International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:9, No:9, 2015

Delay-Independent Closed-Loop Stabilization of Neutral System with Infinite Delays

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***Abstract*—**In this paper, the problem of stability and stabilization for neutral delay-differential systems with infinite delay is investigated. Using Lyapunov method, new delay-independent sufficient condition for the stability of neutral systems with infinite delay is obtained in terms of linear matrix inequality (LMI). Memory-less state feedback controllers are then designed for the stabilization of the system using the feasible solution of the resulting LMI, which are easily solved using any optimization algorithms. Numerical examples are given to illustrate the results of the proposed methods.

***Keywords*—**Infinite delays, Lyapunov method, linear matrix inequality, neutral systems, stability.

I. Introduction

A. Study Background

DIFFERENTIAL equations are important model for harnessing different components into a single system and analyse the inter-relationship that exist between these components which otherwise might continue to remain independent of each other [1]. The most commonly encountered models in the theory of differential equations are those physical systems which express the present states of a situation. However, more realistic physical system models take into account the past states or history of the system, otherwise referred to as time delays, as well as the present state of situations.

Differential equations which involve the present as well as the past states are called delay differential equations or functional differential equations. Delay differential equations are of two broad types: retarded functional differential equations and neutral functional differential equations see [1] and references therein. This paper focuses on the later type, in which the derivatives of the past history or derivatives of functional of the past history are involved as well as the present states of the system.

The existence of time delays in a dynamical system has been the source of poor system performances and even instability. Studies involving different time delays can be found in ship stabilization, control processes for pressure, and heat transfer regulation, but, they are sometimes deliberately introduced into feedback systems to improve system performances see [2] and references therein for details.

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B. Literatures

Several analysis technique have been used in studying the stability and stabilizations of neutral systems which includes the Lyapunov-based, fixed point-based, and spectral radius approach, see [3]-[13]. The Lyapunov-based approaches which are the focus of application in this paper are classified into Razumikhin [6] and the Krasovskii [13] approaches. The Krasovskii's approaches are the most widely used compared to other stability techniques since they often leads to LMI results and can be applied to a wide range of problems that may provide necessary and sufficient conditions. However, they are computationally complex and present poor scalability [14].

Several stability and stabilization of neutral systems results are obtained by the Lyapunov-Krasovskii approach, which leads to LMI results [15], [5], [8]. In particular, [8] considered the stability analysis of neutral delay differential systems and presented sufficient conditions for stability using LMI and Krasovskii approach. Using a similar approach [15] obtained delay-dependent stabilization result of neutral systems with saturating actuators by using static state feedback of systems subject to time-varying delays.

C. Motivation and Contribution

The study of integrodifferential equations with infinite delay has emerged in recent years as an independent branch of modern research because of its wide range of applicable areas see [16] and references therein. Motivated by the works of [1] [4], [5], [8], [15], and the wide range of applicable areas presented by integrodifferential equation with infinite delays especially in the area of epidemics and population growth, this paper investigates the stability and stabilization of neutral functional integrodifferential system with infinite delays by presenting a less conservative results for the stability and stabilization for such systems. By using LMI and the Lyapunov-Krasovskii approach, a new delay-independent condition which is sufficient to make the system uniformly asymptotically stable is developed. Furthermore, a new stabilization criterion and memory-less state feedback controllers are proposed using the same methods and the corresponding design procedures to stabilize the system. The paper extends other known delay independent stability results to neutral functional integrodifferential system with infinite delays; numerical examples are given to illustrate the effectiveness of the proposed methods

D. Organization of the Paper

The rest of the paper is organized as follows: Section II contains mathematical notations, preliminaries and definition on the subject of research. In Section III, the stability results are presented as theorems and proofs which are based on LMI and the Lyapunov-Krasovskii approach. Section IV contains result derived from stabilization conditions and memory-less state feedback designed for the system using the same methods as in Section III. Finally, Section V contains numerical examples which are an illustration of the design procedure and effectiveness of the theoretical results prior to the conclusions.

II. Notations and Preliminaries

A. Notations

Suppose, $h > 0$ is a given number, $E = (-\infty, \infty)$, E^n is a real n – dimensional Euclidean space with norm $|\cdot|$. C= $C([-h, 0], E^n)$ is the space of continuous function mapping the interval $[-h, 0]$ into E^n with the norm $\|\cdot\|$, where $\|\phi\| = \sup_{-h<s\le 0}|\phi(s)|$, I denotes the identity matrix order, and $*$ represents the elements below the main diagonal of a symmetric block matrix.

B. Preliminaries

Consider neutral system

$$\dot{x}(t) - A_0\dot{x}(t-h) = A_1x(t) + A_2x(t-h). \quad (1)$$

This study is based on its extension to neutral functional integrodifferential system with infinite delays of the form

$$\left.\begin{aligned} \dot{x}(t) - A_0\dot{x}(t-h) = A_1x(t) + A_2x(t-h) \\ + \int_{-\infty}^{0} G\big(t, x(t)\big)dt \\ x(t) = \phi(t), t \in [-h, 0] \end{aligned}\right\} \quad (2)$$

and its control base system

$$\dot{x}(t) - A_0\dot{x}(t-h) = A_1x(t) + A_2x(t-h) + Bu(t) + \int_{-\infty}^{0} G\big(t, x(t)\big)dt \quad (3)$$

where $x(t) \in E^n$ is the state vector, $u(t) \in E^m$ is a control variable, and the following assumptions: H_0: A_0, A_1, and A_2 are $n \times n$ constant matrices, H_1: B is an $n \times m$ constant matrix, H_2: $G: (-\infty, 0] \times (-\infty, 0] \times C \to E^n$ is a continuous matrix function which satisfies $\|G(t, x)\| \le M(t, s)\|x\|$ for all $(t, \phi) \in (-\infty, 0] \times C$, where $\int_{-\infty}^{0} M(s)ds = -l < \infty$.

It is assumed that G satisfy enough smoothness conditions to ensure that a solution of (2) exists through each $(t_0, \phi), t \ge t_0 \ge 0$, is unique, and depends continuously upon (t_0, ϕ) and can be extended to the right as long as the trajectory remains in a bounded set $[t_0, \infty) \times C$. These conditions are given in [4].

Lemma 1. For any real vector D and Z with appropriate dimension and any positive scalar τ, then

$$DZ + Z^TD^T \le \tau DD^T + \tau^{-1}Z^TZ$$

Proof: See [17].

Lemma 2. The linear matrix inequality

$$\begin{pmatrix} Z(x) & Y(x) \\ Y^T(x) & W(x) \end{pmatrix} > 0$$

is equivalent to $W(x) > 0$, $Z(x) - Y(x)W^{-1}(x)Y^T(x) > 0$, where $Z(x) = Z^T(x)$, $W(x) = W^T(x)$ and $Y(x)$ depend affinely on x.

Proof: See [18].

III. Stability of Neutral System with Infinite Delays

Here, a delay-independent criterion for the asymptotic stability of (2) in terms of LMI using the standard Lyapunov-Krasovskii approach will be developed and proved.

Theorem 1. The neutral functional integrodifferential system with infinite delays described by (2) is asymptotically stable for all $h \ge 0$ if there exists positive symmetric matrices $P, R > 0$, and some positive scalars $\tau_0, \tau_1, \tau_2 > 0$ which satisfy the following LMI

$$Z(P, R, \tau_0, \tau_1, \tau_2) = \begin{pmatrix} Z_{11} & Z_{12} & (2A_2 + 2XA_1^TA_2) & (2A_0 + 2XA_1^TA_0) \\ * & Z_{22} & 0 & 0 \\ * & * & A_2^TA_2 - R + \tau_1A_2^TA_2 & 2A_2^TA_0 \\ * & * & * & A_0^TA_0 - I + \tau_2A_0^TA_0 \end{pmatrix} < 0 \quad (4)$$

where,

$$Z_{11} = XA_1^T + A_1X - 2lX,$$

$$Z_{12} = [XA_1^T \quad \tau_0XA_1^T \quad XR \quad LX \quad LX \quad LX \quad LX],$$

$$Z_{22} = \mathrm{diag}\{-I, \ -\tau_0I, \ -R, \ -I, \ -\tau_0I, \ -\tau_1I, -\tau_2I\},$$

Proof: Let the Lyapunov function candidate be given by

$$V = V_1 + V_2 + V_3$$

where,

$$V_1 = x^T(t)Px(t), V_2 = \int_{-h}^{0} \dot{x}^T(t+s)\dot{x}(t+s)ds \text{ and}$$

$$V_3 = \int_{-h}^{0} x^T(t+s)Rx(t+s)ds$$

Taking the derivative of V along the solution of (2) gives

$$\dot{V}_1 = x^T(A_1^TP + PA_1)x + 2x^TPA_2x_h + 2x^TPA_0\dot{x}_h + 2x^TP\int_{-\infty}^{0} G(t, x)ds. \quad (5)$$

$$\dot{V}_2 = \dot{x}^T\dot{x} - \dot{x}_h^T\dot{x}_h = x^TA_1^TA_1x + x_h^TA_2^TA_2x_h + \dot{x}_h^TA_0^TA_0\dot{x}_h + \left(\int_{-\infty}^{0} G(t, x)ds\right)^T\int_{-\infty}^{0} G(t, x)ds + 2x^TA_1^TA_2x_h + 2x^TA_1^TA_0\dot{x}_h + 2x_h^TA_2^TA_0\dot{x}_h + 2x^TA_1^T\int_{-\infty}^{0} G(t, x)ds + 2x_h^TA_2^T\int_{-\infty}^{0} G(t, x)ds + 2\dot{x}_h^TA_0^T\int_{-\infty}^{0} G(t, x)ds - \dot{x}_h^T\dot{x}_h. \quad (6)$$

$$\dot{V}_3 = x^TRx - x_h^TRx_h. \quad (7)$$

where x, x_h and $\dot{x}_h$ denote $x(t), x(t-h)$ and $\dot{x}(t-h)$ respectively. The term $\left(\int_{-\infty}^{0} G(t,x)ds\right)^T \int_{-\infty}^{0} G(t,x)ds$ in (6) can be simplified using Jensen's Inequality [19] as follows,

$$\begin{aligned}\left(\int_{-\infty}^{0} G(t,x)ds\right)^T \int_{-\infty}^{0} G(t,x)ds &= \left(\int_{-\infty}^{0} \|G(t,x)ds\|\right)^T \int_{-\infty}^{0} \|G(t,x)ds\| \\ &\leq \left(\int_{-\infty}^{0} m(s)ds\|x\|\right)^T \int_{-\infty}^{0} m(s)ds\|x\| \\ &\leq \left(\int_{-\infty}^{0} |m(s)ds|\right) \int_{-\infty}^{0} |m(s)ds| \|x\|.\|x\| \\ &\leq l \int_{-\infty}^{0} m(s)ds\|x\|^2 \leq l^2\|x\|^2 = l^2x^Tx\end{aligned} \tag{8}$$

Applying Lemma 1 with (8) to the following terms in (5) and (6) gives;

$$2x^TP\int_{-\infty}^{0} G(t,x)ds \leq -2x^TPlx \tag{9}$$

$$\begin{aligned}&2x^TA_1^T \int_{-\infty}^{0} G(t,x)ds \\ &\leq \tau_0 x^TA_1^TA_1x + \tau_0^{-1}\left(\int_{-\infty}^{0} G(t,x)ds\right)^T \int_{-\infty}^{0} G(t,x)ds \\ &\leq \tau_0 x^TA_1^TA_1x + \tau_0^{-1}l^2x^Tx\end{aligned} \tag{10}$$

$$\begin{aligned}&2x_h^TA_2^T \int_{-\infty}^{0} G(t,x)ds \\ &\leq \tau_1 x_h^TA_2^TA_2x_h + \tau_1^{-1}\left(\int_{-\infty}^{0} G(t,x)ds\right)^T \int_{-\infty}^{0} G(t,x)ds \\ &\leq \tau_1 x_h^TA_2^TA_2x_h + \tau_1^{-1}l^2x^Tx\end{aligned} \tag{11}$$

$$\begin{aligned}&2\dot{x}_h^TA_0^T \int_{-\infty}^{0} G(t,x)ds \\ &\leq \tau_2 \dot{x}_h^TA_0^TA_0\dot{x}_h + \tau_2^{-1}\left(\int_{-\infty}^{0} G(t,x)ds\right)^T \int_{-\infty}^{0} G(t,x)ds \\ &\leq \tau_2 \dot{x}_h^TA_0^TA_0\dot{x}_h + \tau_2^{-1}l^2x^Tx\end{aligned} \tag{12}$$

where $\tau_0, \tau_1, \tau_2 > 0$ are scalars to be chosen. The overall derivative of V along the solution of (2) can now be expressed as

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \leq \lambda^T(t)Z(P,R,\tau_0,\tau_1,\tau_2)\lambda(t),$$

where

$$M(P,R,\tau_0,\tau_1,\tau_2) = \begin{pmatrix} M_{11} & (2PA_2 + 2A_1^TA_2) & (2PA_0 + 2A_1^TA_0) \\ * & M_{22} & A_2^TA_0 \\ * & * & A_0^TA_0 - I + \tau_2A_0^TA_0 \end{pmatrix},$$

and $\lambda(t) = [x^T,\ x_h^T,\ \dot{x}_h^T]^T$, so that,

$$\begin{aligned}M_{11} = A_1^TP + PA_1 - 2Pl + A_1^TA_1 + \tau_0A_1^TA_1 + R + l^2I + \tau_0^{-1}l^2I \\ + \tau_1^{-1}l^2I + \tau_2^{-1}l^2I \\ M_{22} = A_2^TA_2 - R + \tau_1A_2^TA_2\end{aligned}$$

Pre and most multiplying $M(\cdot)$ by Γ^{-T} and Γ; and now using the Schur complement gives $Z(X,R,\tau_0,\tau_1,\tau_2)$ where

$$\Gamma = \begin{pmatrix} X & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$$

It then follows that $\dot{V}$ is negative definite since $M(\cdot) < 0$ is equivalent $Z(\cdot) < 0$, which implies that (2) is asymptotically stable see [20].

IV. Stabilization of Neutral System with Infinite Delays Result

Here, a delay - independent stability criterion for the stabilization of the closed loop system is developed.

The interest now is to design a memory-less state feedback controller $u(t)$ for (3) as

$$u(t) = -B^TPx(t) \tag{13}$$

where $P \in E^{m\times n}$ is a positive-definite matrix to be designated.

The closed-loop system design for (3), using (13) is defined by

$$\frac{d}{dt}D(t)x_t = (A_1 - BB^TK)x(t) + A_2x(t-h) + \int_{-\infty}^{0} G(t, x(t))ds \tag{14}$$

The task here is to ensure that (14) is closed-loop asymptotically stable.

Theorem 2. Consider (3) and all its assumptions; if there exists positive symmetric matrices $P,\ R > 0$, some positive scalars $\tau_4, \cdots, \tau_6 > 0$ and a positive-definite symmetric matrix $X \in E^{m\times n}$ which satisfy the following LMI

$$Z(X,R,\tau_4,\cdots,\tau_6) = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ * & Z_{22} & 0 & 0 \\ * & * & Z_{33} & 2A_2^TA_0 \\ * & * & * & A_0^TA_0 - I + \tau_6A_0^TA_0 \end{pmatrix} < 0 \tag{15}$$

so that,

$$Z_{11} = XA_1^T + A_1X - 2BB^T - 2lX - 2XA_1^TBB^T + 2BB^TlX,$$

$$Z_{12} = [XA_1^T \quad BB^T \quad \tau_4XA_1^T \quad XR \quad LX \quad LX \quad LX \quad LX],$$

$$Z_{13} = 2A_2 + 2XA_1^TA_2 - 2BB^TA_2$$

$$Z_{14} = 2A_0 + 2XA_1^T A_0 - 2BB^T A_0$$

$$Z_{22} = \text{diag}\{-I, -I, -\tau_4 I, -R, -I, -\tau_4 I, -\tau_5 I, -\tau_6 I\},$$

$$Z_{33} = A_2^T A_2 - R + \tau_5 A_2^T A_2$$

where $X = P^{-1}$. Then, (3) is closed–loop asymptotically stable, and the input $u(t) = -B^T Px(t)$ is a controller for (3).

Proof: Let the Lyapunov function be given by

$$V = V_1 + V_2 + V_3$$

where,

$$V_1 = x^T(t)Px(t), V_2 = \int_{-h}^{0} \dot{x}^T(t+s)\dot{x}(t+s)ds,$$

and

$$V_3 = \int_{-h}^{0} x^T(t+s)Rx(t+s)ds,$$

Taking the derivative of V along the solution of (3) gives

$$\dot{V}_1 = x^T(A_1^T P + PA_1 - 2PBB^T P)x + 2x^T PA_2 x_h + 2x^T PA_0 \dot{x}_h + 2x^T P \int_{-\infty}^{0} G(t,x)ds \quad (16)$$

$$\begin{aligned}\dot{V}_2 = \dot{x}^T\dot{x} - \dot{x}_h^T\dot{x}_h = x^T A_1^T A_1 x + x^T PBB^T BB^T Px + x_h^T A_2^T A_2 x_h \\ + \dot{x}_h^T A_0^T A_0 \dot{x}_h \\ + \left(\int_{-\infty}^{0} G(t,x)ds\right)^T \int_{-\infty}^{0} G(t,x)ds - 2x^T A_1^T BB^T Px \\ +2x^T A_1^T A_2 x_h + 2x^T A_1^T A_0 \dot{x}_h - 2x^T PBB^T A_2 x_h \\ -2x^T PBB^T A_0 \dot{x}_h + 2x_h^T A_2^T A_0 \dot{x}_h + 2x^T A_1^T \int_{-\infty}^{0} G(t,x)ds \\ -2x^T PBB^T \int_{-\infty}^{0} G(t,x)ds + 2x_h^T A_2^T \int_{-\infty}^{0} G(t,x)ds \\ +2\dot{x}_h^T A_0^T \int_{-\infty}^{0} G(t,x)ds - \dot{x}_h^T \dot{x}_h \quad (17)\end{aligned}$$

$$\dot{V}_3 = x^T Rx - x_h^T Rx_h \quad (18)$$

Applying Lemma 1 with (8) to the term $2x^T PBB^T \int_{-\infty}^{0} G(t,x)ds$ in (17) gives;

$$-2x^T PBB^T \int_{-\infty}^{0} G(t,x)ds \leq 2x^T PBB^T lx \quad (19)$$

Using (19) and inequalities (8) – (12), and by replacing the constants τ_0, τ_1, τ_2 by τ_4, τ_5, τ_6 respectively, the overall derivative of V along the solution of (3) can be expressed as $\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \leq \lambda^T(t)Z(P, R, \tau_4, \cdots, \tau_6)\lambda(t)$, where $Z(P, R, \tau_4, \cdots, \tau_6)$ where

$$M(P, R, \tau_0, \tau_1, \tau_2) = \begin{pmatrix} M_{11} & M_{12} & (2PA_0 + 2A_1^T A_0 - 2PBB^T A_0) \\ * & M_{22} & 2A_2^T A_0 \\ * & * & A_0^T A_0 - I + \tau_6 A_0^T A_0 \end{pmatrix},$$

and $\lambda(t) = [x^T,\ x_h^T,\ \dot{x}_h^T\ ]^T$, so that,

$$M_{11} = A_1^T P + PA_1 - 2PBB^T P - 2Pl - 2A_1^T BB^T P + 2PBB^T l$$

$$+A_1^T A_1 + PBB^T BB^T P + \tau_4 A_1^T A_1 + R + l^2 I + \tau_4^{-1} l^2 I$$

$$+\tau_5^{-1} l^2 I + \tau_6^{-1} l^2 I$$

$$M_{12} = 2PA_2 + 2A_1^T A_2 - 2PBB^T A_2$$

$$M_{22} = A_2^T A_2 - R + \tau_5 A_2^T A_2$$

Pre and most multiplying $M(\cdot)$ by Γ^{-T} and Γ; and now using the Schur complement gives $Z(X, R, \tau_4, \tau_5, \tau_6)$ where

$$\Gamma = \begin{pmatrix} X & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$$

It follows then that $\dot{V}$ is negative definite since $M(\cdot) < 0$ is equivalent $Z(\cdot) < 0$, which implies that (3) is closed-loop asymptotically stable see [20].

Remark 1. The problems in Theorem 1 and 2 are feasibility problems. The solution can be found by solving it in the form of a generalized eigenvalue problem, see [18] for details. In this paper, the solution was found by utilizing the MATLAB's LMI Control Toolbox [21], which implements interior point algorithm.

V. Numerical Examples

Here, numerical examples will be given to illustrate the proposed methods

A. Example 1

Consider the neutral system with infinite delay given by

$$\dot{x}(t) - A_0\dot{x}(t-h) = A_1 x(t) + A_2 x(t-h) + \int_{-\infty}^{0} G(t, x(s))ds \quad (20)$$

where,

$$A_0 = \begin{pmatrix} 0 & 0.4 \\ 0.4 & 0 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad A_2 = \alpha\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$G(t, x(t)) = \begin{pmatrix} 0 \\ -\exp(t-3) \times Sin\, x(t) \cdot x(t) \end{pmatrix}$$

Note that, the function $G(t,x)$ satisfies its conditions with,

$$M(t) = -\exp(t-3) \times Sin\, x(t);$$

$$\int_{-\infty}^{0} M(t)dt = -\exp(-3)/2 = l = -0.02489.$$

Now the bound of α for the asymptotic stability for neutral systems without infinite delay as given in Example 2 of [8] is as follows; [22]: $|\alpha| \leq 0.2$, [23]: $|\alpha| \leq 0.2$, [8]: $|\alpha| \leq 0.9165$, This paper (Theorem 1): $|\alpha| \leq 23700$.

The solutions of the LMI (4) for $\alpha = 23700$ are given as

$$\tau_0 = 0.6662, \tau_1 = 0.0066, \tau_2 = 0.0375, X = \begin{pmatrix} 1.0006 & 0 \\ 0 & 1.0006 \end{pmatrix}$$

$$\text{and } R = \begin{pmatrix} 9.9862 \times 10^8 & 0 \\ 0 & 9.9862x10^8 \end{pmatrix}$$

It is observed that Theorem 1 gives a less conservative bound of α than all the proposed methods in [8].

B. Example 2

Using (20) with the assumption that the systems matrices are equivalent to

$$A_0 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

where G is as defined in Example 1 above. Solving the LMI (15) gives $\tau_4 = 0.1692$, $\tau_5 = 257.3315$, $\tau_6 = 1.5197$, $X = \begin{pmatrix} 0.1011 & 0 \\ 0 & 0.1011 \end{pmatrix}$ and $R = \begin{pmatrix} 1778 & 0 \\ 0 & 1778 \end{pmatrix}$.

Therefore the stabilizing feedback controller $u(t)$ for (20) is

$$u(t) = -B^T P x(t) = -B^T X^{-1} x(t) = -\begin{pmatrix} 9.8949 & 0 \\ 0 & 9.8949 \end{pmatrix} x(t)$$

VI. Conclusion

In this paper, new sufficient conditions are derived for the stability and stabilization of neutral systems with infinite delays. The new stability conditions were obtained by using the Lyapunov stability approach which are then expressed in terms of LMI and solved by using the MATLAB's LMI Toolbox. The stabilization of the system was obtained by designing a memory-less state feedback control law which is presented in terms of LMI and solved by using the MATLAB's LMI Toolbox. Numerical examples were provided to demonstrate the effectiveness of the new sufficient conditions.

References

[1] I. Davies, "Euclidean Null Controllability of Infinite Neutral Differential Systems," *Anziam Journal.* vol. 48, pp. 285-294, 2006.

[2] V. B. Kolmanovskii, and A. Myshkis, *Applied Theory of Functional Differential Equations*. Kluwer Academic Publishers, Boston, 1992.

[3] A. Ardjouni, and A. Djoudi, "Fixed Points and Stability in Linear Neutral Differential Equations with Variable Delays," *Nonlinear Analysis: Theory, Methods & Applications*. Vol.74, pp. 2062-2070, 2011.

[4] M. A. Cruz, and J. K. Hale, "Stability of Functional Differential Equations of Neutral Type," *Journal of Differential Equations*. Vol. 7, pp. 334-355, 1970.

[5] Y. He, M. Wu, and J. She, "Delay Dependent Robust Stability and Stabilization of Uncertain Neutral Systems," *Asian Journal of Control.* vol. 10, pp. 376-383, 2008.

[6] P. Liu, "On the Stability of Neutral-Type Uncertain Systems with Multiple Time Delays," *International Journal of Applied Mathematics and Computer Science.* Vol. 15, pp. 221-229, 2005.

[7] W. K. Liu, and W. Jiang, "Stability of Fractional Neutral Systems," *Advances in Differential Equations.* Vol. 2014: 78, pp. 1-9, 2014.

[8] J. H. Park, and S. Won, "Stability Analysis for Neutral Delay-Differential Systems", *Journal of the Franklin Institute*. Vol. 337, 1-9, 2000.

[9] F. Qiu and Q. Zhang, "Robust Stability of Neutral System with Mixed Time-Varying Delays and Nonlinear Perturbations Using Delay Decomposition Approach," *Abstract and Applied Analysis*. Vol. 2014, pp. 1-11, 2014.

[10] E. I. Verriest, and S. I. Niculescu, Delay-Independent Stability of Linear Neuytral Systems: A Riccati Equation Approach, European Control Conference, Brussels, Belgium. pp. 3632-3636, July, 1997.

[11] T. Vyhlídal, N. Olgac, and V. Kučera, "Delayed Resonator with Acceleration Feedback–Complete Stability Analysis by Spectral Methods and Vibration Absorber Design," *Journal of Sound and Vibration*. Vol. 333, pp. 6781-6795, 2014.

[12] H. Xia, P. Zhao, L. Li, Y, Wang, and A. Wu, "Improved Stability Criteria for Linear Neutral Time-Delay Systems," *Asian Journal of Control*. Vol. 17, pp. 343-351, 2015.

[13] B. Yang, J. Wang and C. Zhong, "A New Delay Dependent Stability Criterion for Neutral Delay Differential Systems," *Asian Journal of Control.* Vol. 5, pp. 433-436, 2003.

[14] C. Briat, (Ed.), "Robust Stability Analysis in the *-Norm and Lyapunov-Razumikhin Functions for the Stability Analysis of Time-Delay Systems," 50th IEEE Conference on Decision and Control and European Control Conference (*CDC-ECC)*. Orlando, FL, USA. pp. 12-15, 2011.

[15] F. El Haoussi, E. Tissir, A. Hmamed, and F. Tadeo, "Stabilization of Neutral Systems with Saturating Actuators," *Journal of Control Science and Engineering.* Vol. 2012, pp. 1-8, 2012.

[16] K. Balachandran, and J. Dauer, "Null Controllability of Nonlinear Infinite Delay Systems with Time Varying Multiple Delays in Control," *Applied Mathematics Letters.* Vol. 9, pp. 115-121, 1996.

[17] P. P. Khargonekar, I. R. Petersen, and K. Zhou, "Robust Stabilization of Uncertain Linear Systems: Quadratic Stabilizability and h∞ Control Theory," *IEEE Transactions on Automatic Control*. Vol. 35, pp. 356-361, 1990.

[18] S. P. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.

[19] K. Gu, J. Chen, and V. L. Kharitonov, *Stability of Time-Delay Systems*. Springer, Birkhäuser, 2003.

[20] J. K Hale, and S. M. Verduyn Lunel, *Introduction to Functional Differential Equations.* Springer, 1993.

[21] P. Gahinet, A. Nemirovskii, A. J. Laub, and M. Chilali (eds.), *The LMI Control Toolbox for User's Guide,* Mathworks, Natick, MA, 1995.

[22] L. M. Li, "Stability of Linear Neutral Delay-Differential Systems," *Bull. Austral. Math. Soc*. Vol. 38, pp. 339-344, 1988.

[23] G. Di Hu, G. Da Hu, "Some Simple Stability Criteria of Neutral Delay-Differential Systems," *Appl. Math. Comput*. Vol. 80, pp. 257-271, 1996.