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Advanced Technologies and Algorithms for Efficient Portfolio Selection

Konstantinos Liagkouras, Konstantinos Metaxiotis

Abstract**—**In this paper we present a classification of the various technologies applied for the solution of the portfolio selection problem according to the discipline and the methodological framework followed. We provide a concise presentation of the emerged categories and we are trying to identify which methods considered obsolete and which lie at the heart of the debate. On top of that, we provide a comparative study of the different technologies applied for efficient portfolio construction and we suggest potential paths for future work that lie at the intersection of the presented techniques.

Keywords**—**Portfolio selection, optimization techniques, financial models, stochastics, heuristics**.**

I. Introduction

SINCE the seminal work by Markowitz [19] and his Mean – Variance theory (MV) the Portfolio Selection problem has attracted considerable attention by both academics and practitioners. People from such diverse fields as finance, engineering, mathematics, computing, operational research, statistics and psychology have attempted to explain the forces behind the financial markets movements [15], [16].

In this paper we taxonomise the various techniques used for constructing efficient portfolios into five distinct categories according to the discipline and the methodological framework followed. We provide a concise presentation of the emerged categories and we are trying to identify which methods considered obsolete and which lie at the heart of the debate. Additionally we present a comparative study of the different technologies applied for efficient portfolio construction and we suggest potential paths for future work that lie at the intersection of the presented techniques.

The rest of the paper is organized as follows. In Section II we provide a concise overview of the emerged classifications of techniques for constructing efficient portfolios. In Section III we present a comparative study of the different technologies applied for efficient portfolio construction. Finally, in Section IV, we present our conclusions from the comparative study of the presented techniques.

K. Metaxiotis is Associate Professor with the Department of Informatics, University of Piraeus (e-mail: kmetax@unipi.gr)

K. Liagkouras is PhD Candidate with the Department of Informatics, University of Piraeus 80, Karaoli & Dimitriou Str., 18534 Piraeus, Greece (e-mail: kliagk@unipi.gr)

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II. Technologies for Efficient Portfolio Construction

A. Financial Approaches

The earliest financial techniques for constructing efficient portfolios were based on analyzing the publicly available stock listings and trying to identify potential patterns and trends. Two major distinct methodological frameworks emerged. The first one, the so called Fundamental Analysis, is based on the analysis of the financial statements. Fundamental Analysis focuses exclusively on the company's performance in order to determine whether or not the stock should be bought or sold. The Technical Analysis on the other hand disregards completely the value of financial statements analysis and focuses exclusively on the movement of the stock prices in order to determine whether to buy or sell a particular stock. While techniques for selecting stocks can be traced back to the 19th century, it was not until the 1952, when Markowitz introduced his pioneering Mean - Variance (MV) portfolio selection model, that the field attracted considerable attention. Markowitz's theory suggests maximizing portfolio expected return for a given amount of portfolio risk or solving its dual problem minimizing portfolio risk for a given level of expected return.

Among the various models, distinguished place have the Black-Scholes model [2] a tool for pricing a stock option. The basic idea behind the Black-Scholes model is that the price of an option is determined implicitly by the price of the underlying stock. The Black-Scholes model displayed the importance of mathematics in the field of finance and it also led to the rapid development of a new field of research the so called financial engineering.

One of the most well-known theories in the field of finance is the Efficient Market Hypothesis (EMH). It was proposed by Eugene Fama [10] in 1960s and shortly states that one cannot achieve returns in excess of the average market returns because share prices always reflect all relevant information. According to the EMH stocks always trade at fair value on exchanges making it impossible to gain by purchasing undervalued stocks. However the stock market crash of 1987 put into question the validity of the EMH and gave the opportunity to other theories to emerge. Although the influence of psychology on the investors and the subsequent effect on markets was known, it was not until the aftermaths of the stock market crash of 1987 that the Behavioural Finance gained wide recognition. Briefly, Behavioural Finance focuses upon how investors interpret and act on information to make informed investment decisions. According to this theory investors do not always behave in a rational and unbiased manner. Behavioural Finance examines how investors' behaviour can lead to various market anomalies.

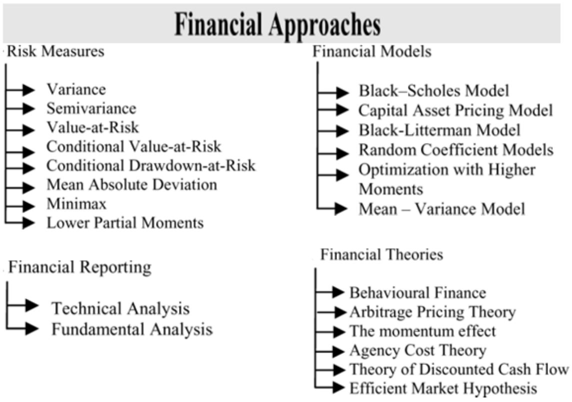

Fig. 1 Financial Methods in Portfolio Management

B. *Mathematical Approaches*

The Mathematical Approaches for constructing Efficient Portfolios can be clustered into four different groups depending on the applied methodology. The first classification deals with portfolio selection problems that are formulated as linear programming (LP) problems. A LP problem is one in which the objectives and all of the constraints are linear functions of the decision variables. The Mean Absolute Deviation (MAD) portfolio optimization model proposed by Konno and Yamazaki [13] is a well-known example of a portfolio selection model that can be formulated as a linear programming problem. Another example is the Minimax (MM) model that can be also formulated as a LP problem. The LP formulation presents some major advantages like its implementation is relatively easy and it guarantees to find optimal solution. The most well-known LP technique is the Simplex method developed by Dantzig [6]. Briefly, the Simplex method is a method that proceeds from one basic feasible solution (BFS) or extreme point of the feasible region of a LP problem expressed in tableau form to another BFS, in such a way to continually increase (or decrease) the value of the objective function until optimality is reached. The second classification of Portfolio Selection problems formulation concerns the Quadratic Programming (QP) problems. A QP problem has an objective which is a quadratic function of the decision variables, and constraints which are all linear functions of the variables. The Mean-Variance (MV) portfolio selection model introduced by Markowitz [19] is the most well-known example of this category of problems. The classical MV model is a convex QP problem which can be solved by a number of algorithms with a moderate computational effort even for relatively large number of portfolio stocks. However, if we apply to the classical MV model some realistic constraints like for instance cardinality constraint and buy-in threshold the problem is no longer a convex optimization problem because of the non-convexity of its feasible region. The third classification of portfolio selection problems deals with Nonlinear Programming (NLP) problems. A NLP is one in which the objective or at least one of the constraints is a nonlinear function of the decision variables. The nonlinear function may be convex or non-convex. In case where the objective function and all constraints are convex functions a NLP problem can be solved efficiently up to large portfolio sizes. However, if the objective function or any constraints are non-convex, the problem may have multiple feasible regions and multiple locally optimal points within these regions. In such case a NLP problem can be quite difficult to be solved. Finally, techniques like Fourier Transformation and Fibonacci Sequence have been applied to the Portfolio Selection problem.

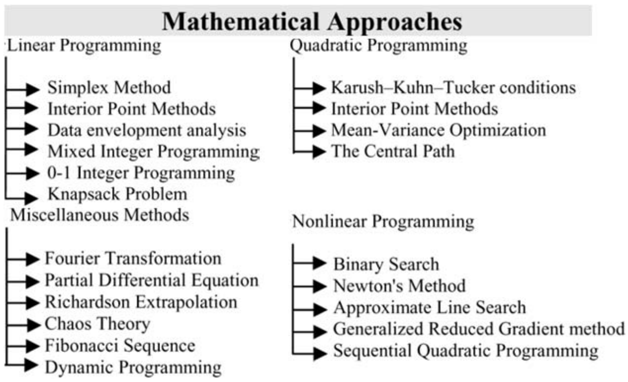

Fig. 2 Mathematical Techniques in Portfolio Management

C. *Stochastic Approaches*

The Stochastic models have been extensively used in modern financial risk analysis. Stochastic model is a technique of financial modeling in which one or more variables within the model are random and are used in order to estimate probability distributions of potential outcomes. Brownian motion is a well-known example of a continuous stochastic process that has been widely used in finance for modeling the random fluctuations in a stock's price. A stochastic process related to Brownian motion is the Random walk. In reality Brownian motion is the continuous analog to the Random walk. Another popular technique in financial modeling is the Markov Chain which is a sequence of stochastic events where the current state of a variable is independent of all past states, except the current state. A number of models assume that stock prices follow a Markov process. This assumption is consistent with the weak form of market efficiency, which claims that all past prices of a stock are reflected in today's stock price. Thus, Technical analysis cannot be used to predict and beat the market. Fuzzy logic is among the techniques that have been used extensively in financial modeling. Briefly, Fuzzy logic is a type of logic that recognizes degrees of truth rather than the well-established "true" or "false" Boolean logic. Fuzzy logic has been developed in order to represent real world problems that cannot be easily represented using the two-valued logic: 1 or 0. From this point of view Fuzzy logic seems closer to the way the human brain works, which means that by considering all available information, takes the best possible decision. Many other stochastic methods have been used in financial modeling; the table below indicates some of the most well-known stochastic methods in financial modeling.

D. *Computational Approaches*

Computer Science not only provided a fast and reliable way of calculating computationally demanding financial models but also revolutionized the financial modeling research field itself by developing innovative algorithmic approaches for solving difficult financial problems that in many cases cannot be solved using exact methods. The Computational Approaches dealing with financial modeling can be clustered into four different groups depending on the applied methodology.

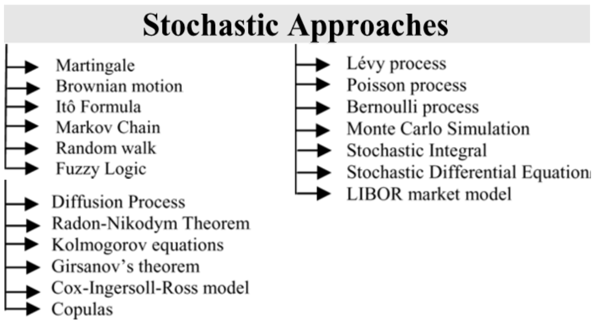

Fig. 3 Stochastic Techniques in Portfolio Management

The first one, the so called Evolutionary algorithms (EA) are population based stochastic optimization heuristics inspired by Darwin's Evolution Theory. An EA searches through a solution space in parallel by evaluating a set (population) of possible solutions (individuals). An EA starts with a random initial population. Then the 'fitness' of each individual is determined by evaluating the objective function. After the best individuals are selected, new individuals for the next generation are created. The new individuals are generated by altering the individuals through random mutation and by mixing the decision variables of multiple parents through crossover. Then the generational cycle repeats until a breaking criterion is fulfilled. EAs have been applied successfully to a wide range of problems such as engineering, biology, genetics, finance etc. Genetic Algorithms (GA) which belong to the family of EAs have been proved very effective for solving constrained portfolio optimization problems [4], that cannot be solved with exact methods. Genetic Programming (GP) belongs to EAs as well and its main difference between with the GAs is the representation of the solution. GP creates computer programs as the solution while GAs create a string of numbers that represent the solution. Evolutionary Programming (EP) is a stochastic optimization strategy developed by Fogel [11]. EP algorithms are similar to GAs, but do not incorporate crossover, instead they rely on mutation and the survival of the fittest. Evolutionary Strategy (ES) is similar to GAs but use recombination to exchange genetic material or information between population members instead of crossover, and most of the times use a different type of mutation as well. Neuroevolution is the use of GA to train artificial neural networks. Neuroevolution is used to construct and adapt artificial neural networks (ANN) through reinforcement learning to decide on optimal portfolio selection strategies.

The second classification of algorithmic approaches for the construction of efficient portfolios concerns the Swarm Algorithms. Swarm Intelligence (SI) is inspired from the biological examples provided by social insects like ants, bees, termites, wasps and by swarming, flocking and herding behaviors in vertebrates. Evolution has produced swarming in so many different contexts because the synergy and interaction of an agent with the group within a swarm provides considerable benefits. Swarm Intelligence is a decentralized, self-organized system in which the agents through their collective behavior find coherent solutions to the arisen problems. Ant Colony Optimization (ACO) is an optimization procedure inspired by ants' ability to identify optimal paths by depositing pheromone on the ground. Similarly in ACO a number of artificial ants identify the optimal path by marking solutions as they move on the graph. Another popular swarm intelligence technique is the Particle Swarm Optimization (PSO). It is an optimization technique that explores a large number of candidate solutions in order to find the optimum solution. The individual or particle exchanges information with the neighboring members, in order to adjust its trajectory towards the best attained position. Both ACO and PSO techniques have been applied to solve the constrained portfolio selection problem [1], [7], [8], [12]. Many others swarm algorithm techniques have been devised during the last years, some of the most well-known are illustrated in the table below.

The third classification of computational approaches for the solution of the portfolio selection problem concerns the Local Search Algorithms techniques. These algorithms try to improve an initial solution by applying iteration in order to create the neighborhood of the current solution. Then the best solution of the neighborhood is selected for the next iteration. The process continues until a solution considered optimum is found. Simulated Annealing (SA) is a well-known local search technique developed to deal with highly nonlinear problems. SA is a search method, inspired by the metals' process of annealing, an initial solution is randomly generated, and then applying an iteration process a neighbour is found. Suppose we are searching for the global maximum all uphill points are accepted while some downhill points are accepted as well depending on probabilistic criteria. By accepting points that reduce the objective, the algorithm avoids being trapped in local maxima. SA techniques have been applied extensively for the solution of the portfolio selection problem [1], [3], [5], [9], [18]. Hill Climbing (HC) is another local search techniques applied to the portfolio optimization problem. HC starts with a random solution to a problem, then attempts to move to a better position by using an evaluation function to assign score to each successor. If one of the successors has a better score than the current solution then set the new current state to be the successor with the best score. The process is repeated until no further improvement can be made. Tabu Search (TS) is a trajectory based optimization technique. TS provides enhanced efficiency of the exploration process by keeping track not only of the local information, but also information related to the exploration process. Once a candidate solution has been identified, it is marked as a tabu solution, thus it is avoided in the next iterations and we get rid of the cycling effect.

Finally the last classification of computational approaches for the solution of the portfolio selection problem concerns the

Multiobjective Evolutionary Algorithms (MOEAs). Multiobjective optimization (MO) is the problem of maximizing / minimizing a set of conflicting objectives functions subject to a set of constraints. In MO there is not a single solution that maximizes / minimizes each objective to its fullest. This happens because the various objectives functions in the problem are usually in conflict with each other. Therefore, the objective in MO is to find the Pareto front of efficient solutions that provide a tradeoff between the various objectives. MOEAs can be useful in the solution of complex problems for which no efficient deterministic algorithm exists (i.e. there is no deterministic algorithm that can solve them in polynomial time) [21], [22], [25], [26]. In finance there are several NP-hard problems for which the use of a heuristic is clearly justified [23], [24]. Portfolio Selection belongs to this category of problems, because of the simultaneous optimization of several conflicting objectives subject to a set of constraints imposed to the problem. Over the past years researchers developed several approaches for the solution of multi-objective optimization problems with the use of EAs. The first implementation of a MOEA dates back to the mid-1980s [20]. Since then, a considerable amount of research has been done in this area, now known as evolutionary multiobjective optimization.

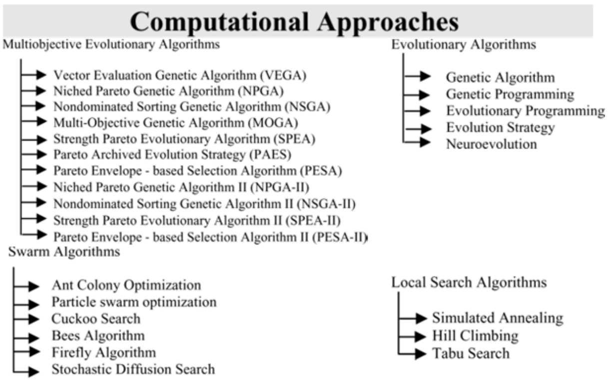

Fig. 4 Computational Techniques for the Solution of Portfolio Optimization Problem

III. Comparative Study of Technologies Applied for Efficient Portfolio Construction

In the previous pages we covered a considerable number of techniques applied for efficient portfolio construction. We saw that such diverse fields as finance, mathematics, stochastics and computer science have been utilized for finding reliable answers to the questions posed by the portfolio selection problem complexity. Below we will attempt to provide a comprehensive comparative study of the different technologies applied for constructing efficient portfolio. The purpose of this comparison is to highlights the strengths and weaknesses of the various technologies.

A. *Financial Approaches*

The strong point of the financial techniques is that most of the times they provide a theoretical framework that interpret the empirical results. The framework is usually based on actual practices and regulations of the financial markets. To state it otherwise for financial people it is not good enough the model to provide reliable results but they also need to confirm these results from a financial perspective.

The main criticism of the financial approaches is focused on the simplified nature of the various financial models. To put in other words the financial theories and models are simplified representations of the actual world. However, because of this abstraction the financial models loose a significant part of their ability to tackle efficiently real world portfolio selection problems. A characteristic example is the Mean-Variance (MV) model introduced by Markowitz (1952). The MV model was criticized for unrealistic assumptions, such as i. No transaction costs in buying and selling securities, ii. An investor can take any position of any size in any security he wishes, iii. Investor does not consider taxes and is indifferent to receiving dividends when making investment decisions, iv. Variance penalizes any dispersion from the expected return. The MV model was far from being the final answer to the problem of portfolio selection. However, it was Markowitz's work that triggered the interest of scholars in this particular research field.

B. *Mathematical Approaches*

Financial mathematics provides rigorous formulas for assets pricing. In other words mathematical approaches are mainly concerned with the determination of the fair value of the various financial instruments. That way it becomes possible to estimate whether or not an asset is fairly valued or it is undervalued or overvalued and correspondently respond the potential investor. Mathematical approaches are characterized by their deterministic nature. To put it in other words randomness is not included in the model construction.

The criticism about the mathematical approaches is focused mainly in the abstract structure of these models. In other words most of the times the mathematical models are simplified approaches of the real world. This is happening because if we apply in the portfolio selection problem some real world constraints and multiple objectives the problem become so complicate that in many cases cannot be solved with exact methods. Another criticism of the mathematical techniques is that they do not provide adequate theoretical reasoning of the derived results.

C. *Stochastic Approaches*

The Stochastic approaches disciplinary belong to the study of Financial Mathematics, however are distinguished from the mathematical approaches because in the model construction take into consideration the randomness. Stochastic models are used extensively in financial markets for assets pricing. The main advantage of stochastic techniques is that they allow us to incorporate into the portfolio selection models some real world parameters and to obtain estimates of the potential outcomes.

The main limitation of the stochastic approaches has been identified to be the complexity associated with this kind of problems. The complexity varies according to the formulation of each individual stochastic model. Thus, different formulations of the stochastic models lead to considerably different complexities. However the constant improvement of the new generations of computers guarantees that this problem will be of less concern with the pass of time. Another weakness of the stochastic approaches is that they provide multiple solutions. However, it is the nature of the stochastic processes that allows the answer to vary from run to run within a range determined by the convergence criteria. Finally, as another issue of concern with stochastic processes, has been identified the convergence level. The convergence level of the solution must be quite high to achieve an accurate solution but no so high as to increase the processing time significantly.

D. Computational Approaches

Computational technologies applied to the portfolio selection problem have become increasingly popular relatively recently [15], [16]. Not only because they provide a fast and reliable way of calculating computationally demanding financial models but also why revolutionized the financial modeling research field itself by developing innovative algorithmic approaches for solving difficult financial problems that in many cases cannot be solved with exact methods. A considerable number of computational techniques have been devised and applied successfully for constructing efficient portfolios. Computational techniques are better able to address the shortcomings of other approaches like for instance and financial and mathematic approaches. This is because computational technologies like the Evolutionary Algorithms (EAs) can deal simultaneously with a set of possible solutions (population) which allows finding several members of the Pareto optimal set in a single run of the algorithm, instead of having to perform a series of separate runs as in the case of the traditional mathematical programming techniques. Moreover, EAs are less susceptible to the shape or continuity of the Pareto front (they can easily deal with discontinuous and concave Pareto fronts), whereas these two issues are known problems with mathematical programming techniques. MOEAs can be particularly useful in the solution of complex portfolio selection problems for which no efficient deterministic algorithm exists [14], [17], [27]. An issue that is related to some computational approaches is the computational time required for finding the solution; however new generations of computers and more efficient algorithms guarantee that this problem will be of less concern in the years to come.

IV. Conclusions

The purpose of this paper is to provide an insight into the current state of research in the technologies and algorithms applied for constructing efficient portfolios. For that purpose, we classify the various technologies and algorithms applied for the solution of the portfolio selection problem according to the discipline and the methodological framework followed. We provide a concise presentation of the emerged categories and we are trying to identify which methods considered obsolete and which lie at the heart of the debate. In particular, we notice a shift of the research interest from the financial approaches towards more numerate techniques such as financial mathematics and stochastic approaches. The last decade computational approaches gained popularity as the result of their ability to solve complicate portfolio selection problems that cannot be solved with exact methods. The incorporation in the portfolio selection models of some real world constraints and multiple objectives made them difficult to be solved with exact methods. Computational approaches such as Evolutionary Algorithms in general and MOEAs in particular, can be useful in the solution of complex problems for which no efficient deterministic algorithm exists. Portfolio Selection belongs to this category of problems, because of the simultaneous optimization of several conflicting objectives subject to a set of constraints imposed to the problem.

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