International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:9, No:7, 2015

Parameters Optimization of the Laminated Composite Plate for Sound Transmission Problem

Yu T. Tsai, Jin H. Huang

***Abstract*—**In this paper, the specific sound Transmission Loss (TL) of the Laminated Composite Plate (LCP) with different material properties in each layer is investigated. The numerical method to obtain the TL of the LCP is proposed by using elastic plate theory. The transfer matrix approach is novelty presented for computational efficiency in solving the numerous layers of dynamic stiffness matrix (D-matrix) of the LCP. Besides the numerical simulations for calculating the TL of the LCP, the material properties inverse method is presented for the design of a laminated composite plate analogous to a metallic plate with a specified TL. As a result, it demonstrates that the proposed computational algorithm exhibits high efficiency with a small number of iterations for achieving the goal. This method can be effectively employed to design and develop tailor-made materials for various applications.

***Keywords*—**Sound transmission loss, laminated composite plate, transfer matrix approach, inverse problem, elastic plate theory, material properties.

I. Introduction

THE sound transmission characteristics [1] of a multi-layer composite plate (LCP) can be observed by investigating the level of sound absorption, reflection, and transmission of the composite plate, which is related the material proprieties of the various layers of composite plate. The sound transmission phenomena can be obtained from several numerical analytic methods through the analysis of the wave propagation and the sound radiation of the plate. The wave characteristics of wave propagation in an infinite plate by using classical thin plate and Timoshenko-Mindlin plate theory [2], [3] have investigated by many of researchers [4]-[6]. Moreover, the computation of the finite anisotropic plate is complicated for classifying the wave type in a finite anisotropic plate [7]-[10] which is not as simple as that in an infinite isotropic plate. Many of exact and analytical numerical methods have been described and developed from elasticity theory and applied for isotropic laminated plates [11]. Applying the method of dynamic stiffness matrix for analyzing the wave propagation in different layer media in acoustic field, [12] gave the theoretical descriptions of wave propagation and sound radiation from layered composite plate. The dynamic stiffness of a multi-layer composite plate can be obtained by assembling the dynamic stiffness matrices from all of individual layers, which is expressed as a frequency-dependent matrix equation with the spectral displacements and spectral tractions on the layer surfaces. In the light of the simplicity of the dynamic stiffness method, the present work yields this method to analyze the level of sound transmission phenomena of the multi-layer composite plate. Although determining the dynamic stiffness matrix of a composite plate, the level of TL coefficient can be obtained by calculating the surface displacement and surface stress of the composite plate. On the other hand, [13] applied the classical method of a transfer matrix to study the transmission of elastic waves through a stratified solid medium. The transfer matrix method is based on the relationship between the pressure and bulk flow of two ends of a sound propagating route, which has been expressed by a suitable matrix and has been considered as an inherent and invariant property of a material of an acoustic structure. Most of the acoustical properties of the material in an acoustic structure can be obtained, provided the transfer matrix for a material is known. Additionally, leveraging the continuity of sound pressure and sound velocity, the matrices of individual laminates can be joined together into a single matrix for predicting the acoustical properties of multilayer acoustic material.

The purpose of this study is to obtain compatible material combinations for achieving the desired TL for the composite plate. The TL has significantly contributed to the evaluation of the efficiency of sound attenuation in numerous acoustic applications, including the creation of porous materials and sound absorption walls. In most cases, it is possible to construct a sound attenuation panel from different composite materials with differing acoustical attributes. A prevalent issue in utilizing a composite material is the difficulty of estimating the overall TL when the materials are laminated. The specific TL can be assessed from the measured incident, reflected, and transmitted waves of the plate or obtained by theoretical calculations. The effective use of materials should be clearly evaluated, and the feasibility of these materials for a compatible TL response should be determined.

II. Methods

A. Direct Problem

The sound transmission through plate involves sound waves in air on both sides of plate and elastic waves through solid, involving simultaneous conversion at fluid-solid interface. The stress-displacement relation can be given as

Yu T. Tsai is with the Bachelor's Program in Precision System Design, Feng Chia University, No. 100, Wenhwa Rd., Seatwen Taichung, Taiwan 40724, R.O.C. (e-mail: ytrich@mail.fcu.edu.tw)

Jin H. Huang is with the Department of Mechanical and Computer-Aided Engineering, Feng Chia University, No. 100, Wenhwa Rd., Seatwen Taichung, Taiwan 40724, R.O.C. (corresponding author to provide phone: +886-24517250; e-mail: jhhuang@fcu.edu.tw).

$$\begin{bmatrix} D_{11}^{(1)} & D_{12}^{(1)} & D_{13}^{(1)} & D_{14}^{(1)} \\ D_{21}^{(1)} & D_{22}^{(1)} & D_{23}^{(1)} & D_{24}^{(1)} \\ D_{31}^{(1)} & D_{32}^{(1)} & D_{33}^{(1)} & D_{34}^{(1)} \\ D_{41}^{(1)} & D_{42}^{(1)} & D_{43}^{(1)} & D_{44}^{(1)} \end{bmatrix} \begin{bmatrix} u^{(1)} \\ w^{(1)} \\ u^{(2)} \\ w^{(2)} \end{bmatrix} = \begin{bmatrix} \sigma_{zx}^{(1)} \\ \sigma_{zz}^{(1)} \\ \sigma_{zx}^{(2)} \\ \sigma_{zz}^{(2)} \end{bmatrix} \tag{1}$$

for LCP, the **D**-matrix as given in (1) is employed by combining individual **D**-matrices. As a representative example, **D**-matrix for double layer LCP can be written as:

$$\begin{bmatrix} D_{11}^{(1)} & D_{12}^{(1)} & D_{13}^{(1)} & D_{14}^{(1)} & 0 & 0 \\ D_{21}^{(1)} & D_{22}^{(1)} & D_{23}^{(1)} & D_{24}^{(1)} & 0 & 0 \\ D_{31}^{(1)} & D_{32}^{(1)} & D_{33}^{(1)}+D_{11}^{(2)} & D_{34}^{(1)}+D_{12}^{(2)} & D_{13}^{(2)} & D_{14}^{(2)} \\ D_{41}^{(1)} & D_{42}^{(1)} & D_{43}^{(1)}+D_{21}^{(2)} & D_{44}^{(1)}+D_{22}^{(2)} & D_{23}^{(2)} & D_{24}^{(2)} \\ 0 & 0 & D_{34}^{(1)}+D_{12}^{(2)} & D_{32}^{(2)} & D_{33}^{(2)} & D_{34}^{(2)} \\ 0 & 0 & D_{41}^{(2)} & D_{42}^{(2)} & D_{43}^{(2)} & D_{44}^{(2)} \end{bmatrix} \begin{bmatrix} u^{(1)} \\ w^{(1)} \\ u^{(2)} \\ w^{(2)} \\ u^{(3)} \\ w^{(3)} \end{bmatrix} = \begin{bmatrix} \sigma_{zx}^{(1)} \\ \sigma_{zz}^{(1)} \\ \sigma_{zx}^{(2)} \\ \sigma_{zz}^{(2)} \\ \sigma_{zx}^{(3)} \\ \sigma_{zz}^{(3)} \end{bmatrix} \tag{2}$$

consequently, the **D**-matrix of n-layers system can be as follows:

$$\mathbf{D} \begin{bmatrix} u^{(1)} \\ w^{(1)} \\ u^{(n+1)} \\ w^{(n+1)} \end{bmatrix} = \begin{bmatrix} \sigma_{zx}^{(1)} \\ \sigma_{zz}^{(1)} \\ \sigma_{zx}^{(n+1)} \\ \sigma_{zz}^{(n+1)} \end{bmatrix} \tag{3}$$

Now the **D**-matrix from (3) can be used for computation of the LCP by treating it as a single-layer composite plate. In (3), let **M** be the inverse of dynamic stiffness **D**, *i.e.*, **M** = $\mathbf{D}^{-1}$. Then, $\sigma_{zz}^{(n+1)} / w^{(n+1)} = 1/ \mathbf{M}(4,4)$ and $w^{(1)} / w^{(n+1)} = \mathbf{M}(2,4) / \mathbf{M}(4,4)$. Referring to the acoustic wave [1], incident, reflected, and transmitted waves are given by $Ae^{-i\gamma_{n+1}(z-H)}e^{i(kx-\omega t)}$, $Be^{-i\gamma_{n+1}(z-H)}e^{i(kx-\omega t)}$, and $Ce^{-i\gamma_{n+1}(z-H)}e^{i(kx-\omega t)}$, respectively with $\gamma_{n+1} = \sqrt{(\omega / c_0)^2 - k^2}$ then:

$$w^{(n+1)} = \left.\frac{\partial \phi}{\partial z}\right|_{z=H} = -i\gamma_{n+1}(A-B) \tag{4}$$

and

$$\sigma_{zz}^{(n+1)} = -\rho_{n+1}\omega^2 (A+B) \tag{5}$$

This leads to incident and reflected wave as given:

$$A = -\frac{1}{2}\left(\frac{w^{(N+1)}}{i\sqrt{(\omega / c_{N+1})^2 - k^2}} + \frac{\sigma_{zz}^{(N+1)}}{\rho_{N+1}\omega^2} \right) \tag{6}$$

$$B = \frac{1}{2}\left(\frac{w^{(N+1)}}{i\sqrt{(\omega / c_{N+1})^2 - k^2}} + \frac{\sigma_{zz}^{(N+1)}}{\rho_{N+1}\omega^2} \right) \tag{7}$$

Also, the transmission wave can be derived as $C = w^{(1)} / -i\gamma$. Above equation can be used to calculate the acoustic transmission in a variety of plate. Then, the sound transmission, reflection, and absorption can be obtained by:

$$\text{Sound absorption (dB): } 10\log_{10}\left((1 - |B/A|^2 - |C/A|^2) \right) \tag{8}$$

TL calculation was proposed to analyze the sound transmissions from the laminated composite plate. In general, if the values of **D** and $\sigma_{zz}^{(n+1)}$ of a composite plate are available, then the theoretical method can be used to solve the surface displacements **u** = $[u^{(1)}, u^{(n+1)}, w^{(1)}, w^{(n+1)}]^T$. Thus, the transmitted/incident wave ratio $T(\theta,\omega)$ can be determined easily. However, if the surface displacement vector **u** is known, then the solution of the material properties in matrix **D** can be treated as an optimization problem.

B. Optimization

In the previous section, TL calculation was proposed to analyze the sound transmissions from the laminated composite plate. In general, if the values of **D** and $\sigma_{zz}^{(n+1)}$ of a composite plate are available, then the theoretical method can be used to solve the surface displacements $u = [u^{(1)}, u^{(n+1)}, w^{(1)}, w^{(n+1)}]^T$. Thus, the transmitted/incident wave ratio $T(\theta,\omega)$ can be determined easily. However, if the surface displacement vector u is known, then the solution of the material properties in matrix **D** can be treated as an inverse problem.

To solve the inverse problem, first, an objective function J must be defined through the estimated and measured values of the surface displacements. Considering that is only dependent on $w^{(1)}$ and $w^{(n+1)}$ in the z-axis, $u^{(1)}$ and $u^{(n+1)}$ can be neglected. Therefore, the problem can be regarded as an optimization problem of the material properties involving the determination of the desired TL.

$$J = \sum_{\omega=\omega_0}^{\omega_n} \left\| \frac{\hat{w}^{(1)}}{\hat{w}^{(n+1)}} - \frac{w^{(1)}}{w^{(n+1)}} \right\|^2 \tag{9}$$

To solve the optimization problem, first, an objective function J must be defined through the estimated and measured values of the surface displacements. Considering that $T(\theta,\omega)$ is only dependent on $w^{(1)}$ and $w^{(n+1)}$ in the *z*-axis, $u^{(1)}$ and $u^{(n+1)}$ can be neglected. Therefore, the problem can be regarded to use an optimization method for determining the material properties involving the determination of the desired TL.

III. Numerical Simulation and Results

In this section, the ability of the proposed optimization method to obtain the material properties of a composite plate to a reasonable solution is evaluated. The purpose of the designed optimization case is to find another feasible and efficiency composite material plate to replace the thick aluminum plate. The estimated composite material plate should have the same TL results of the thick aluminum plate. From the chosen of initial guesses for desire materials properties, the TL curves can be calculated.

The calculation method of sound transmission of the multi-layer composite plate as shown in Fig. 1 is described in above section. The case shows in Fig. 2 to explore the TL of the composite plate by object values. As an illustrated example to show how our method can cope with the optimization problem, a case of a 0.6 cm thin plate is illustrated. Table I lists the initial guesses and optimal values of the material properties for a 2-layers composite plate. Fig. 3 shows that the optimization TL obtained by our method is matched with the objective TL. Fig. 4 shows that the optimization curve calculated by our estimated materials property is matched the objective curve. Fig. 5 shows that this computational algorithm also exhibited high efficiency with a small number of iterations for achieving the goal. Thus, the proposed optimization method has high potential for designing a composite material plate with reference to the solid plate.

TABLE I
THE LIST OF OBJECT VALUES, INITIAL GUESSES AND OPTIMAL VALUES

items	Object values (1-layer)	Initial guesses (2-layers)		Optimal values (2-layers)	
E (Pa)	6E9	1.6E9	7E9	3E9	2E9
h (cm)	3	1.3	0.3	2.5	1.27
v	0.4	0.80	0.40	0.4	0.28
P (kg/m^3)	600	800	200	300	93

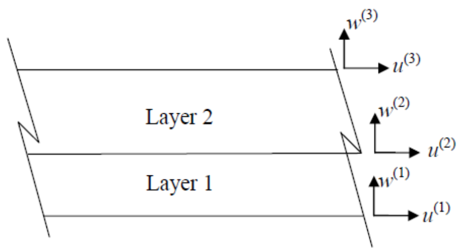

Fig. 1 The illustration of the multilayer composite plate

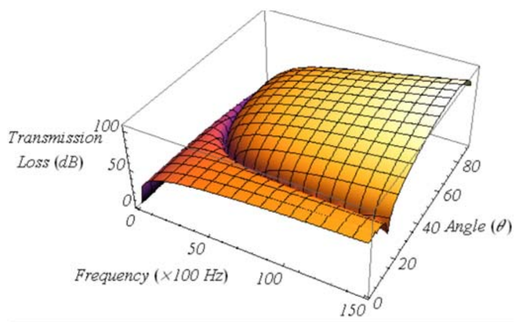

Fig. 2 Sound Absorption level of the objected LCP

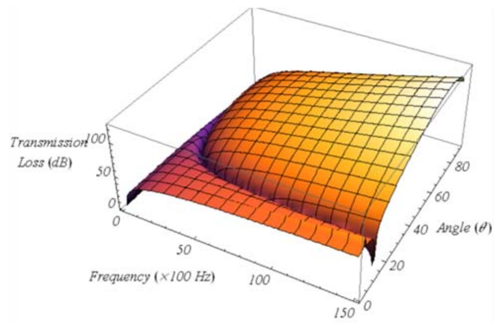

Fig. 3 Sound Absorption level of the LCP from optimization

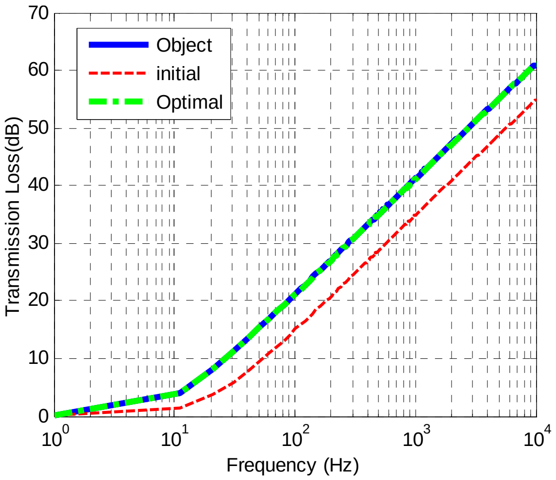

Fig. 4 Comparison of the transmission values for incident angles θ = 10∘

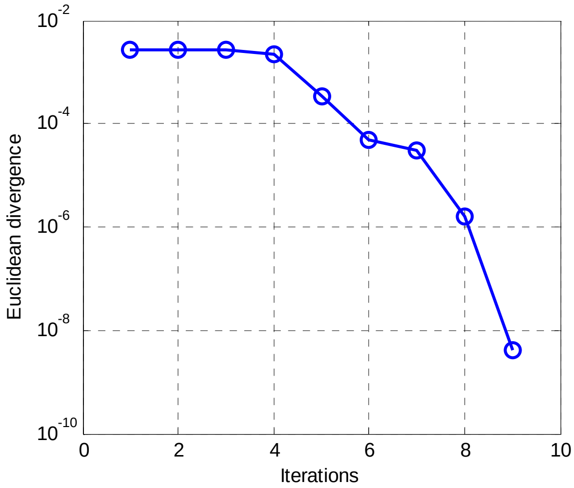

Fig. 5 Comparison of the convergence of the objective function for the result as shown in Fig. 4

IV. CONCLUSION

The work has demonstrated successful use of conjugate gradient optimization method for material properties estimation. Over a given range of frequencies, it is demonstrated that the optimal solution and the object yielded very similar results. The theoretical approach has found to be computationally efficient owing to use of small number of iterations during parameter optimization. Specifically, the dip in the response (coincidence frequency) or the reduction in the transmission loss of at the resonance is concluded to depend on the angle of incident. The design of laminated composite plate analogous to metallic plate with the specified transmission loss has established that this approach can be effectively employed for design and development of tailor made material for varied applications.

ACKNOWLEDGMENT

The authors would like to thank the National Science Council of Taiwan for financing under Contract Nos. NSC 101-2622-E-035-017-CC2 and NSC 101-2221-E-035-004-MY3. We are also thankful for partial financial support from Merry Electronic Co. Taichung, Taiwan.

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Yu T. Tsai received his M.Sc. degree from Electroacoustic Graduate Program at Feng Chia University in Taichung, Taiwan, in 2010. He was conferred upon the Gold Medal Award from the Evaluation Committee for the Merry Electro-acoustic thesis award by Merry Electronics, Co. Ltd. He was also selected an honorary member of the Phi Tau Pai Scholastic Honor Society of the Republic of China. He was finished for his doctorate in the Ph.D. Program of Mechanical and Aeronautical Engineering at Feng Chia University in 2010. The areas of research he planned to pursue include the following: mechanical and electrical integration, cyber-physical systems (CPSs), computer-aided optimization design, controller software development, precision measurement and analysis systems, inverse operation in sophisticated systems, and abnormal vibration sound processing technology.

Jin H, Huang is the founder and Director of the Electroacoustic Graduate Program at Feng Chia University since September 2006. He worked for the Department of Mechanical Engineering of the Feng Chia University (FCU) from 1993 till now. Presently, he is also working as Dean, College of Engineering, FCU. He has earned a PhD degree in Mechanical Engineering from Northwestern University in 1992. His research interest is in the areas of electroacoustics, MEMS Transducers, inverse problem of acoustics, and acoustics of fluid-structure interactions. He is using B&K Pulse, Sound Check, and Klippel measurements since 2005 for research and education in sound-structure interactions and electroacoustic engineering analysis.

He has published more than 100 scientific papers in international journals and nearly 100 scientific papers in international conference worldwide. He has authored 3 technical books. He is also working for various reputed international journals. He has been involved in active academic and industrial consultancy since his inception at FCU. He is actively involved for organization of seminars, workshops, visits for students of Electroacoustic Graduate Program.