

MP-SMC-I Method for Slip Suppression of Electric Vehicles under Braking

Tohru Kawabe

Abstract—In this paper, a new SMC (Sliding Mode Control) method with MP (Model Predictive Control) integral action for the slip suppression of EV (Electric Vehicle) under braking is proposed. The proposed method introduces the integral term with standard SMC gain, where the integral gain is optimized for each control period by the MPC algorithms. The aim of this method is to improve the safety and the stability of EVs under braking by controlling the wheel slip ratio. There also include numerical simulation results to demonstrate the effectiveness of the method.

Keywords—Sliding Mode Control, Model Predictive Control, Integral Action, Electric Vehicle, Slip suppression

I. INTRODUCTION

OVER the past decades, the automobile population has been increasing rapidly in the developing countries. With the wide spread of automobiles all over the world, especially internal-combustion engine vehicles (ICEVs), the environment and energy problems: air pollution, global warming, oil resource exhaustion and so on, are going severely [1]. As a countermeasure to these problems, the development of next-generation vehicles have been focused. EVs run on electricity only and they are zero emission and eco-friendly. So EVs have attracted great interests as a powerful solution against the problems mentioned above [2], [3].

EVs are propelled by electric motors, using electrical energy stored in batteries or another energy storage devices. Electric motors have several advantages over (internal-combustion engines) ICEs:

- 1) The input/output response is faster than for gasoline/diesel engines.
- 2) The torque generated in the wheels can be detected relatively accurately
- 3) Vehicles can be made smaller by using multiple motors placed closer to the wheels.

The travel distance per charge for EV has been increased through battery improvements and using regeneration brakes, and attention has been focused on improving motor performance. The following facts are viewed as relatively easy ways to improve maneuverability and stability of EVs. It's, therefore, important to research and development to achieve high-performance EV traction control.

Much research has been done on the stability of general automobiles, for example, ABS (Anti-lock-Braking Systems), TCS (Traction-Control-Systems), and ESC (Electric-Stability-Control)[4] as well as VSA

(Vehicle-Stability-Assist)[5] and AWC (All-Wheel-Control) [6]. What all of these have in common is that they maintain a suitable tire grip margin and reduce drive force loss to stabilize the vehicle behavior and improve drive performance. With gasoline/diesel engines, however, the response time from accelerator input until the drive force is transmitted to the wheels is slow and it is difficult to accurately determine the drive torque, which limits the vehicle's control performance.

This paper deals with the traction control of EV for slip prevention during braking. EVs have a fast torque response and the motor characteristics can be used to accurately determine the torque, which makes it relatively easy and inexpensive to realize high-performance traction control. This is expected to improve the safety and stability of EV under braking.

Several methods have been proposed for the traction control [7], [8], [9] by using slip ratio of EVs, such as the method based on Model Following Control (MFC) in [7] and Sliding Mode Control (SMC) method [10] by us. Moreover, we have been also proposed Model Predictive PID method (MP-PID) in [11].

These methods show good performances under the nominal conditions where the situations, for example, mass of vehicle, road condition, and so on, are not changed. To meet the high performance even variation happened in such conditions, it is significant to construct the robust control systems against the changing of situation. About this point, SMC performs good robustness against the uncertainties and nonlinearities of the systems.

However, for slip suppression with the conventional SMC [12], the control performance will get degradation due to the chattering which always occurs when switching the control inputs due to the structure of SMC. To overcome such disadvantages, the SMC method introducing the integral action with gain to design the sliding surface (SMC-I) has been proposed in [13], where the integral gain is derived by trial and error. In order to get better control performance and save more energy for slip suppression of EVs with changing the mass of vehicle and road condition, the optimal gain derived on-line is expected.

This paper, therefore, proposes the Model Predictive Sliding Mode Control with Integral action (MP-SMC-I), which determines the integral gain adaptively at each step by MPC algorithm [14] for slip suppression of EVs under braking improving the safety and stability. The numerical examples show the effectiveness of the proposed method for slip suppression of EVs under braking.

T. Kawabe is with the Division of Information Engineering, Faculty of Engineering, Information and Systems University of Tsukuba, Tsukuba 305-8573 JAPAN (phone: +81-29-853-5507; e-mail: kawabe@cs.tsukuba.a.jp).

II. ELECTRIC VEHICLE DYNAMICS

A. One wheel car model

As a first step toward practical application, this paper restricts the vehicle motion to the longitudinal direction and uses direct motors for each wheel to simplify the one-wheel model to which the drive force is applied. In addition, braking was not considered this time with the subject of the study being limited to only when driving.

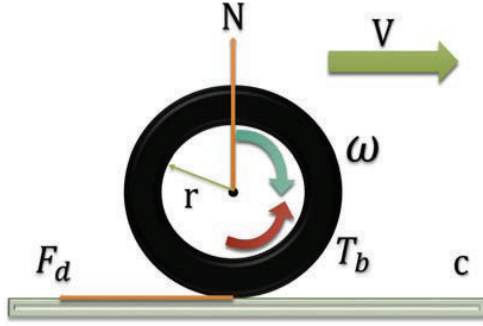


Fig. 1. One-wheel car model

From Fig. 1, the vehicle dynamical equations are expressed as (1) to (3).

$$M \frac{dV}{dt} = -F_d(\lambda) + F_a \quad (1)$$

$$J \frac{d\omega}{dt} = rF_d(\lambda) - T_b \quad (2)$$

$$F_d = \mu(c, \lambda)N \quad (3)$$

Where M is the vehicle weight, V is the vehicle body velocity, F_d is the driving force, J is the wheel inertial moment, F_a is the resisting force from air resistance and other factors on the vehicle body, T_b is the braking torque, ω is the wheel angular velocity, r is the wheel radius, c is road surface condition coefficient, and λ is the slip ratio. The slip ratio is defined by (4) from the wheel velocity ($V_\omega := r\omega$) and vehicle body velocity (V) as

$$\lambda = \frac{V - V_\omega}{V} \quad (\text{braking}). \quad (4)$$

λ is belong to $[-1, 0]$, $\lambda = 0$ indicates that wheel and vehicle velocities are the same and $\lambda = -1$ means the wheel is completely skidding.

B. Magic Formula

The frictional forces that are generated between the road surface and the tires are the force generated in the longitudinal direction of the tires and the lateral force acting perpendicularly to the vehicle direction of travel, and both of these are expressed as a function of λ . The frictional force generated in the tire longitudinal direction is expressed as μ , and the relationship between μ and λ is shown by (5) below, which is a formula called the Magic-Formula[15] and which was approximated from experimental data.

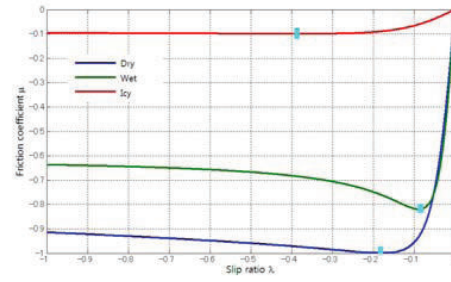


Fig. 2. $\mu - \lambda$ Curve by Magic-Formula

$$\mu(\lambda) = D \sin(C \arctan(B\lambda - E(B\lambda - \arctan(B\lambda)))) \quad (5)$$

where B, C, D and E are the coefficients that determined by experimental data, the values of these coefficients corresponding dry, wet and icy road are shown in Table I and the $\mu - \lambda$ curve is shown in Fig. 2

TABLE I
TYPICAL VALUES OF COEFFICIENTS

Coefficient	Name	Dry	Wet	Icy
B	Stiffness	10	12	4
C	Shape	1.9	2.3	2
D	Peak	1	0.82	0.1
E	Curvature	0.97	1	1

From Fig. 2, the optimal slip ratios (λ^*) under dry, wet and icy road conditions are different. They are -0.18 , -0.08 and -0.39 respectively.

III. MP-SMC WITH INTEGRAL ACTION DESIGN FOR SLIP SUPPRESSION

A. SMC with Integral Action (SMC-I) Method

In this section, the previous proposed control strategy based on SMC with integral action (SMC-I) [13] is explained. Without loss of generality, one wheel car model in fig. 1 is used for the design of the control law. The nonlinear system dynamics can be presented by a differential equation as

$$\dot{\lambda} = f + bT_m \quad (6)$$

where $\lambda \in R$ is the state of the system representing the slip ratio of the driving wheel which is defined as (4) for the case of braking, T_m is the control input. f describes the nonlinearity of system and b is the input gain, and they are all time-varying. Differentiating (4) with respect to time gives

$$\dot{\lambda} = \frac{-\dot{V} + (1 - \lambda)\dot{V}_w}{V_w} \quad (7)$$

and the following equations can be attained,

$$f = -\frac{g}{V_w} \left[1 + (1 - \lambda) \frac{r^2 M}{J_w} \right] \mu(c, \lambda), \quad (8)$$

$$b = \frac{(1 - \lambda)r}{J_w V_w}. \quad (9)$$

The sliding mode controller is described to maintain the value of slip ratio λ at the desired value λ^* .

Referring to [13], in order to reduce the undesired chattering effect for which it is possible to excite high frequency modes, and guarantee zero steady-state error, an integral action with gain has been introduced to the design of sliding surface. By adding an integral item to the difference between the actual and desired values of the slip ratio, the sliding surface function s is given by

$$s = \lambda_e + K_{in} \int_0^t \lambda_e(\tau) d\tau, \quad (10)$$

where λ_e is defined as $\lambda_e = \lambda - \lambda^*$ and K_{in} is the integral gain, $K_{in} > 0$.

The sliding mode occurs when the state reaches the sliding surface defined by $s = 0$. The dynamics of sliding mode is governed by

$$\dot{s} = 0. \quad (11)$$

By using (6) to (11), the sliding mode control law is derived by adding a switching control input T_{msw} to the nominal equivalent control input T_{meq_n} as in [13]

$$T_m = T_{meq_n} + T_{msw}, \quad (12)$$

$$T_{meq_n} = \frac{1}{b} [-f_n - K_{in} \lambda_e], \quad (13)$$

$$T_{msw} = \frac{1}{b} \left[-K \text{sat} \left(\frac{s}{\Phi} \right) \right], \quad (14)$$

$$\text{sat} \left(\frac{s}{\Phi} \right) = \begin{cases} -1 & s < -\Phi \\ \frac{s}{\Phi} & -\Phi \leq s \leq \Phi \\ 1 & s > \Phi \end{cases}, \quad (15)$$

where “ n ” is used to indicate the estimated model parameters. f_n is the estimation of f calculated by using the nominal values of vehicle mass M_n and road surface condition coefficient c_n . $\Phi > 0$ is a design parameter which defines a small boundary layer around the sliding surface. The sliding gain $K > 0$ is selected as

$$K = F + \eta \quad (16)$$

by defining Lyapunov candidate function in [13], where $F = |f - f_n|$ and η is a design parameter.

By using (12), (13), (14) and (16), the control law of SMC-I can be represented as

$$T_m = \frac{1}{b} \left[-f_n - K_{in} \lambda_e - (F + \eta) \text{sat} \left(\frac{s}{\Phi} \right) \right]. \quad (17)$$

B. Model Predictive SMC with Integr Action (MP-SMC-I) Method

Generally, MPC algorithm is used to predict the future state behavior base on the discrete-time state space model [14]. The continuous time state space model for the slip ratio control represented by (6) can not be dealt with in the same way. It is transformed to the discrete time state space model at sampling

time $t = kT$, T is the sampling period. The torque input is defined by

$$T_m(t) = T_m(kT), \quad kT \leq t < (k+1)T. \quad (18)$$

For convenience, we will omit T in the following equations.

The controlled object of vehicle dynamics can be described by a nonlinear difference equation as follows.

$$\lambda(k+1) = f_d(k, \lambda(k)) + b_d(k, \lambda(k)) T_m(k) \quad (19)$$

where $\lambda(k)$ is the state variable representing the slip ratio at time k . $f_d(k, \lambda(k))$ describes the nonlinearity of the discrete time system, $b_d(k, \lambda(k))$ is the input gain, and they are give by

$$f_d(k, \lambda(k)) = -\frac{g}{V_w(k)} \left\{ 1 + [1 - \lambda(k)] \frac{r^2 M}{J_w} \right\} \times \mu(c, \lambda(k)) \quad (20)$$

$$b_d(k, \lambda(k)) = \frac{[1 - \lambda(k)] r}{J_w V_w(k)}. \quad (21)$$

The control input $T_m(t)$ given by (17) can be rewritten as

$$T_m(k) = \frac{1}{b_d(k, \lambda(k))} \left\{ f_{dn}(k, \lambda(k)) - K_{in}(k) [\lambda(k) - \lambda^*] - [F_d(k, \lambda(k)) + \eta] \text{sat} \left(\frac{s(k, \lambda(k), K_{in}(k))}{\Phi} \right) \right\} \quad (22)$$

where λ^* is the reference slip ratio, η is the design parameter, and both of them are constants. $f_{dn}(k, \lambda(k))$ is the estimation of $f_d(k, \lambda(k))$ and is defined as

$$f_{dn}(k, \lambda(k)) = \frac{-g\mu(c_n, \lambda(k))}{V_w(k)} \left\{ 1 + [1 - \lambda(k)] \frac{r^2 M_n}{J_w} \right\}. \quad (23)$$

Now, we set current time to k . For a prediction horizon H_p , the predicted slip ratios $\hat{\lambda}(k+i)$ for $i = 1, \dots, H_p$ depend on the known values of current slip ratios, current torque input and future torque inputs. By using (19), the predicted slip ratios can be represented as

$$\begin{aligned} \hat{\lambda}(k+H_p) &= f_d(k+H_p-1, \hat{\lambda}(k+H_p-1)) \\ &+ b_d(k+H_p-1, \hat{\lambda}(k+H_p-1)) \\ &\times \hat{T}_m(k+H_p-1) \end{aligned} \quad (24)$$

where $\hat{T}_m(k+i)$, $i = 0, \dots, H_p-1$ are predicted control inputs. $\hat{T}_m(k+i)$ is given by

$$\begin{aligned} \hat{T}_m(k+i) &= \frac{f_{dn}(k+i, \hat{\lambda}(k+i)) - K_{in} [\hat{\lambda}(k+i) - \lambda^*]}{b_d(k+i, \hat{\lambda}(k+i))} \\ &- \frac{[F_d(k+i, \hat{\lambda}(k+i)) + \eta] \text{sat} \left(\frac{s(k+i, \hat{\lambda}(k+i), K_{in})}{\Phi} \right)}{b_d(k+i, \hat{\lambda}(k+i))} \end{aligned} \quad (25)$$

Where

$$F_d(k, \lambda(k)) = \frac{g}{|V_w(k, \lambda(k))|} \left| \mu(c_{max}, \lambda(k)) - \mu(c_n, \lambda(k)) \right| + \frac{g(1-\lambda(k)) \frac{r^2}{J_w}}{|V_w(k, \lambda(k))|} \left| M_{max} \mu(c_{max}, \lambda(k)) \right| - \frac{g(1-\lambda(k)) \frac{r^2}{J_w}}{|V_w(k, \lambda(k))|} \left| M_n \mu(c_n, \lambda(k)) \right|, \quad (26)$$

and where M_n is the estimated value of vehicle mass M and c_n is estimated for the viscous friction coefficient c . Here, we define the estimated values of these parameters respectively as the arithmetic mean of the value of the bounds.

$$c_n = \frac{c_{min} + c_{max}}{2} \quad (27)$$

$$M_n = \frac{M_{min} + M_{max}}{2}. \quad (28)$$

Actually, the mass of the car often changes with the number of passengers and the weight of luggage. Besides, the car has to always travel on various road surfaces. Then the ranges of variation in parameter c and parameter M are assumed to be defined as

$$c_{min} \leq c \leq c_{max} \quad (29)$$

$$M_{min} \leq M \leq M_{max}. \quad (30)$$

Here, the objective function J for deciding the value of K_{in} can be written as

$$J = \sum_{i=0}^{H_p-1} \left\{ q |\hat{\lambda}(k+i+1) - \lambda^*| + r |\hat{T}_m(k+i)| \right\} \quad (31)$$

where q, r are the positive weights. By using (24) and (25), both $\hat{\lambda}$ and \hat{T}_m can be expressed by K_{in} , thus J can be represented by a function of K_{in} . Our aim is to find the parameter K_{in} that minimizes this objective function $J(K_{in})$. In a nutshell, the optimization problem is given by

$$\begin{aligned} \min_{K_{in}} J \\ \text{s.t. } (24) \text{ and } (25) \\ i = 0, \dots, H_p - 1. \end{aligned} \quad (32)$$

At time k , the optimal $K_{in}(k)$ can be found by solving (32) with some optimization method (here, a grid search method is made to the discretized K_{in}) by MPC Algorithm. Once the optimal $K_{in}(k)$ is determined, it is used as the continuous $K_{in}(t)$ for $kT \leq t < (k+1)T$, then the continuous control input $T_m(t)$ can be calculated by (17). At the next sampling time $k+1$, the optimal $K_{in}(k+1)$ is calculated as the previous step. At each sampling period, the same operation is repeated. Therefore, using the MP-SMC-I method could determine the optimal parameter K_{in} by solving the optimization problem.

IV. NUMERICAL EXPERIMENTS

This section shows the numerical simulation results to demonstrate the effectiveness of the proposed MP-SMC-I method. The performance of the proposed MP-SMC-I method is compared with the previously proposed SMC-I methods.

A. Experimental setup

In the simulation examples, the vehicle starts from rest and accelerates on icy road, wet asphalt road and dry asphalt road respectively with the parameters of dynamics shown in Table II. And the car is slamming on the brakes with the speed from 100[km/h] to 0.25[km/h] (because the body velocity cannot be 0 due to the formula of slip ratio).

TABLE II
PARAMETERS USED IN THE SIMULATIONS.

J_w : Inertia of wheel	21.1[kg/m ²]
r : Radius of wheel	0.26[m]
λ^* : Desired slip ratio	0.13
g : Acceleration of gravity	9.81[m/s ²]

The controller parameters Φ, η as well as the integral gain K_{in} are listed in Table III.

TABLE III
CONTROLLER SETTINGS

SMC-I	$\Phi = 1, \eta = 5$ $K_{in} = 10$
MP-SMC-I	$\Phi = 1, \eta = 5$ $0 \leq K_{in} \leq 200, \Delta K_{in} = 1$ $q = 1.0 \times 10^8$ $r = 1.0$

In order to evaluate the electric energy consumption (ignore the driver behavior and thermal losses) is presented to evaluate the power saving performance, it is defined as

$$E = \int_0^t T_b(t) \omega(t) dt \quad (33)$$

where $T_b(t)$ is the braking torque and $\omega(t)$ the angular velocity of wheel.

B. Simulation results

Figs. 3 ~ 5 show time response of slip ratio on dry, wet and icy road surface respectively.

From these figures, we can see that the proposed MP-SMC-I method can keep the value of λ to almost around -0.1 in any situations compared with the SMC-I method.

Table IV shows the stopping time, braking distance and electric energy consumption of two methods. From Table. IV, we can see that the proposed MP-SMC-I method shows good performance in almost all categories compared with SMC-I method.

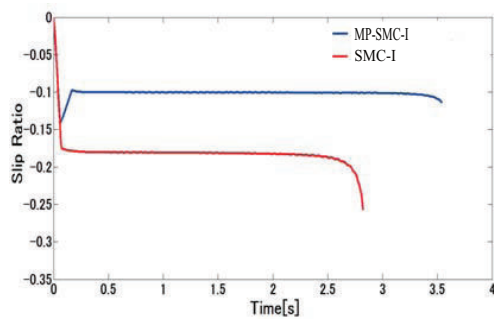


Fig. 3. Time Response of Slip Ratio (Dry)

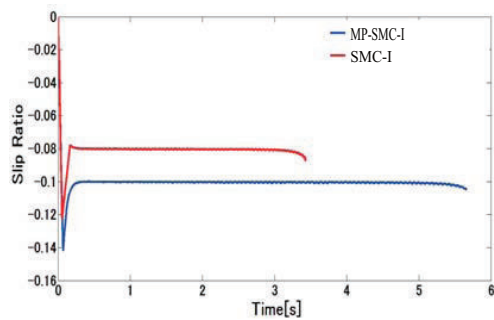


Fig. 4. Time Response of Slip Ratio (Wet)

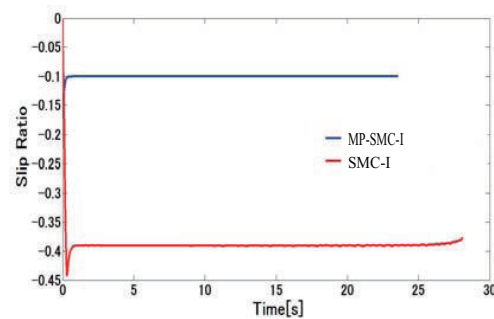


Fig. 5. Time Response of Slip Ratio (Icy)

TABLE IV
COMPARISON OF PERFORMANCE INDEXES

Road condition	performance Index	SMC-I	MP-SMC-I
Dry	Time [s]	3.54	2.82
	Distance [m]	49.83	29.88
	Energy [Kwh]	0.035	0.035
Wet	Time [s]	5.66	3.40
	Distance [m]	80.00	48.50
	Energy [Kwh]	0.038	0.038
Icy	Time [s]	23.57	24.82
	Distance [m]	330.60	335.97
	Energy [Kwh]	0.048	0.050

V. CONCLUSION

In this paper, the MP-SMC-I method has been proposed for slip suppression for EV traction control under braking. This method focuses on improving the braking performance by determining the sliding surface parameter k_{in} adaptively. The effectiveness of the method has been confirmed by numerical simulations.

The proposed method can suppress the slip ratio to the desired value on three different road conditions (dry asphalt road, wet asphalt road and icy road) during the braking.

In future work, it is expected that MP-SMC-I method could be expanded for overall driving modes (i.e., acceleration, cruise and deceleration). It is also intended to prepare and apply to more detailed vehicle model for making practical high performed robust traction control systems with low energy consumption of EVs by promoting further progress.

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