavy Structures Lab of the University of Leeds. During tests the mid-span deflection, creep coefficient and shrinkage strains were monitored for duration of 90 days. The obtained results are set against the values predicted by Eurocode 2 and the tools within an FE commercial package (i.e. Midas FEA) to yield that existing knowledge and practise is at times over-conservative.

**Keywords**—Eurocode2, midas fea, repeated, sustained loading.

## I. INTRODUCTION

NATURALLY concrete is a brittle material with little capacity to carry loads in flexure without cracking. To support structures in tension and increase ductility, steel reinforcement is used. Globally, reinforced concrete structures have been successfully used in all types of infrastructure such as bridges, houses, airports, etc. With time, and possibly as a result of overloading the structures or due to the environmental conditions (hot in summer and cold in winter), the long-term behavior of reinforced concrete members can be affected.

It is clear that creep, shrinkage and loss of tension stiffening increase deflection of spanning elements with time [1]. In design it is therefore necessary to consider these factors in order to control the deflection, to increase the lifespan of reinforced concrete elements, and control crack propagation. In flexural members, however, when the load is applied, primary cracks occur below the neutral axes (i.e. for sagging members) when the concrete reaches its tensile strength. The concrete capacity to carrying tensile stresses between cracks is usually called tension stiffening [2]. With time under sustained loading about 50% of the short term tension stiffening disappear in a period of 30 days [3]. In the 1970's, the Concrete in the Oceans (CiO) research program assigned the

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long-term deterioration technique focusing on the RC section capacity rather than load types [4]. However, [5] presented an experimental study on a lightly reinforced concrete bridge deck subjected to fatigue loading. As the number of load cycles increased there was a progressive loss in tension stiffening. [6]. The additional deformations in both tension and compression zones caused by repeated loading mostly occur within the first 10 days [4].

Civil engineers usually employ design codes for the analysis of concrete structures [7]. In this study the accuracy of long-term deflections predicted using EC2 and Midas FEA will be verified using experimental data gathered at the University of Leeds.

## II. EXPERIMENTAL PROGRAMME

### A. Shrinkage and Creep Tests

Both creep and shrinkage tests were carried out on specimens with dimensions of 200 x 75 x 75 mm as shown in Figs. 1 (a) and (b). A set of DEMountable MECHANical (DEMEC) points were attached 150 mm apart on both sides of the prisms. Specimens were stored in a controlled room at  $21 \pm 1^\circ\text{C}$  and a relative humidity of  $60 \pm 5\%$ .

### B. Long-Term Deflection Test

Two reinforced concrete beams were cast and tested. The first beam was made from normal concrete under sustained loading while the second one was normal concrete under repeated loading (NC-SUS and NC-REP, respectively). Both beams had the same properties; compressive strength,  $f_{ck,cube} = 55 \text{ MPa}$ , indirect tensile strength,  $f_{ct} = 3.6 \text{ MPa}$  and a modulus of elasticity  $E_{cm} = 33.7 \text{ GPa}$ . Both beams had the same dimensions; 300 mm width, 150 depth and 4200 mm length. Three bars  $\varnothing 16$  with a yielding strength of (510 MPa) were used as bottom longitudinal reinforcement. To avoid shear failure,  $\varnothing 8$  mm shear links were placed at 150 mm c/c and two bars  $\varnothing 10$  were used to hold the links.

Three strain gauges were placed at the bottom of the reinforcement of each beam. Also, two LVDTs (Linear Variable Differential Transformers) were placed under each beam to monitor the deflection along the beam. Four sets of DEMECs were placed on both sides of the beams. The bottom and top rows of DEMEC points were at the level of the reinforcement. Both beams were preloaded to 19 kN so that a stabilised crack pattern was produced. The steel stresses were checked at that load and was 200 MPa. The load was sustained on the first beam whereas the second beam was subjected to repeated loads cycled under 0.2 Hz frequency and +/-2.5kN amplitude. All readings were taken at 19 kN.



(a)



(b)

Fig. 1 Test specimens (a) Shrinkage (b) Creep

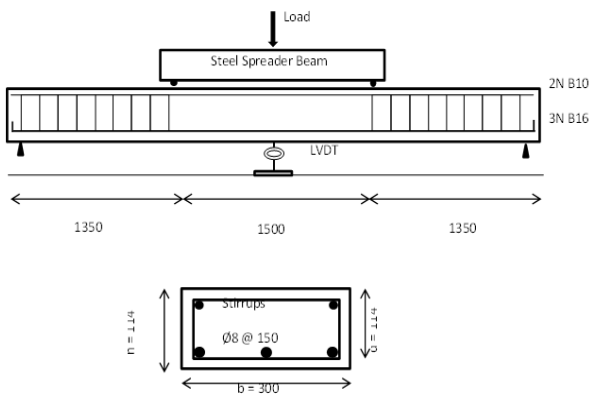


Fig. 2 Beam Dimensions and Experimental Setup

### III. CALCULATION OF CURVATURE, CREEP COEFFICIENT, AND SHRINKAGE BASED ON EC2

#### A. Curvature

Eurocode 2 [8] predicts the long term deflection by superposition of creep and shrinkage curvatures. The creep curvature was calculated by modifying the modulus of elasticity of the section:

$$E_{c\text{eff}} = \frac{E_{co}}{1 + \phi(t)} \quad (1)$$

where  $E_{co}$  = the elastic modulus;  $\phi(t)$  = the creep coefficient; whereas the shrinkage curvature was calculated using:

$$1/r_{cs} = \epsilon_{cs} \alpha_e S/I \quad (2)$$

where  $1/r_{cs}$  = the shrinkage curvature;  $\epsilon_{cs}$  = shrinkage strain;  $\alpha_e$  = effective modular ratio ( $\frac{E_s}{E_{c\text{eff}}}$ );  $S$  = first moment of area of the reinforcement about the centroid of the section;  $I$  = second moment of area of section (cracked or uncracked as appropriate); however, the curvature  $1/r$  is calculated as:

$$1/r = \xi(1/r)_{cr} + (1 - \xi)(1/r)_{uc} \quad (3)$$

where  $1/r$  = average curvature;  $(1/r)_{cr}$  = values of curvature calculated for the cracked and uncracked section respectively;  $\xi$  = is the distributed coefficient allowing for tension stiffening given by  $\xi = 1 - \beta \frac{M_{cr}}{M_a}$ ;  $\beta$  = the coefficient taking account the duration of loading (0.5 for sustained or cyclic loading and 1 for single short term load);  $M_{cr}$  = cracking moment;  $M_a$  = applied moment.

#### B. Creep Coefficient

There are many factors which effect creep [9] i.e.: Moisture content, magnitude of the applied stress, concrete compressive strength, temperature and aggregate content. The strain due to creep at the end of loading is about three to four times the elastic strain [10]. Eurocode 2 [8] suggests an equation to predict the creep coefficient based on compressive strength of the concrete, relative humidity and type of cement used. This equation is:

$$\varphi(t, t_0) = \varphi_0 * \beta_c(t, t_0) \quad (4)$$

where  $\varphi_0$  = is the notional creep coefficient which can be calculated from  $\varphi_0 = \varphi RH * \beta(f_{cm}) * \beta(t_0)$ ;  $\varphi RH$  = is a factor to allow for the effect of relative humidity on the notional creep coefficient.

$$\varphi RH = \left[ 1 + \frac{1 - RH/100}{0.1 * \sqrt[3]{h_0}} * \alpha_1 \right] \alpha_2$$

where  $RH$  = is the relative humidity %;  $h_0$  = is the notational size of member in mm;  $h_0 = \frac{2A_c}{u}$ ;  $A_c$ : Section area;  $u$ : Perimeter in contact with atmosphere.

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$$

where  $f_{cm}$ : mean compressive strength of concrete;  $\beta(t_0)$  is a factor to allow for the effect of concrete age at loading on the notional creep coefficient.

$$\beta(t_0) = \frac{1}{(0.1 + t_0^{0.2})}$$

$$\beta_c(t, t_0) = \left[ \frac{(t - t_0)}{(\beta_H + t - t_0)} \right]^{0.3}$$

$\beta_H$  = A coefficient depending on the relative humidity ( $RH$  in %) and the notional member size. It may be estimated from:

$$\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250\alpha_3$$

where  $\alpha_1 = \left[\frac{35}{f_{cm}}\right]^{0.7}$ ;  $\alpha_2 = \left[\frac{35}{f_{cm}}\right]^{0.2}$ ;  $\alpha_3 = \left[\frac{35}{f_{cm}}\right]^{0.5}$ .

### C. Shrinkage

Shrinkage can be considered at two stages, plastic and drying shrinkage. [11]. Plastic shrinkage usually happens within the first few hours of producing the concrete and is due to cement hydration. Whereas drying shrinkage is due to the reduction in concrete size due to evaporation of water after the concrete has achieved final set. The shrinkage suggested by EC2 can be calculated from:

$$\varepsilon_{cd,0} = 0.85 \left[ (220 + 110 * \alpha_{ds1}) * \exp\left(-\alpha_{ds1} * \frac{f_{cm}}{f_{cm0}}\right) \right] 10^{-6} * \beta_{RH} \quad (5)$$

where  $\alpha_{ds1}$ ,  $\alpha_{ds2}$  are factors depend on type of cement (4 and 0.12 respectively);  $f_{cm0} = 10$  Mpa;  $\beta_{RH} = 1.55 \left[ 1 - \left( \frac{RH}{RH_0} \right)^3 \right]$ ;  $RH_0 = 100$ .

## IV. NUMERICAL MODELLING

The analytical approaches to predict the long-term deflection of cracked members is usually complicated [12]. A commercial FE package Midas FEA was used to simulate the experimental behaviour of the beam shown in Fig. 2.

For the long term behaviour, "construction stages" was defined in the analysis of the beam, to reflect the effect of the evolving material properties with time (creep and shrinkage) displacements. Thus concrete should be considered as an elastic material to activate (creep/shrinkage) functions. However, Midas FEA is flexible about the code which will be used (CEB-FIP, ACI, PCA, Combined ACI, PCA and AASHTO) when it comes to defining shrinkage strain and creep coefficient. In this study CEB-FIP Model Code 1990 was used.

According to CEB-FIP Model Code 1990 [13], for elastic analysis of concrete, a reduced modulus of elasticity should be used to reflect the initial plastic strain. Thus, the modulus of elasticity of concrete was multiplied by 0.85.

For the case of repeated loading, it is not possible in Midas FEA to add fatigue analysis to the construction stages.

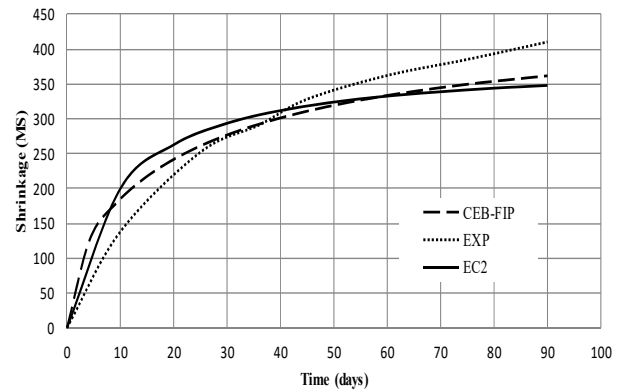
## V. RESULT AND DISCUSSION

### A. Shrinkage and Creep Coefficient

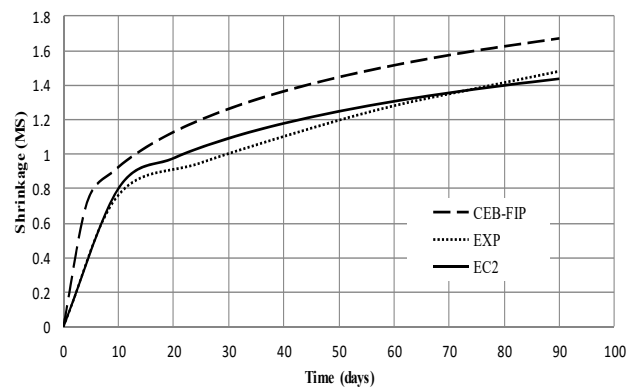
Figs. 3 (a) and (b) show the shrinkage and creep coefficient development with time, respectively.

It can be observed that both EC2 and the CEB-FIP Model Code 1990 overestimate the shrinkage in the first 20 days. It also shows that the CEB-FIP Model Code 1990 and EC2 have a good agreement with the experimental shrinkage after 90 days. Still the rate of shrinkage (i.e. gradient in Fig. 3 (a)) is much greater when at 90 days for the experiment. This means that with time the small differences will tend to amplify.

The creep coefficient development with time is shown in Fig. 3 (b). This figure indicates that EC2 predicts the creep coefficient correctly whereas the CEB-FIP Model Code 1990 overestimates the creep coefficient.



(a)



(b)

Fig. 3 (a) Shrinkage Development with Time, (b) Creep Coefficient with Time

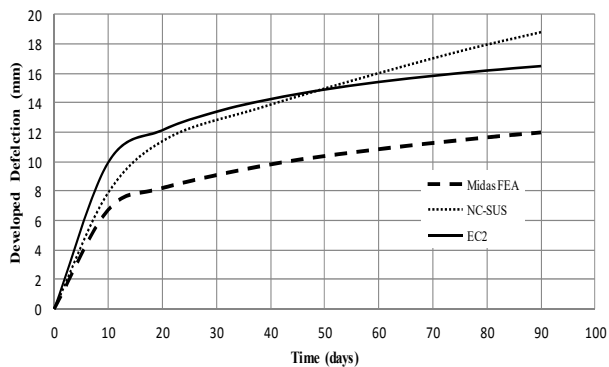
### B. Mid-Span Deflection

Fig. 4 (a) compares the developed mid-span deflection (NC-SUS) with that calculated from the EC2 and Midas FEA. It is clear that both EC2 and Midas FEA underestimate the mid-span deflection at 90 days. However EC2 also over-predicts up to 50 days this behavior might come from the shrinkage curvature (as the shrinkage strain was overestimating in the first 40 days and underestimated at the end of the test). Whereas Midas FEA assumes concrete is elastic and therefore the shrinkage curvature was only from the uncracked section (the shrinkage is uniformly distributed all over the cross section).

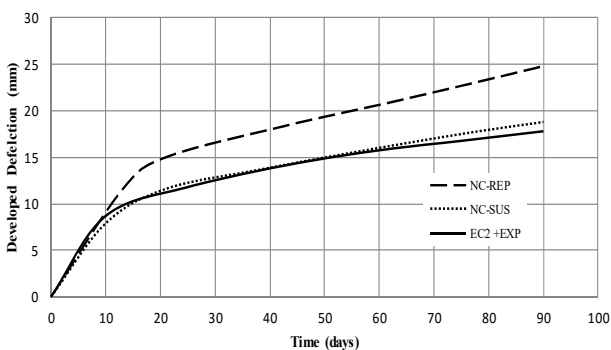
The deflection predicted by the software, thus, is underestimated. However, after 90 days of loading, EC2 prediction is 12 % less than that in NC-SUS (16.5 mm vs 18.8 mm). The relevant figure for the Midas FEA estimation is 12 mm, being 36% less than the NC-SUS.

Fig. 4 (b) compares the mid-span developed deflection of the two tested beams and that predicted by EC2 using creep and shrinkage experimental data (i.e. actual data obtained from this experimental programme). It is clear that the deflection of the beam under sustained load is predicted well by EC2. However, the beam under repeated loading has more deflection than the beam under sustained loading as the

concrete is debonded from the steel due to the repeated loading. This extra deflection of the repeating loading beam case is in agreement with [5] and is therefore explained by loss of tension stiffening / debonding / cyclic creep [4]. (Note, debonding may have a greater influence in this investigation as the beams were loaded to achieve a stabilized crack pattern; in practice, many beams are nowhere near this condition and only exhibit possible a third of the cracks present in a stabilized crack pattern.) However both beams have approximately the same deflection after 20 days.



(a)



(b)

Fig. 4 (a) Mid-Span Deflection with Time (NC-SUS, EC2 and Midas FEA), (b) Mid-Span Deflection with Time (NC-REP, NC-SUS and EC2+EXP)

Fig. 4 (b) shows that the suggested EC2 equation for long term deflection is missing parameters to reflect loading history.

## VI. CONCLUSIONS

- Both EC2 and CEB-FIP Model Code 1990 overestimate shrinkage in the first 20 days.
- EC2 predicts the sustained long-term deflection correctly when the experimental creep and shrinkage data were used.
- The deflection in the case of repeated loading is higher than that in the case of sustained load. Due to the loss of tension stiffening in the early stages.
- For the long term deflection, more coefficients should be involved to reflect the repeated loading in EC2 equations.
- Midas FEA has a limited capability for long term loading (i.e. sustained load only).
- The finite element software Midas FEA does not predict the shrinkage curvature accurately in case of sustained loading as concrete assumed elastic.

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## REFERENCES

- Scott, R.H. and A.W. Beeby, Long-term tension-stiffening effects in concrete. *Aci Structural Journal*, 2005. 102(1): p. 31-39.
- Pirayeh Gar, S., M. Head, and S. Hurlbaas, Tension Stiffening in Prestressed Concrete Beams Using Moment-Curvature Relationship. *Journal of Structural Engineering*, 2011. 138(8): p. 1075-1078.
- Beeby, A.W. and R.H. Scott, Mechanisms of long-term decay of tension stiffening. *Magazine of Concrete Research*, 2006. 58(5): p. 255-266.
- Higgins, L., et al., Behaviour of cracked reinforced concrete beams under repeated and sustained load types. *Engineering Structures*, 2013. 56(0): p. 457-465.
- Zanuy, C., P. de la Fuente, and L. Albajar, Estimation of parameters defining negative tension stiffening. *Engineering Structures*, 2010. 32(10): p. 3355-3362.
- Whaley, C. and A. Neville, Non-elastic deformation of concrete under cyclic compression. *Magazine of Concrete Research*, 1973. 25(84): p. 145-154.
- Gribniak, V., et al., Long-term deflections of reinforced concrete elements: accuracy analysis of predictions by different methods. *Mechanics of Time-Dependent Materials*, 2013. 17(3): p. 297-313.
- Eurocode 2, Design of Concrete Structures. 1992: ENV 1992-1-1.
- Domone, P.L. and J.M. Illston, eds. *Construction Materials: Their Nature and Behaviour*. Fourth edition ed. 2010, Spon Press: London.
- Acker, P. and F.-J. Ulm, Creep and shrinkage of concrete: physical origins and practical measurements. *Nuclear Engineering and Design*, 2001. 203(2-3): p. 143-158.
- Nawy, E.G., *Concrete construction engineering handbook*. 2010: CRC press.
- Deflection of Cracked RC Beams under Sustained Loading. *Journal of Structural Engineering*, 2000. 126(6): p. 708-716.
- Du béton, F., *CEB-FIP Model Code 1990: Design Code*. 1993: T. Telford.