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Transient Heat Conduction in Nonuniform Hollow Cylinders with Time Dependent Boundary Condition at One Surface

Sen Yung Lee, Chih Cheng Huang, Te Wen Tu

***Abstract*—**A solution methodology without using integral transformation is proposed to develop analytical solutions for transient heat conduction in nonuniform hollow cylinders with time-dependent boundary condition at the outer surface. It is shown that if the thermal conductivity and the specific heat of the medium are in arbitrary polynomial function forms, the closed solutions of the system can be developed. The influence of physical properties on the temperature distribution of the system is studied. A numerical example is given to illustrate the efficiency and the accuracy of the solution methodology.

***Keywords*—**Analytical solution, nonuniform hollow cylinder, time-dependent boundary condition, transient heat conduction.

I. Introduction

THE applications of heat conduction in nonuniform hollow cylinders with time-dependent boundary conditions can be widely found in numerous engineering fields, such as barrel of cannon, tube in heat exchanger, time variation heating on walls of circular structure and heat treatment on hollow cylinders. Therefore, an accurate solution methodology is very helpful for relevant developments.

It is well-known that the problem of heat conduction with time-dependent boundary conditions cannot be solved directly by the separation of variables method. In most of the analyses, the integral transformation method has been used to remove the time-dependent term; however, taking the inverse integral transformation is always tedious. Moreover, for the problem of heat conduction in uniform hollow cylinders with time-dependent boundary conditions, the associated governing differential equation is a second-order Bessel differential equation with constant coefficients. After conducting a Laplace transformation, the analytical solution can be found in [1].

When the structure is a functionally graded hollow cylinder, i.e., the cylinder is a nonuniform medium, the associated governing differential equation is a second-order regular singular differential equation with variable coefficients. For problems with time-independent boundary conditions, only numerical methods, such as: the perturbation method [2], the finite difference method [3], and the finite element method [4] can be found. Later, Jabbari, Sohrabpour, and Eslami derived analytical solutions for thermal stresses of functionally graded hollow cylinders whose material properties vary with the power product form of the radial coordinate variable due to radially symmetric loads [5] and non-axisymmetric loads [6]. By using the Laplace transformation and a series expansion of Bessel functions, [7] analyzed a one-dimensional transient thermo elastic problem with the material properties varying with the power-law form of the radial coordinate variable. Zhao, Ai, Li, and Zhou [8] analyzed the temperature change when the thermal and thermo elastic properties are assumed to vary exponentially in the radial direction. And further, [9] considered the material properties to be nonlinear with a power law distribution through the thickness, while the temperature distribution was derived analytically using the Bessel functions.

The study of heat conduction in functionally graded hollow cylinders with time-dependent boundary conditions is quite limited. Shao and Ma [10] employed Laplace transform technique and the series solving method to study thermo mechanical stresses in functionally graded hollow cylinders with linearly increasing boundary temperatures. Wang and Liu [11] used the method of separation variables to develop the analytical solution of transient temperature fields for two-dimensional transient heat conduction in a fiber-reinforced multilayer cylindrical composite. Recently, for problems with time-dependent temperatures at boundaries, [12] developed exact solutions for heat conduction of a hollow cylinder with thermal conductivity and specific heat in power functions with different orders. However, their solution methodology is not applicable to problems with time-dependent heat flux boundary conditions.

From the literature, it can be seen that the exact solutions for heat conduction in nonuniform hollow cylinders with variable time-dependent boundary conditions have not yet been developed. In this paper, the shifting function method [12]-[14] is modified and extended to the problem of heat conduction in nonuniform hollow cylinders with time-dependent heat flux boundary condition at the outer surface. When the thermal conductivity and the specific heat of the medium are in polynomial function forms, the exact solution of the system can be developed. The proposed solution methodology is simple and free of Laplace transformation. Finally, the numerical analysis is given to depict the procedure of the solution methodology.

Sen Yung Lee and Chih Cheng are with the Mechanical Engineering Department, National Cheng Kung University, Tainan, Taiwan, Republic of China (phone: +886-6-2757575 ext.62150; e-mail: sylee@mail.ncku.edu.tw, huang.adsl@msa.hinet.net).
Te Wen Tu is with the Mechanical Engineering Department, Air Force Institute of Technology, Kaohsiung, Taiwan, Republic of China (tutewen@gmail.com; tu123@ms17.hinet.net).

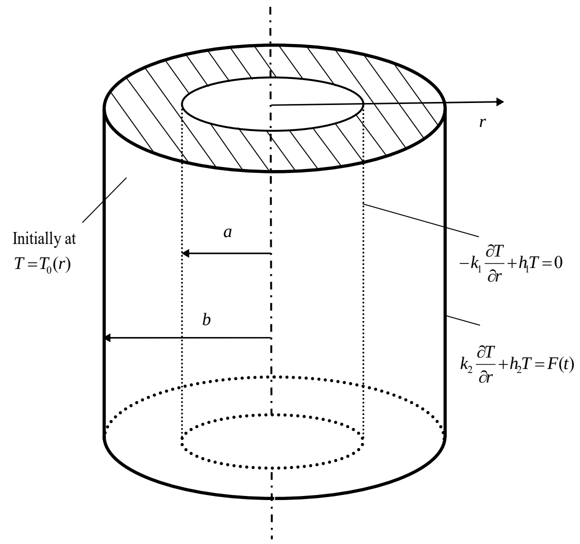

Fig. 1 A hollow cylinder with time-dependent boundary condition at outer surface

II. Mathematical Modeling

Consider the transient heat conduction in a nonuniform hollow cylinder with time-dependent boundary condition at the outer surface, as shown in Fig. 1. No heat is generated within the hollow cylinder. The governing differential equation, boundary conditions, and the initial condition of the boundary value problem are:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[rk(r)\frac{\partial T}{\partial r}\right] = \rho\, c(r)\frac{\partial T}{\partial t}, a < r < b, \quad t > 0, \tag{1}$$

$$-k_1\frac{\partial T}{\partial r} + h_1 T = 0, \text{ at } \ r = a, \tag{2}$$

$$k_2\frac{\partial T}{\partial r} + h_2 T = F(t), \quad \text{at } \ r = b, \tag{3}$$

$$T(r, 0) = T_0(r), \quad \text{at } \ t = 0. \tag{4}$$

Here, r is the space variable, $k(r)$ is the thermal conductivity function, $T(r,t)$ is the temperature function, ρ is the mass density, $c(r)$ is the specific heat function, t is the time variable, and a, b are inner and outer radii, respectively. k_1 and k_2, along with h_1 and h_2 are the thermal conductivities and the heat transfer coefficients at the inner and outer surfaces, respectively. $F(t)$ is the time-dependent heat flux function at the outer surface and $T_0(r)$ is the initial temperature function. In terms of the following dimensionless quantities:

$$\xi = \frac{r}{b}, \quad K(\xi) = \frac{k(r)}{k(b)}, \theta(\xi,\tau) = \frac{T(r,t)}{T_r}, \quad C(\xi) = \frac{c(r)}{c(b)},$$
$$\tau = \frac{k(b)t}{c(b)\rho b^2}, \quad \bar{r} = \frac{a}{b}, \quad \bar{K}_1 = \frac{k_1}{k(b)}, \quad \bar{H}_1 = \frac{h_1 b}{k(b)},$$

$$\bar{K}_2 = \frac{k_2}{k(b)}, \quad \bar{H}_2 = \frac{h_2 b}{k(b)}, \quad f(\tau) = \frac{F(t)b}{k(b)T_r}, \theta_0(\xi) = \frac{T_0(r)}{T_r}, \tag{5}$$

the boundary value problem now becomes

$$\frac{1}{\xi}\frac{\partial}{\partial\xi}\left[\xi K(\xi)\frac{\partial\theta}{\partial\xi}\right] = C(\xi)\frac{\partial\theta}{\partial\tau}, \quad \bar{r} < \xi < 1, \ \tau > 0, \tag{6}$$

$$-\bar{K}_1\frac{\partial\theta}{\partial\xi} + \bar{H}_1\theta = 0, \quad \text{at } \ \xi = \bar{r}, \tag{7}$$

$$\bar{K}_2\frac{\partial\theta}{\partial\xi} + \bar{H}_2\theta = f(\tau), \quad \text{at } \ \xi = 1, \tag{8}$$

$$\theta(\xi,0) = \theta_0(\xi), \quad \text{at } \ \tau = 0, \tag{9}$$

where T_r is a reference temperature.

III. The Solution Methodology

A. Change of Variable

To find the solution for the second-order differential equation with a non-homogeneous boundary condition, the shifting function method developed by [12]-[14] was extended, by taking:

$$\theta(\xi,\tau) = v(\xi,\tau) + g(\xi)\, f(\tau), \tag{10}$$

where $v(\xi,\tau)$ is the transformed function and $g(\xi)$ is a shifting function to be specified.

Substituting (10) into (6-9) yields the following partial differential equation along with the boundary conditions and initial condition:

$$\frac{1}{\xi}\frac{\partial}{\partial\xi}\left[\xi K(\xi)\frac{\partial v}{\partial\xi}\right] - C(\xi)\frac{\partial v}{\partial\tau} = F(\xi,\tau), \tag{11}$$

$$-\bar{K}_1\left[\frac{\partial v(\bar{r},\tau)}{\partial\xi} + \frac{dg(\bar{r})}{d\xi}f(\tau)\right] + \bar{H}_1[v(\bar{r},\tau) + g(\bar{r})f(\tau)] = 0, \tag{12}$$

$$\bar{K}_2\left[\frac{\partial v(1,\tau)}{\partial\xi} + \frac{dg(1)}{d\xi}f(\tau)\right] + \bar{H}_2[v(1,\tau) + g(1)f(\tau)] = f(\tau), \tag{13}$$

$$v(\xi,0) = \theta(\xi,0) - g(\xi)f(0), \tag{14}$$

where $F(\xi,\tau)$ is defined as:

$$F(\xi,\tau) = C(\xi)g(\xi)\frac{df(\tau)}{d\tau} - f(\tau)\frac{1}{\xi}\frac{d}{d\xi}\left[\xi K(\xi)\frac{dg(\xi)}{d\xi}\right] \tag{15}$$

B. The Shifting Function

To simplify the analysis, the shifting function is specifically chosen such that it satisfies the following differential equations:

$$-\bar{K}_1 \frac{dg(\bar{r})}{d\xi} + \bar{H}_1 g(\bar{r}) = 0, \tag{16}$$

$$\bar{K}_2 \frac{dg(1)}{d\xi} + \bar{H}_2 g(1) = 1. \tag{17}$$

Here, the shifting function given by [12] is modified and set in the following form

$$g(\xi) = a_1\xi + a_2\xi^2. \tag{18}$$

Hence, two constants in the shifting function can be easily determined as:

$$a_1 = \frac{\bar{r}(-2\bar{K}_1 + \bar{r}\bar{H}_1)}{(1-\bar{r})(2\bar{K}_1\bar{K}_2 - \bar{r}\bar{H}_1\bar{H}_2) + (1-2\bar{r})\bar{K}_1\bar{H}_2 - \bar{r}(2-\bar{r})\bar{H}_1\bar{K}_2}, \tag{19}$$

$$a_2 = \frac{\bar{K}_1 - \bar{r}\bar{H}_1}{(1-\bar{r})(2\bar{K}_1\bar{K}_2 - \bar{r}\bar{H}_1\bar{H}_2) + (1-2\bar{r})\bar{K}_1\bar{H}_2 - \bar{r}(2-\bar{r})\bar{H}_1\bar{K}_2}, \tag{20}$$

C. Reduced Homogeneous Problem

With the shifting function, (18), the two boundary conditions, (12), (13), are reduced to homogeneous ones:

$$-\bar{K}_1 \frac{\partial v(\bar{r},\tau)}{\partial\xi} + \bar{H}_1 v(\bar{r},\tau) = 0, \quad \text{at } \xi = \bar{r}, \tag{21}$$

$$\bar{K}_2 \frac{\partial v(1,\tau)}{\partial\xi} + \bar{H}_2 v(1,\tau) = 0, \quad \text{at } \xi = 1. \tag{22}$$

Here the transformed initial condition now is:

$$v(\xi,0) = \theta_0(\xi) - g(\xi)f(0) = v_0(\xi). \tag{23}$$

D. Solution of Transformed Function

To find the solution, $v(\xi,\tau)$, we use the series expansion theorem and assume the solution to be in the form

$$v(\xi,\tau) = \phi(\xi)q(\tau). \tag{24}$$

The dimensionless space variable $\phi(\xi)$ satisfies the following equations:

$$\frac{1}{\xi}\frac{\partial}{\partial\xi}\left[\xi K(\xi)\frac{d\phi(\xi)}{d\xi}\right] + \lambda^2 C(\xi)\phi(\xi) = 0, \tag{25}$$

$$-\bar{K}_1 \frac{d\phi(\bar{r})}{d\xi} + \bar{H}_1\phi(\bar{r}) = 0, \quad \text{at } \xi = \bar{r}, \tag{26}$$

$$\bar{K}_2 \frac{d\phi(1)}{d\xi} + \bar{H}_2\phi(1) = 0, \quad \text{at } \xi = 1, \tag{27}$$

and the separation equation for the dimensionless time variable $q(\tau)$ is:

$$\frac{dq(\tau)}{d\tau} = -\lambda^2 q(\tau), \tag{28}$$

where λ's denote the corresponding eigenvalues. Now, let $X_i(\xi), i = 1, 2$ be the two linearly independent fundamental solutions of the boundary value problem; then, the solution of (25) can be written as:

$$\phi(\xi) = C_1 X_1(\xi) - C_2 X_2(\xi), \tag{29}$$

where C_1 and C_2 are constants to be determined from the homogeneous boundary conditions, (26), (27).

After substituting solutions, (29), into the boundary conditions, (26), (27), we will obtain the following characteristic equation:

$$\begin{aligned}&\bar{H}_1\{\bar{H}_2[X_1(1)X_2(\bar{r}) - X_1(\bar{r})X_2(1)] + \bar{K}_2[X_1'(1)X_2(\bar{r}) - X_1(\bar{r})X_2'(1)]\}\\ &+ \bar{K}_1\{\bar{H}_2[X_1'(\bar{r})X_2(1) - X_1(1)X_2'(\bar{r})] + \bar{K}_2[X_1'(\bar{r})X_2'(1) - X_1'(1)X_2'(\bar{r})]\} = 0\end{aligned} \tag{30}$$

Consequently, the eigenvalues $\lambda_n (n = 1, 2, 3, \cdots\cdots)$ can be determined. The associated n-th eigenfunction $\phi_n(\xi)$ is determined as:

$$\begin{aligned}\phi_n(\xi) = {}&[\bar{H}_2 X_2(1) + \bar{K}_2 X_2'(1)]X_{n,1}(\xi) - \\ &[\bar{H}_2 X_1(1) + \bar{K}_2 X_1'(1)]X_{n,2}(\xi)\end{aligned}, \quad n = 1, 2, 3, \cdots\cdots \tag{31}$$

where $X_{n,1}(\xi)$ and $X_{n,2}(\xi)$ are respectively defined as:

$$X_{n,1}(\xi) = X_1(\lambda_n, \xi), \quad X_{n,2}(\xi) = X_2(\lambda_n, \xi). \tag{32}$$

The eigenfunctions $\phi_n(\xi)$ constitute an orthogonal set in the interval $\bar{r} \le \xi \le 1$ with respect to a weighting function $\xi C(\xi)$:

$$\int_{\bar{r}}^{1} \xi C(\xi)\phi_m(\xi)\phi_n(\xi)d\xi = \begin{cases} 0, & \text{for } m \ne n \\ N_n, & \text{for } m = n \end{cases} \tag{33}$$

where

$$N_n = \int_{\bar{r}}^{1} \xi C(\xi)\phi_n^2(\xi)d\xi. \tag{34}$$

In terms of eigenfunctions, the transformed function $v(\xi,\tau)$ can be expressed as:

$$v(\xi,\tau) = \sum_{n=1}^{\infty}\phi_n(\xi)q_n(\tau). \tag{35}$$

Substituting solution from (35) into differential (11), multiplying it by $\xi C(\xi)\phi_m(\xi)$ and integrating ξ from $\bar{r}$ to 1, the resulting differential equation becomes:

$$\frac{dq_n(\tau)}{d\tau} + \lambda_n^2 q_n(\tau) = -\gamma_n(\tau), \tag{36}$$

where $\gamma_n(\tau)$ is:

$$\gamma_n(\tau) = \frac{1}{N_n}\int_{\bar{r}}^{1} \xi C(\xi)\phi_n(\xi)F(\xi,\tau)d\xi \cdot \quad (37)$$

As a result, the solution for $q_n(\tau)$ in (36) is:

$$q_n(\tau) = e^{-\lambda_n^2\tau}[q_n(0) - \int_0^{\tau} e^{\lambda_n^2\zeta}\gamma_n(\zeta)d\zeta], \quad (38)$$

where $q_n(0)$ is determined from the initial condition as:

$$q_n(0) = \frac{1}{N_n}\int_{\bar{r}}^{1} \xi C(\xi)\phi_n(\xi)v_0(\xi)d\xi \cdot \quad (39)$$

After substituting the solution of the transformed function (35) and the shifting function (18) back into (10), the exact solution for the system is obtained.

E. Fundamental Solutions

In general, the closed-form fundamental solutions of a regular singular second-order differential with variable coefficients are not available. However, if the physical properties of the system can be expressed in arbitrary polynomial function forms, then a power series representation of the fundamental solutions can be constructed via the Frobenius method.

IV. Verification and an Example

To illustrate the previous analysis and the accuracy of the solution methodology, one examines the following example.

Consider the heat conduction in a non-uniform medium with the exponentially time-dependent heat flux at the outer surface,

$$f(\tau) = (1 - e^{-\beta\tau})\eta, \quad (40)$$

where β and η are two parameters. The coefficients of thermal conductivity and the specific heat functions are chosen as:

$$K(\xi) = \sum_{j=0}^{4} c_j\xi^j, \qquad c_0 \neq 0, \quad (41)$$

and

$$C(\xi) = \sum_{j=0}^{4} e_j^*\xi^j, \qquad e_0^* \neq 0 \cdot \quad (42)$$

The boundary value problem of the heat conduction in dimensionless form is:

$$\frac{1}{\xi}\frac{\partial}{\partial\xi}\left[\xi K(\xi)\frac{\partial v}{\partial\xi}\right] - C(\xi)\frac{\partial\theta}{\partial\tau} = 0, \quad \bar{r} < \xi < 1, \quad \tau > 0, \quad (43)$$

$$-\bar{K}_1\frac{\partial\theta(\bar{r},\tau)}{\partial\xi} + \bar{H}_1\theta(\bar{r},\tau) = 0, \qquad \text{at } \xi = \bar{r}, \quad (44)$$

$$\bar{K}_2\frac{\partial\theta(1,\tau)}{\partial\xi} + \bar{H}_2\theta(1,\tau) = (1 - e^{-\beta\tau})\eta, \text{ at } \xi = 1, \quad (45)$$

$$\theta(\xi, 0) = 0 \cdot \quad (46)$$

Now, $F(\xi,\tau)$ can be calculated as:

$$F(\xi,\tau) = \eta[\beta P_2(\xi) + P_1(\xi)]e^{-\beta\tau} - \eta P_1(\xi), \quad (47)$$

where $P_1(\xi)$ and $P_1(\xi)$ are

$$P_1(\xi) = a_1\xi^{-1} + 2(a_1 + 2a_2) + 3(a_1 + 2a_2)\xi + 4(a_1 + 2a_2)\xi^2, \quad (48)$$
$$+5(a_1 + 2a_2)\xi^3 + 6(2a_2)\xi^4$$

$$P_2(\xi) = a_1\xi + (a_1 + a_2)\xi^2 + (a_1 + a_2)\xi^3 + (a_1 + a_2)\xi^4. \quad (49)$$
$$+(a_1 + a_2)\xi^5 + a_2\xi^6$$

The eigenvalues λ_n and the associated eigenfunctions $\phi_n(\xi)$ are obtained from (30), (31) by numerical analysis. The two coefficients in (37), (38) are derived as:

$$\gamma_n(\tau) = \bar{\gamma}_{n1}(\tau)e^{-\beta\tau} + \bar{\gamma}_{n2}(\tau), \quad (50)$$

$$q_n(\tau) = \left[\frac{\bar{\gamma}_{n1}(\tau)}{\lambda_n^2 - \beta} + \frac{\bar{\gamma}_{n2}(\tau)}{\lambda_n^2}\right]e^{-\lambda_n^2\tau} - \frac{\bar{\gamma}_{n1}(\tau)}{\lambda_n^2 - \beta}e^{-\beta\tau} - \frac{\bar{\gamma}_{n2}(\tau)}{\lambda_n^2}, \quad (51)$$

where $\bar{\gamma}_{n1}(\xi)$ and $\bar{\gamma}_{n2}(\xi)$ are:

$$\bar{\gamma}_{n1}(\tau) = \frac{\eta}{N_n}\left(\int_{\bar{r}}^{1}\{\xi C(\xi)\phi_n(\xi)[\beta P_2(\xi) + P_1(\xi)]\}d\xi\right), \quad (52)$$

$$\bar{\gamma}_{n2}(\tau) = \frac{-\eta}{N_n}\left(\int_{\bar{r}}^{1}[\xi C(\xi)\phi_n(\xi)P_1(\xi)]d\xi\right) \cdot \quad (53)$$

Consequently, the exact solution for the system can be derived as:

$$\theta(\xi,\tau) = \sum_{n=1}^{\infty}[\phi_n(\xi)q_n(\tau)] + (a_1\xi + a_2\xi^2)(1 - e^{-\beta\tau})\eta \cdot \quad (54)$$

For convenience in the numerical analysis, we chose the parameters, $\beta = 10$ and $\eta = 4$, and set $\bar{K}_1 = \bar{H}_1 = 1$ at the inner surface of the hollow cylinder. The temperatures of the nonuniform hollow cylinder along the radial direction from $\xi = 0.8$ to $\xi = 1.0$ at the time $\tau = 0.2$ is investigated and shown in Table I. In the case, the thermal conductivity function and the specific heat function are chosen as $K(\xi) = C(\xi) = 1 + \xi + \xi^2 + \xi^3 + \xi^4$. We found that the trend of the temperature distribution is all the same in different $\bar{K}_2$ and $\bar{H}_2$. Moreover, it can be observed that the temperature of the medium increases as the thermal conductivity $\bar{K}_2$ increases for the same heat transfer coefficient $\bar{H}_2$.

TABLE I
TEMPERATURES OF A NONUNIFORM HOLLOW CYLINDER WITH A TIME-DEPENDENT HEAT FLUX BOUNDARY CONDITION
[$K(\xi) = C(\xi) = 1 + \xi + \xi^2 + \xi^3 + \xi^4$, $\bar{K}_1 = \bar{H}_1 = 1$, $\beta = 10$, $\eta = 4$, $\tau = 0.2$]

$\bar{K}_2$	$\bar{H}_2$	ξ				
		0.80	0.85	0.90	0.95	1.00
1	1	1.258	0.855	0.648	0.826	0.616
2	1	1.317	0.893	0.676	0.867	0.648
3	1	1.385	0.937	0.709	0.916	0.686
1	2	1.465	0.987	0.746	0.973	0.731
1	3	1.559	1.046	0.789	1.040	0.783

The influence of the heat transfer coefficient $\bar{H}_2$ at the outer surface on the temperature of a nonuniform hollow cylinder is shown in Fig. 2. When $\bar{K}_2 = 1$ and $\xi = 0.9$, the temperature of the medium decreases as the heat transfer coefficient $\bar{H}_2$ increases. The temperatures in all cases reach the steady state as time goes to infinite.

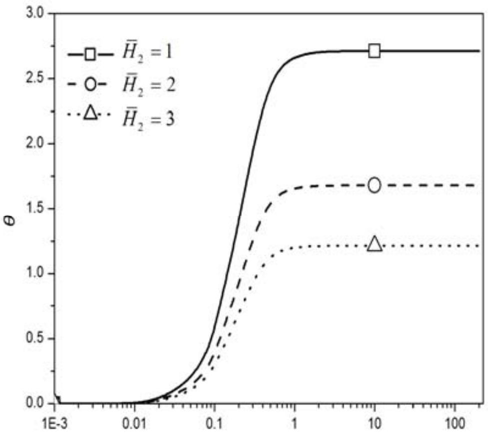

Fig. 2 Influence of heat transfer coefficient $\bar{H}_2$ on the temperature of a nonuniform hollow cylinder
[$K(\xi) = C(\xi) = 1 + \xi + \xi^2 + \xi^3 + \xi^4$, $\bar{K}_1 = \bar{H}_1 = \bar{K}_2 = 1$, $\beta = 10$, $\eta = 4$, $\xi = 0.9$]

Table II offers the temperature variations of various nonuniform hollow cylinders at the position, $\xi = 0.9$. The thermal conductivity function $K(\xi)$ and specific heat function $C(\xi)$ are specified by the same polynomial function, and we set $\bar{K}_2 = \bar{H}_2 = 1$. When $K(\xi)$ and $C(\xi)$ of the medium are increased, the temperature of the medium will increase except for $\tau < 0.5$. From Table II, one can find that the system reaches to the steady state as $\tau > 10$.

V. CONCLUSION

The shifting function method was proposed to develop exact solutions for the transient heat conduction in nonuniform hollow cylinders with time-dependent heat flux boundary condition at one surface. This work sets the thermal conductivity and the specific heat of the medium in polynomial function forms, therefore, the exact solutions of the system can be explicitly developed. The influence of physical properties on the temperature field of the heat conduction system was also investigated. Numerical analysis showed the efficiency of the proposed solution methodology.

TABLE II
TEMPERATURE VARIATIONS OF VARIOUS NONUNIFORM HOLLOW CYLINDERS
[$\bar{K}_1 = \bar{H}_1 = \bar{K}_2 = \bar{H}_2 = 1$, $\beta = 10$, $\eta = 4$, $\xi = 0.9$]

τ	$K(\xi) = C(\xi)$				
	$1+\xi+\xi^2+\xi^3+\xi^4$	$1+\xi+\xi^2+\xi^3$	$1+\xi+\xi^2$	$1+\xi$	1
0	0	0	0	0	0
0.02	0.027	0.025	0.020	0.036	0.030
0.05	0.175	0.166	0.136	0.205	0.174
0.08	0.397	0.377	0.315	0.444	0.382
0.1	0.564	0.537	0.453	0.617	0.536
0.2	1.385	1.322	1.163	1.390	1.252
0.5	2.514	2.411	2.285	2.212	2.107
1	2.706	2.599	2.545	2.294	2.215
10	2.711	2.604	2.558	2.295	2.217
20	2.711	2.604	2.558	2.295	2.217

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REFERENCES

[1] Őzisik, M.N., *Boundary value problems of heat conduction*, first ed., International Textbook Company, Scranton, 1968, pp. 125-163.
[2] Obata, Y. and Noda, N., “Steady Thermal Stresses in a Hollow Circular Cylinder and a Hollow Sphere of a Functionally Gradient Material”, *Journal of Thermal stresses*, vol. 17, 1994, pp. 471– 487.
[3] Awaji, H. and Sivakumar, R., “Temperature and Stress Distributions in a Hollow Cylinder of Functionally Graded Material: The Case of Temperature- Dependent Material Properties”, *Journal of the American Ceramic Society*, vol. 84, 2001, pp. 1059 – 1065.
[4] Praveen, G.N. and Reddy, J.N., “Nonlinear Transient Thermoelastic Analysis of Functionally Graded Ceramic-Metal Plates”, *International Journal of Solids and Structures*, vol. 35, pp. 4457– 4476, 1998.
[5] Jabbari, M., Sohrabpour, S. and Eslami, M.R., “Mechanical and Thermal Stresses in a Functionally Graded Hollow Cylinder due to Radially Symmetric Loads”, *International Journal of Pressure Vessels and Piping*, vol. 79, 2002, pp. 493 – 497.
[6] Jabbari, M., Sohrabpour, S. and Eslami, M.R., “General Solution for Mechanical and Thermal Stresses in a Functionally Graded Hollow Cylinder due to Nonaxisymmetric Steady-State Loads”, *ASME Journal of Applied Mechanics,* vol. 70, 2003, pp. 111 – 118.
[7] Ootao, Y. and Tanigawa, Y., “Transient Thermoelastic Analysis for a Functionally Graded Hollow Cylinder”, *Journal of Thermal Stresses,* vol. 29, 2006, pp. 1031 – 1046.
[8] Zhao, J., Ai, X., Li, Y.Z. and Zhou, Y.H., “Thermal Shock Resistance of Functionally Gradient Solid Cylinders”, *Materials Science and Engineering*, vol. 418, 2006, pp. 99 – 110.
[9] Hosseini, S.M., Akhlaghi, M. and Shakeri, M., “Transient Heat Conduction in Functionally Graded Thick Hollow Cylinders by Analytical Method”, *Heat Mass Transfer*, vol. 43, 2007, pp. 669 – 675.
[10] Shao, Z.S. and Ma, G.W., “Thermo-Mechanical Stresses in Functionally Graded Circular Hollow Cylinder with Linearly Increasing Boundary Temperature”, *Composite Structures*, vol. 83, 2008, pp. 259 – 265.
[11] Wang, H.M. and Liu, C.B., “Analytical solution of two-dimensional transient heat conduction in fiber-reinforced cylindrical composites”, *International Journal of Thermal Sciences*, vol. 69, 2013, pp. 43-52.
[12] Lee, S.Y. and Huang, C.C., “Exact solutions for Heat Conduction in Functionally Graded Circular Hollow Cylinders with Time Dependent

Boundary Conditions", *Mathematical Problems in Engineering*, vol. 2013, Article ID 816385, 8 pages.
[13] Lee, S.Y. and Huang, T.W., "Exact solutions for Heat Conduction in Non-uniform Mediums with General Time-Dependent Boundary Conditions", *J. Chinese Society of Mechanical Engineering*, vol.34, no.6, 2013, pp.475-485.
[14] Lee, S.Y. and Lin, S.M., "Dynamic analysis of nonuniform beams with time-dependent elastic boundary conditions", *ASME Journal of Applied Mechanics*, vol. 63, 1996, pp. 474-478.

Sen Yung Lee is a Distinguished Professor of Department of Mechanical Engineering, National Cheng Kung University, Taiwan, R. O. C.. He received his Ph.D. from State University of New York at Buffalo, USA in 1984. His research interests are solid mechanics, heat conduction, and creative design with more than 80 patients.