Fuzzy Gauge Capability (C_g and C_{gk}) through Buckley Approach

Seyed Habib A. Rahmati, Mohsen Sadegh Amalnick

***Abstract*—**Different terms of the Statistical Process Control (SPC) has sketch in the fuzzy environment. However, Measurement System Analysis (MSA), as a main branch of the SPC, is rarely investigated in fuzzy area. This procedure assesses the suitability of the data to be used in later stages or decisions of the SPC. Therefore, this research focuses on some important measures of MSA and through a new method introduces the measures in fuzzy environment. In this method, which works based on Buckley approach, imprecision and vagueness nature of the real world measurement are considered simultaneously. To do so, fuzzy version of the gauge capability (C_g and C_{gk}) are introduced. The method is also explained through example clearly.

***Keywords*—**SPC, MSA, gauge capability, C_g, C_{gk}.

I. Introduction

MEASUREMENT SYSTEM ANALYSIS (MSA) is an important method which quantifies measurement variation including the variation resulting from the measurement system, the operators, and the parts themselves, AIAG [1]. In the literature of the MSA, [2] carried out MSA studies on two or more measurement devices and proposed a procedure for estimating the sensitivity of the measurement devices. Moreover, [3] assessed MSA through design of experiments. Many other crisp studies are also done on the MSA measures. Nevertheless, the number of the fuzzy studies is so rare. Specifically, on the capability measures of the MSA, there is not any fuzzy study in the literature. These measures in the Statistical Process Control (SPC) are called C_p and C_{pk} that are developed in the fuzzy environment vastly [4]-[7]. However, those of the MSA, called C_g and C_{gk} are not in the fuzzy environment. Meanwhile, since in practical systems different type of measurements are usually estimated by taking sample, validation of the measurements and analyzes obtained from them are severely relied on the quality of obtained data. Usually, in the statistical systems, obtained data have two special features; 1) uncertainty and 2) vagueness. The first features force us to use different statistical concepts, like calculating expected value of measures; estimating point and interval estimate of measures and other similar concept to prevent bad effect of uncertainty on the systems. On the other hand, the second features force us to prevent another type of bad effects on our systems which are usually due to; 1) having no data or 2) having imprecise data. To cope with second feature usually a concept which was developed by Zadeh is applied. Therefore, in this paper because of importance of fuzzy MSA as a popular statistical method, fuzzy assessment is developed. The implemented method of this research is based on Buckley approach [8]-[13].

Buckley introduced a method to create fuzzy numbers for parameters of different distributions discrete and continuous, in which both of the uncertainty and vagueness features are considered. In his method, confidence intervals for a parameter are used as a family *offcuts* of a triangular shaped fuzzy number. In this paper, developing his approach some measures of queuing systems is estimated [12], [13].

The structure of the rest of the paper is organized as follows. Section II explains C_g and C_{gk} in the both crisp and fuzzy. Section III through a numeral example demonstrates the considered fuzzy method of metrics. Finally, in Section V concludes the paper.

II. The Crisp and Fuzzy Capability Study of MSA

Measurement system is the collection of instruments or gauges, standards, operations, methods, fixtures, software, personnel, environment and assumption used to quantify a unit of measure or fix assessment. Correspondingly, MSA is a collection of statistical methods for the analysis of measurement system capability. It seeks to describe, categorize, evaluate the quality of measurements; improve the usefulness, accuracy, precision, meaningfulness of measurements; and propose methods for developing better measurement instruments by Montgomery. Some stated goals of MSA are estimated components of measurement error, estimate the contribution of measurement error to the total variability of a process or equipment parameter, determines stability of a metrology tool over time, and to compare and correlate multiple metrology tools. Measurement process is a kind of production process that its output is number. Fig. 1 presents measurement process with its inputs and outputs.

Generally, MSA ensures suitability of the measurement devices by focusing on some measures classified in accuracy, precision, and stability categories. Meanwhile, Cg and Cgk are the specific metric for evaluating capability of the measurement system. These metric are formulated in (1) and (2). In these equations, Xm denotes master value and T denotes part tolerance. Besides, μ_g represents, the master value, $\overline{x}_g$ denotes the average of observed value and σ_g and

Seyed Habib A. Rahmati is with the Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran (corresponding author to provide e-mail: sdh.rahmati@gmail.com).
Mohsen Sadegh Amalnick is with the University of Tehran, College of Engineering, Faculty of Industrial Engineering, Tehran, Iran (e-mail: amalnick@ut.ac.ir).

S_g denote the standard deviation and its estimated value, respectively.

$$C_g = \frac{0.2T}{6\sigma_g} = \frac{0.2T}{6S_g} \tag{1}$$

$$C_{gk} = \frac{0.1T - |\mu_g - x_m|}{3\sigma_g} = \frac{0.1T - |\bar{x}_g - x_m|}{3S_g} \tag{2}$$

In what follows, with modification, fuzzy estimation based on Buckley's approach is presented. First some notations are introduced. A triangular shaped fuzzy number N is a fuzzy subset of the real numbers R satisfying:

1) Normality: N(x) =1 for exactly one x R
2) Convexity: one way for creating a convex shape for a fuzzy number is to create an L-R (Left-Right) number. A L-R number is a number which its left side is created by an increasing function and its right side is created by a decreasing number. L-R function in this paper is created by a closed and bounded interval.

In this paper, triangular shaped fuzzy number is used and created for parameter estimation and following definition are considered:

X: a random variable with p. d .f. (p .m .f) for single parameter

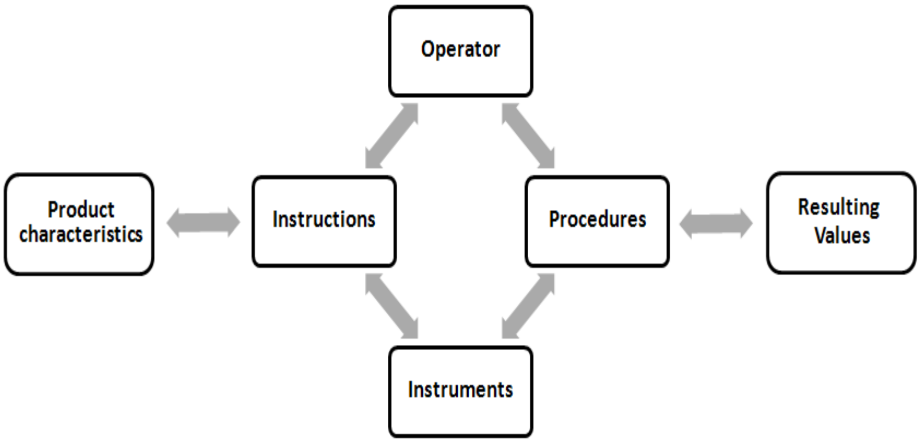

Fig. 1 Measurement system analysis process

Given the values of our random variable, one can obtain a point estimate. In most of statistical studies, since we never expect this point estimate be exactly equal to main parameter, we also compute a confidence interval. Then, a family of confidence intervals for the estimating parameter is obtained. Placing these confidence intervals, one on top of the other, a triangular shaped fuzzy number is obtained. Hence by using this method we have more information about rather than a point estimate, or just a single interval estimate. It is easy to generalize Buckley's method.

Shortly explaining, according to the Buckley method the metrics are developed in the fuzzy environments. This method considers $(1-\beta)100\%$ confidence intervals of a parameter as a family of $\alpha - cuts$ and suggests a triangular shaped fuzzy number. In other words, his method suggests a Left-Right (L-R) number $N_\alpha = [n_1(\alpha), n_2(\alpha)]$, where $n_1(\alpha)$ is increasing and $n_2(\alpha)$ is decreasing continuous functions. Therefore, considering X as a random variable in the function $f(x;\theta)$ for single parameter θ that is via a random sample $X_1, X_2, X_3, \ldots, X_n$, the obtained Buckley approach is shown in Figs. 1 and 2.

The confidence intervals of these metrics are as (3).

$$(C_g)_\alpha = \left[\frac{0.2T}{6\frac{(n-1)S_g^2}{\chi^2_{(1-\alpha/2),(n-1)}}}, \frac{0.2T}{6\frac{(n-1)S_g^2}{\chi^2_{(\alpha/2),(n-1)}}}\right] \tag{3}$$

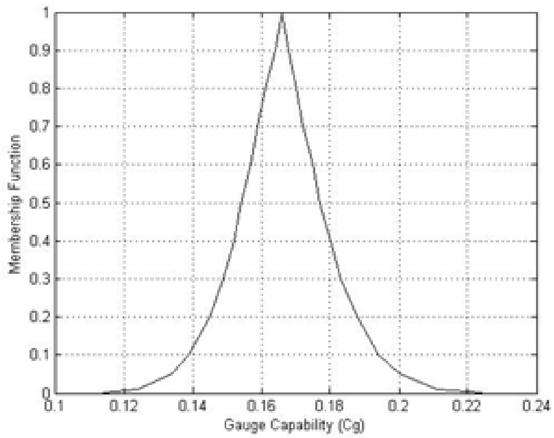

Fig. 2 The membership function of the fuzzy Cg

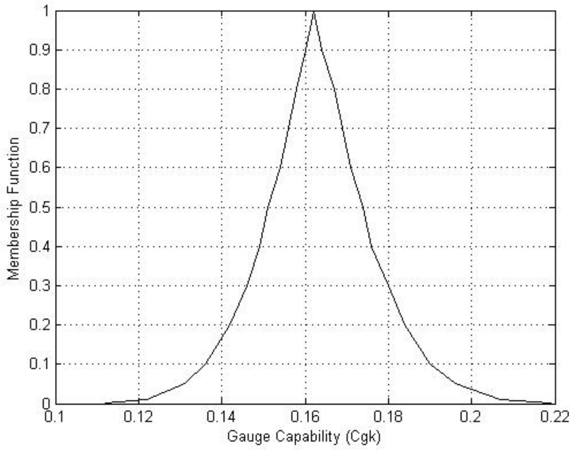

Fig. 3 The membership function of the fuzzy C_{gk}

$$(C_{gk})_\alpha = [\frac{0.1T - |\bar{x}_g - x_m|}{3\frac{(n-1)S_g^2}{\chi^2_{(1-\alpha/2),(n-1)}}}, \frac{0.1T - |\bar{x}_g - x_m|}{3\frac{(n-1)S_g^2}{\chi^2_{(\alpha/2),(n-1)}}}] \quad (4)$$

Therefore, according to Fig. 4, putting confidence intervals of the parameters on each other creates a triangular fuzzy number.

III. The Experimental Results

The explaining example is related to a part with the technical specification 17.05 ± 0.25 and the master value 17.05 mm (X_m =17.05). The other crisp specifications of this example are (5). The input data are presented in Table I.

$$\begin{aligned} &T = 0.5; n = 50; X_m = 17.05 \\ &\bar{x}_g = \frac{\sum_{i=1}^{n}(x_i)}{n} = 17.049 \\ &S_g = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n-1}} = \sqrt{\frac{\sum_{i=1}^{50}(x_i - 17.049)}{49}} = 0.00998 \\ &B_g = \bar{x}_g - X_m = -0.001 \\ &C_g = \frac{0.2T}{6S_g} = \frac{0.2 * 0.5}{6 * 0.00998} = 1.67 \\ &C_{gk} = \frac{0.1T - |\bar{x}_g - x_m|}{3S_g} = \frac{0.1(0.5) - |17.049 - 17.05|}{3 * 0.00998} = 1.636 \end{aligned} \quad (5)$$

1- Consider y=u($X_1, X_2, X_3, \ldots, X_n$) as a statistic estimator of θ and the values of these random variable, e.g. $X_i = x_i$, $1 \le i \le n$, to obtain the point estimate $\hat{\theta} = \hat{y} = u(x_1, x_2, x_3, \ldots, x_n)$ for θ.

2- Calculate of $(1-\beta)100\%$ confidence interval for θ.

3- Put the obtained intervals, one on top of the other to obtain triangular shaped fuzzy as $\theta_\alpha = [\theta_1(\alpha), \theta_2(\alpha)]$ for $0 < \alpha < 1, \theta_0 = \ddot{\theta}$ and $\theta_1 = [\hat{\theta}, \hat{\theta}]$.

Fig. 4 The developed Buckley method for C_g and C_{gk}

Therefore, the fuzzy calculations of the developed fuzzy numbers are as (6) and (7).

$$(C_g)_\alpha = [\frac{0.2(0.5)}{6\frac{(49)(0.00998)^2}{\chi^2_{(1-\alpha/2),(49)}}}, \frac{0.2(0.5)}{6\frac{(49)(0.00998)^2}{\chi^2_{(\alpha/2),(49)}}}] \quad (6)$$

$$(C_{gk})_\alpha = [\frac{0.1(0.5) - |17.049 - 17.05|}{3\frac{(49)(0.00998)^2}{\chi^2_{(1-\alpha/2),(49)}}}, \frac{0.1(0.5) - |17.049 - 17.05|}{3\frac{(49)(0.00998)^2}{\chi^2_{(\alpha/2),(49)}}}] \quad (7)$$

These numbers are also plotted as Figs. 2 and 3.

IV. Conclusion

This study focused on reinforcing the MSA capability by developing Cg and Cgk in fuzzy environment. These, metrics are not study in this study yet. To do so, Buckley approach is implemented. In this method, after generating first point estimation and the confidence intervals of these point estimations of the considered metrics, placing them one on top of another, triangular shaped fuzzy numbers were obtained as fuzzy estimator of Cg and Cgk. The method was also explained by numerical example explicitly.

TABLE I
THE OBSERVATIONS OF THE FIRST EXAMPLE

		Batch Number									
		1	2	3	4	5	6	7	8	9	10
Sample Size	1	17.05	17.05	17.05	17.04	17.05	17.04	17.03	17.04	17.04	17.05
	2	17.06	17.04	17.05	17.04	17.04	17.04	17.05	17.06	17.03	17.07
	3	17.04	17.06	17.07	17.06	17.05	17.06	17.05	17.05	17.05	17.06
	4	17.00006	17.05	17.05	17.05	17.03	17.04	17.06	17.07	17.06	17.06
	5	17.04	17.04	17.05	17.06	17.05	17.04	17.05	17.04	17.05	17.05

REFERENCES

[1] Automotive Industry Action Group (AIAG), Measurement Systems Analysis Reference Manual. 3rd ed., Chrysler, Ford, General Motors Supplier Quality Requirements Task Force, 2002.
[2] J. M. Juran, F. M. Gyrna, Quality Planning and Analysis, McGraw-Hill, New York, 1993.
[3] S. Senol, Measurement system analysis using designed experiments with minimum α-β Risks and n. *Measurement*, 36, 131–141, 2004.
[4] H. T. Lee, C_{pk} index estimation using fuzzy numbers. *European Journal of Operational Research*, 129, 683-688, 2001.
[5] P. K. Leung, F. Spiring, Adjusted action limits for C_{pm} based on departures from normality. *International Journal of Production Economics*, 107, 237-249, 2007.
[6] A. Parchami, M. Mashinchi, A.R. Yavari, and H.R. Maleki, Process Capability Indices as Fuzzy Numbers. *Austrian Journal of Statistics*, 34(4), 391–402, 2005.
[7] A. Parchami, M. Mashinchi. Fuzzy estimation for process capability indices. *Information Sciences*, 177, 1452–1462, 2007.
[8] J.J. Buckley, Elementary Queuing Theory based on Possibility Theory. *Fuzzy Sets and Systems*37 1990, pp. 43–52.
[9] J.J. Buckley, Y. Qu, (1990), On Using α -Cuts to Evaluate Fuzzy Equations. Fuzzy Sets and Systems 38,309–312.
[10] J.J. Buckley, T. Feuring, Y. Hayashi, (2001), Fuzzy Queuing Theory Revisited. *International Journal of Uncertainty, Fuzziness, and Knowledge-based Systems* 9,527–537.
[11] J.J. Buckley, E. Eslami, *Uncertain Probabilities I*: The Discrete Case; Soft Computing 2003a, pp. 500-505.
[12] J.J. Buckley, E. Eslami, *Uncertain Probabilities II*: The Continuous Case; Soft Computing 2003b, pp. 500-505.
[13] J.J. Buckley, Fuzzy Statistics: *Hypothesis Testing; Soft Computing*, 2005a, (9) pp. 512-518.