

# Second-Order Slip Flow and Heat Transfer in a Long Isothermal Microchannel

Huei Chu Weng, Chien-Hung Liu

**Abstract**—This paper presents a study on the effect of second-order slip and jump on forced convection through a long isothermally heated or cooled planar microchannel. The fully developed solutions of thermal flow fields are analytically obtained on the basis of the second-order Maxwell-Burnett slip and Smoluchowski jump boundary conditions. Results reveal that the second-order term in the Karniadakis slip boundary condition is found to contribute a negative velocity slip and then to lead to a higher pressure drop as well as a higher fluid temperature for the heated-wall case or to a lower fluid temperature for the cooled-wall case. These findings are contrary to predictions made by the Deissler model. In addition, the role of second-order slip becomes more significant when the Knudsen number increases.

**Keywords**—Microfluidics, forced convection, gas rarefaction, second-order boundary conditions.

## I. INTRODUCTION

NOWADAYS, Microelectromechanical Systems (MEMS) have developed a large number of microfluidic devices in physical, chemical, biological, medical, engineering, and energy-related fields. A fundamental understanding of physical aspects of microscale flow and heat transfer, which may deviate from those of macroscale flow and heat transfer, is required for the technological demands.

Forced convection is often encountered in technically relevant microscale flow and heat transfer problems. Tunc & Bayazitoglu [1] analytically studied the fully developed forced convection in an isoflux rectangular microchannel by solving the Navier-Stokes and energy equations subject to the first-order Maxwell slip and local heat flux boundary conditions. Renksizbulut et al. [2] numerically investigated the developing forced convection in an isothermal rectangular microchannel by using the first-order Maxwell slip and Smoluchowski jump boundary conditions. Shojaeian & Dibaji [3] performed a numerical study of first-order fully developed forced convection in an isothermal triangular microchannel. Sadeghi & Saidi [4] examined the role of viscous dissipation in first-order fully developed forced convection by considering planar and annular microchannels with asymmetric wall heat fluxes. Recently, Çetin [5] modeled the fully developed forced convection in isoflux planar and circular microchannels based on the second-order Deissler and Karniadakis slip and local

heat flux boundary conditions. The Maxwell-Burnett slip law, however, has been shown to be an adequate way to model the second-order slip flow on the wall surfaces at the microscale [6]. Weng [7] analytically solved the Navier-Stokes and energy equations subject to the second-order Maxwell-Burnett slip and local heat flux boundary conditions for the fully developed forced convection in an isoflux heated or cooled planar microchannel.

In this paper, a study on forced convection in a long isothermally heated or cooled planar microchannel is conducted. The Navier-Stokes and energy equations subject to the second-order Maxwell-Burnett slip and Smoluchowski jump boundary conditions are analytically solved for the fully developed flow. The calculated results are presented for air at the standard reference state with complete accommodation. The Deissler and Karniadakis slip models are then tested via the comparisons of predictions made by them with those obtain by the present slip model, so as to see how well these two slip models describe the flow and heat transfer behavior.

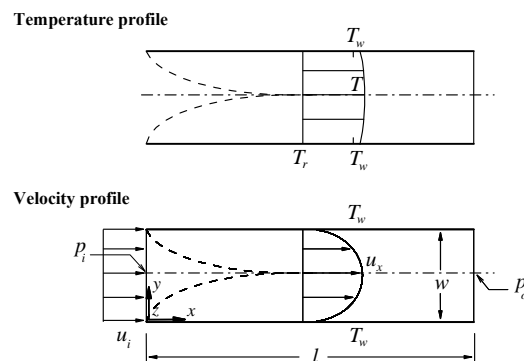


Fig. 1 Diagram of thermal flow configuration and coordinate system

## II. PROBLEM DESCRIPTION

Consider a long symmetrically heated or cooled stationary horizontal planar microchannel of length  $l$  and width  $w$ , whose temperature is  $T_w$ , as shown in Fig. 1. The rarefied gas flow in the microchannel originates from a reservoir at a reference state and terminates in a discharge area of lower pressure. In the system considered, the flow enters the channel with a uniform velocity  $u_i$ . Let  $x$  and  $y$  denote the usual rectangular coordinates, let  $u_x$  and  $u_y$  denote the components of velocity in the  $x$  and  $y$  directions, let  $T$  denote the temperature, and let the subscripts  $i$  and  $o$  denote the inlet and outlet values, respectively. For a sufficiently long

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microchannel, we assume that a hydrodynamically and thermally fully developed flow prevails in the isothermal microchannel, obeying the limit:  $u_y = 0$ ,  $\partial u_x / \partial x = 0$ , and  $\partial T / \partial x = 0$ . The simplified field equations for steady two-dimensional incompressible flow of constant material properties with negligible gravitational field and internal heat generation are

$$0 = -\frac{dp}{dx} + \mu_r \frac{d^2 u_x}{dy^2}, \quad (1)$$

$$0 = k_r \frac{d^2 T}{dy^2} + \mu_r \left( \frac{du_x}{dy} \right)^2, \quad (2)$$

where the subscript  $r$  denotes the reference-state values,  $p$  is the pressure,  $\mu$  is the viscosity, and  $k$  is the thermal conductivity. It should be noted that for a low-speed flow, the field equations could be simplified to incompressible ones. In addition, a small temperature difference between the wall and the reservoir supports the assumption of constant material properties [8].

The corresponding second-order Maxwell-Burnett slip and Smoluchowski jump boundary conditions [9] are

$$\left. \begin{aligned} u_x(0) &= a_1 \frac{2 - \sigma_m}{\sigma_m} \lambda_r \frac{du_x(0)}{dy} + a_2 \lambda_r^2 \frac{d^2 u_x(0)}{dy^2}, \\ u_x(w) &= -a_1 \frac{2 - \sigma_m}{\sigma_m} \lambda_r \frac{du_x(w)}{dy} + a_2 \lambda_r^2 \frac{d^2 u_x(w)}{dy^2}, \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} T(0) &= T_w + b_1 \frac{2 - \sigma_e}{\sigma_e} \lambda_r \frac{dT(0)}{dy}, \\ T(w) &= T_w - b_1 \frac{2 - \sigma_e}{\sigma_e} \lambda_r \frac{dT(w)}{dy}, \end{aligned} \right\} \quad (4)$$

where  $\sigma_m$  and  $\sigma_e$  are the tangential momentum and thermal accommodation coefficients, respectively,  $\lambda$  is the molecular mean free path, related to  $T$  and  $p$  by

$$\lambda = \frac{\sqrt{\pi \hat{R} T / 2 \mu}}{p}, \quad (5)$$

and

$$a_1 = 1, \quad a_2 = -\frac{9}{4\pi} \frac{\gamma_r - 1}{\gamma_r} \text{Pr}, \quad b_1 = \frac{2\gamma_r}{\gamma_r + 1} \frac{1}{\text{Pr}}. \quad (6)$$

Here  $\hat{R}$  is the specific gas constant,  $\gamma$  is the ratio of the constant-pressure specific heat  $c_p$  to the constant-volume specific heat  $c_v$ , and Pr is the Prandtl number. Note that the values of the second-order slip coefficient  $a_2$  used by [5] are 0.5, on the basis of the Karniadakis slip law [10], and  $-1.125$ , on the basis of the Deissler slip law [11]. The comparisons of predictions made by the Karniadakis and Deissler slip laws

with those obtain by the present slip law, which can describe the actual slip flow behavior, could be done to verify the validation of the two second-order boundary conditions.

Equations (1)–(4) can be non-dimensionalized by using the following parameters:

$$\left. \begin{aligned} X &= \frac{x}{l_c \text{Re}}, \quad Y = \frac{y}{l_c}, \quad U = \frac{u_x}{u_c}, \quad \Theta = \frac{T - T_c}{T_w - T_c}, \quad P = \frac{p}{p_c}, \\ \text{Re} &= \frac{\rho_r u_c l_c}{\mu_r}, \quad \text{Br} = \frac{\mu_r u_c^2}{k_r (T_w - T_c)}, \quad \text{Pr} = \frac{c_{p,r} \mu_r}{k_r}, \quad \text{Kn} = \frac{\lambda_r}{l_c}, \end{aligned} \right\} \quad (7)$$

where Re is the Reynolds number, Br is the Brinkman number, and Kn is the Knudsen number. Here  $l_c$ ,  $u_c$ ,  $T_c$ , and  $p_c$  are the characteristic length, velocity, temperature, and pressure, respectively, and defined as:

$$l_c = w, \quad u_c = u_i, \quad T_c = T_r, \quad p_c = \rho_r u_c^2. \quad (8)$$

Thus, the dimensionless field equations can be written as

$$\frac{d^2 U}{dY^2} = \frac{dP}{dX}, \quad (9)$$

$$\frac{d^2 \Theta}{dY^2} = -\text{Br} \left( \frac{dU}{dY} \right)^2, \quad (10)$$

and the corresponding dimensionless boundary conditions are given by

$$\left. \begin{aligned} U(0) &= a_1 \frac{2 - \sigma_m}{\sigma_m} \text{Kn} \frac{dU(0)}{dY} + a_2 \text{Kn}^2 \frac{d^2 U(0)}{dY^2}, \\ U(1) &= -a_1 \frac{2 - \sigma_m}{\sigma_m} \text{Kn} \frac{dU(1)}{dY} + a_2 \text{Kn}^2 \frac{d^2 U(1)}{dY^2}, \end{aligned} \right\} \quad (11)$$

$$\left. \begin{aligned} \Theta(0) &= 1 + b_1 \frac{2 - \sigma_e}{\sigma_e} \text{Kn} \frac{d\Theta(0)}{dY}, \\ \Theta(1) &= 1 - b_1 \frac{2 - \sigma_e}{\sigma_e} \text{Kn} \frac{d\Theta(1)}{dY}. \end{aligned} \right\} \quad (12)$$

The velocity solution of (9) as a function of only  $Y$  is possible only if the pressure gradient  $dP/dX$  is a constant. In addition, a dimensionless conservation condition for the flow rate is given by

$$\int_0^1 U dY = 1. \quad (13)$$

Solving (9) and (10) subject to the boundary conditions (11) and (12) and flow-rate conservation condition (13) gives the following velocity, temperature, and pressure gradient solutions:

$$U(Y) = \frac{1}{2} \frac{dP}{dX} Y^2 + A_1 Y + A_0, \quad (14)$$

$$\Theta(Y) = -Br \left( \frac{1}{12} \left( \frac{dP}{dX} \right)^2 Y^4 + \frac{1}{3} A_1 \frac{dP}{dX} Y^3 + \frac{1}{2} A_1^2 Y^2 \right) + B_1 Y + B_0, \quad (15)$$

$$\frac{dP}{dX} = - \frac{12}{1 + 6 \left( a_1 \frac{2 - \sigma_m}{\sigma_m} - 2a_2 Kn \right) Kn}, \quad (16)$$

where

$$\left. \begin{aligned} A_0 &= -\frac{1}{2} \left( a_1 \frac{2 - \sigma_m}{\sigma_m} - 2a_2 Kn \right) Kn \frac{dP}{dX}, \\ A_1 &= -\frac{1}{2} \frac{dP}{dX}, \\ B_0 &= 1 + b_1 \frac{2 - \sigma_e}{\sigma_e} Kn B_1, \\ B_1 &= - \frac{Br \left( \frac{1}{12} \left( \frac{dP}{dX} \right)^2 + \frac{1}{3} A_1 \frac{dP}{dX} + \frac{1}{2} A_1^2 \right) + b_1 \frac{2 - \sigma_e}{\sigma_e} Kn \left( \frac{1}{3} \left( \frac{dP}{dX} \right)^2 + A_1 \frac{dP}{dX} + A_1^2 \right)}{1 + 2b_1 \frac{2 - \sigma_e}{\sigma_e} Kn} \end{aligned} \right\} \quad (17)$$

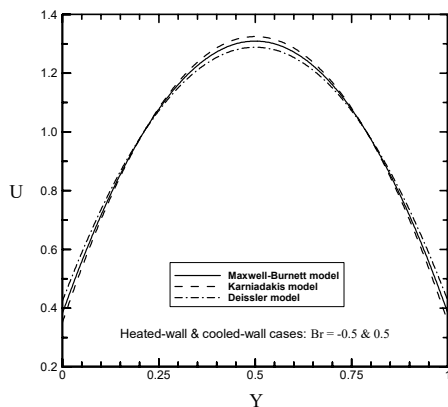


Fig. 2 Velocity distribution for different second-order slip models with  $Kn = 0.1$

### III. RESULTS AND DISCUSSION

Air is used in many engineering application fields. We now pay attention to the influence of second-order slip on the forced convection of air at the standard reference state ( $T_r = 25^\circ\text{C}$  and  $p_r = 1\text{ atm}$ ) with complete accommodations ( $\sigma_m = 1$  and  $\sigma_e = 1$ ). The physical properties at this state can be found in [8]. The parametric analysis of this problem is performed over the range  $-0.5 \leq Br \leq 0.5$ , and the chosen reference value of  $Kn$  (or  $w$ ) for the analysis is 0.1 (or  $0.667\text{ }\mu\text{m}$ ).

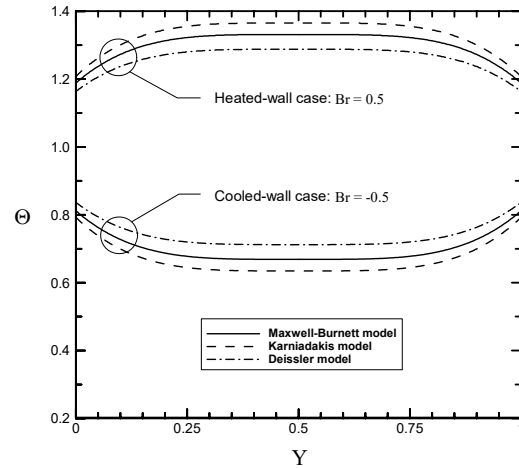


Fig. 3 Temperature distribution for different second-order slip models with  $Kn = 0.1$

In Figs. 2–4, we investigate the influence of second-order slip on the velocity, temperature, and pressure gradient relative to the heated-wall case  $Br > 0$  and the cooled-wall case  $Br < 0$  at a microscale level ( $Kn = 0.1$ ). Fig. 2 illustrates the velocity profiles for the Maxwell-Burnett model ( $a_2 = -0.145$ ), the Karniadakis model ( $a_2 = 0.5$ ), and the Deissler model ( $a_2 = -1.125$ ). Comparisons with the Karniadakis and Deissler solutions show that the Karniadakis model predicts a significantly relatively small velocity slip while the Deissler model predicts a significantly relatively large slip. However, it is seen that the Deissler model predicts a significantly relatively large velocity close to the center while the Deissler model predicts a significantly relatively small velocity. Fig. 3 illustrates the temperature profiles for the three models. It should be noted that the viscous dissipation in a flowing fluid may lead to the rise of the temperature  $T$ , so that, from the figure, the dimensionless gas temperature  $\Theta$  increases in a heated microchannel but decreases in a cooled microchannel. It is observed that the second-order term in the Karniadakis slip boundary condition causes more viscous dissipation but the second-order Deissler term causes less viscous dissipation. Fig. 4 illustrates the variations of the pressure gradient  $dP/dX$  with the Knudsen number  $Kn$ . It is found that the second-order term in the slip boundary condition could play an important role in the slip flow regime ( $0.01 < Kn \leq 0.1$ ). The Karniadakis model predicts relatively small pressure gradient values, while the Deissler model predicts relatively large values. The smaller (larger) pressure gradient means that the second-order slip flow displays a higher (lower) pressure drop. When the value of  $Kn$  increases; the effects of the second-order Karniadakis and Deissler slips increase.

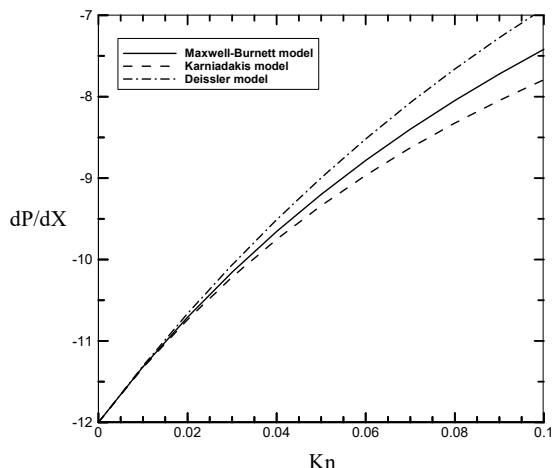


Fig. 4 Pressure gradient versus Kn

#### IV. CONCLUSIONS

An analytical study on forced convection in a long heated or cooled planar microchannel with symmetric wall temperatures has been made by solving the Navier-Stokes and energy equations subject to the second-order Maxwell-Burnett slip and Smoluchowski jump boundary conditions. The fully developed solutions of velocity, temperature, and pressure gradient were presented for air at the standard reference state with complete accommodations. The Deissler and Karniadakis models were tested via the comparisons of predictions made by them with those obtain by the present model. For flow analysis, it was found that the second-order term in the Karniadakis slip boundary condition contributes a negative velocity slip and then leads to a higher pressure drop, while the Deissler slip model predicts a positive slip and then a lower drop. As for heat transfer analysis, it was observed that, for the heated-wall case, the second-order term in the Karniadakis slip boundary condition results in a higher temperature rise, while the Deissler slip model predicts a lower rise. The conclusions for the cooled-wall case were found to be contrary to the heated-wall predictions. The effects of the second-order Karniadakis and Deissler slips can be enhanced by increasing the Knudsen number.

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**Huei Chu Weng** is currently an Associate Professor in the Department of Mechanical Engineering at Chung Yuan Christian University and has taught a variety of undergraduate and graduate courses in all the major disciplines, including Thermodynamics, Fluid Mechanics, Heat Transfer, Introduction of Fluid Mechanics, Convective Heat Transfer, and Theory and Applications of Nanofluids.

His primary research interests are in the fields of thermal-fluid and energy sciences. Research topics include Smart Nanomaterial Science, Micro/Nanoscale Thermal-Fluid Science, Power and Energy Science. A summary of academic contributions is given below:

##### 1. Smart nanomaterial science:

A magnetic fluid is a colloidal magnetic nanofluid and the importance arises from scientific, industrial, and commercial applications. Weng proposed a modification in the magnetization equation (the WC model) in the dynamics of magnetic fluids, initiated the study of the influence of the spin of magnetic moments within particles on the non-Newtonian flow, and reviewed the arguments for different field equations in hydrodynamics of fluids with micro/nanostructure and stated the deterministic nature. Recently, he performed an analysis for the effects of particles and magnetic field on biomedical magnetic fluid flow to study the targeted magnetic-particle delivery in a blood vessel.

##### 2. Micro/nanoscale thermal-fluid science:

Micro/nanotechnology develops a large number of microfluidic and nanofluidic systems. The importance of micro/nanoflow arises from new applications in these system devices. Weng initiated the studies of buoyancy-driven gas microflow and thermocreep-driven gas microflow. He also placed emphasis on the importance of thermal creep and examined the roles of variable physical properties and wall-surface curvature. Recently, he found that based on the Navier-Stokes (NS) equations subject to the second-order slip boundary conditions, continuum modeling can be valid for the Knudsen numbers up to 1.60 and provided a more detailed microscale theory of magnetogasdynamics (MGD) and developed a mathematical model of pressure-driven gas flow and heat transfer through a heated microchannel in the presence of an applied electric and magnetic field.

As a scientist and researcher Prof. Weng have published more than 24 peer reviewed journal papers and hold the following memberships:

- ASME: American Society of Mechanical Engineers
- CSME: Chinese Society of Mechanical Engineers
- STAM: Society of Theoretical and Applied Mechanics
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