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# Study of Mixed Convection in a Vertical Channel Filled with a Reactive Porous Medium in the Absence of Local Thermal Equilibrium

Hamid Maidat, Khedidja Bouhadef, Djamel Eddine Ameziani, Azzedine Abdedou

***Abstract*—**This work consists of a numerical simulation of convective heat transfer in a vertical plane channel filled with a heat generating porous medium, in the absence of local thermal equilibrium. The walls are maintained to a constant temperature and the inlet velocity is uniform. The dynamic range is described by the Darcy-Brinkman model and the thermal field by two energy equations model. A dimensionless formulation is developed for performing a parametric study based on certain dimensionless groups such as, the Biot interstitial number, the thermal conductivity ratio and the volumetric heat generation, q '''. The governing equations are solved using the finite volume method, gave rise to a multitude of results concerning in particular the thermal field in the porous channel and the existence or not of the local thermal equilibrium.

***Keywords*—**Mixed convection, porous medium, power generation, local thermal non equilibrium model.

## I. INTRODUCTION

THE convective heat transfer in porous media has a big importance in the case of technologies such as geothermal exploitation, cooling of electronic components and the treatment of radioactive waste. In fact, its involvement extends throughout geophysics as well as all the environmental sciences. Nield D. A. and Bejan A. [1] and Kambiz Vafai [2] provide a summary of numerical and experimental research already accomplished on the subject. Different models are used for modeling the dynamic and thermal fields. There may be mentioned in this context the Darcy-Brinkman model for the dynamic field and LTE (Local Thermal Equilibrium) and LTNE (Local Thermal Non Equilibrium) models for the thermal field.

Among the closest studies of the case, there may be mentioned the following works. Thus, [3] presented a numerical study of heat and mass transfer in a cylinder filled with a reactive porous medium. A numerical and analytical study of forced convection to a porous layer heated from below in the absence of local thermal equilibrium was investigated by [4]. B. Alazmi and K. Vafai [5] presented a study on the different transport models in porous media. An analytical investigation of fully developed forced convection in a channel limited by two plates subject to constant flux, and filled with a porous medium is presented by [6]. Yasser Mahmoudi and Mehdi Maerefat [7] have conducted an analytical investigation of heat transfer in a partially filled porous medium channel in the absence of local thermal equilibrium. A numerical investigation of mixed convection jet for a cooling heated surface immersed in a channel filled with a porous medium is carried out by [8]. A numerical analysis of the effect of the solid particle size on forced convection around a cylinder immersed in a horizontal bed of spherical particles is presented by [9]. Finally, [10] studied analytically and numerically the influence of the thermal non equilibrium in the mixed convection in a vertical channel filled with a porous medium.

In the present work, we are interested in the study of heat transfer in a vertical channel, open at both ends and filled with a reactive porous medium, in the absence of local thermal equilibrium. Some simplifying assumptions are adopted to allow the mathematical modeling of the problem.

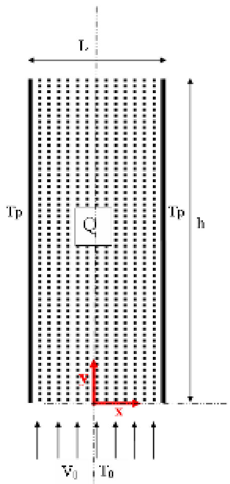



Fig. 1 Schematic of the physical model

## II. MATHEMATICAL FORMULATION

We consider a plane channel limited by two vertical walls (Fig. 1). Channel dimensions are its width L (the spacing between the two walls) and its height h. The channel is filled with a homogeneous and isotropic porous media, the fluid is injected at the channel inlet with uniforms speed and temperature $V_0$ and $T_0$, the duct walls are maintained at temperature $T_P$ higher than $T_0$, which gives the field symmetry about the median plane located midway between the plates.

Hamid Maidat, Khedidja Bouhadef, Djamel Eddine Ameziani, and Azzedine Abdedou are with Faculty of Mechanical and Process Engineering (FGMGP) USTHB. Laboratory of Multiphase Transport and Porous Media (LTPMP), Algeria (e-mail: hamidmaidat@yahoo.fr, khedbouh@yahoo.fr, ameziani.de@gmail.fr, abdedou.azzedine@gmail.com).

The analysis is made in terms of dimensionless parameters that reflect successfully all effects influencing transport. For this purpose, the setting in dimensionless form of the equations governing transfers is performed on the basis of the reference variables such as the width L, the input speed and the inlet temperature. Hence the following dimensionless parameters:

$$X=\frac{x}{L};\ Y=\frac{y}{L};\ U=\frac{u}{V_0};\ V=\frac{v}{V_0};\ \theta=\frac{T-T_0}{T_p-T_0}$$
$$H=\frac{h}{L};\ P=\frac{\varepsilon p}{\rho_f V_0^{\,2}} \tag{1}$$

Under these conditions, the dimensionless conservation equations are written as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{2}$$

$$\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\cdot\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}\right)U-\frac{1}{\mathrm{Re}.Da}U \tag{3}$$

$$\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}\right)V-\frac{1}{\mathrm{Re}.Da}V+\frac{Ra.\varepsilon}{\mathrm{Re}^2.\mathrm{Pr}}\theta_f \tag{4}$$

$$U\frac{\partial\theta_f}{\partial X}+V\frac{\partial\theta_f}{\partial Y}=\frac{1}{\mathrm{Re}.\mathrm{Pr}}\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}\right)\theta_f+\frac{6(1-\varepsilon)}{\varepsilon^2}\frac{Bi}{\mathrm{Re}_p.\mathrm{Pr}}\left(\theta_s-\theta_f\right) \tag{5}$$

$$0=\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}\right)\theta_s+Rc.q'''-\frac{6BiRc}{\varepsilon}\cdot\left(\frac{\mathrm{Re}}{\mathrm{Re}_p}\right)\left(\theta_s-\theta_f\right) \tag{6}$$

where:

$$\mathrm{Re}=\frac{\rho_f L V_0}{\mu_f};\ Da=\frac{K}{\varepsilon L^2};\ \mathrm{Pr}=\frac{\nu_f}{\alpha_f};\ \alpha_f=\frac{k_f}{\left(\rho c_p\right)_f};$$
$$Ra=\frac{g\beta\Delta T_{ref}L^3}{\nu_f\alpha_f};\ \nu_f=\frac{\mu_f}{\rho_f};\ q'''=\frac{Q}{Q_{ref}}\quad Q_{ref}=\frac{k_f\Delta T_{ref}}{L^2};$$
$$\Delta T_{ref}=T_p-T_0;\ Bi=\frac{h_{sf}L}{k_f};\ Rc=\frac{k_f}{k_s}$$

Thus, the non-dimensional boundary conditions can be expressed mathematically as:

At the inlet ( $Y=0$ ):

$$U(X,0)=0;\ V(X,0)=V_0;\ \theta_s(X,0)=\theta_f(X,0)=0 \tag{7}$$

At the exit ( $Y=H$ ):

$$\frac{\partial U(X,H)}{\partial Y}=\frac{\partial V(X,H)}{\partial Y}=0;\quad \frac{\partial\theta_s(X,H)}{\partial Y}=\frac{\partial\theta_f(X,H)}{\partial Y}=0 \tag{8}$$

In the middle ( $X=0$ ):

$$\frac{\partial U(0,Y)}{\partial X}=\frac{\partial V(0,Y)}{\partial X}=0;\quad \frac{\partial\theta_s(0,Y)}{\partial X}=\frac{\partial\theta_f(0,Y)}{\partial X}=0 \tag{9}$$

At the walls ( $X=0.5$ ):

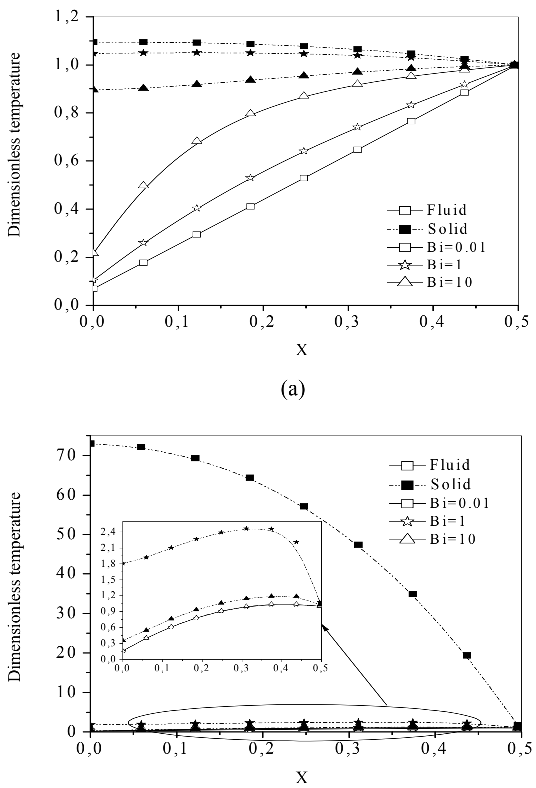
$$U(0.5,Y)=V(0.5,Y)=0;\quad \theta_s(0.5,Y)=\theta_f(0.5,Y)=1 \tag{10}$$

## III. Numerical Procedure

Solving the equations that govern our problem is through the use of a numerical method. This method consists in developing ways of solving these equations and involves the concept of discretization. The finite volume method [11] is used for the equations discretization. The power law scheme is used to evaluate the physical quantities at the interfaces of control volumes. The SIMPLER algorithm [12] is applied to determine the velocity and pressure fields and due to the symmetry about the median plane located midway between the walls, calculations will be made on half of the area.

## IV. Results and Discussion

For all the following results some dynamic and thermo-physical parameters, characteristics of the study, are held constant. These are the porosity ε=0.9, the particle Reynolds number, $\mathrm{Re}_p$=1 and the Prandtl number, Pr = 0.7.




(a)

(b)

Fig. 2 Effect of Biot number and thermal conductivity ratio on the dimensionless temperature for: $q'''$=100, Re=10, Da=$10^{-4}$, Ra=$10^4$
(a) Rc=0 .01, (b) Rc=10

The curves in Fig. 2 illustrate the effect of the Biot interstitial number on the temperature distribution for both fluid and solid phases for different values of the thermal conductivities ratio "Rc", with q''' = 100.

It appears that for high energy generation in the solid, with a relatively high thermal conductivity ratio of the order of Rc = 10 (Fig. 2 (b)), the axial temperature of the solid phase is increased sufficiently to reach high values for low values of Biot number and the temperature difference between both solid and fluid phases is very important, this is due to low values of the interstitial transfer coefficient between fluid and solid phases in coexistence; which makes very small internal thermal communication. Therefore, the more Biot increases, the temperature difference between the solid and fluid phases decreases, until the two profiles are overlap (Bi = 10). Indeed, when the thermal conductivity ratio is low (Rc = 0.01, i,e the conductivity of the solid is a hundred times higher than that of the fluid), the effect of the latter parameter tends to counterbalance the influence of increase in energy generation. This is consistent in governing equations (see (6)) and can be physically explained by the fact that heat generation is highly evacuated by conduction in the solid, allowing this phase remain at temperatures which are below the threshold value to the wall (Fig. 2 (a)).

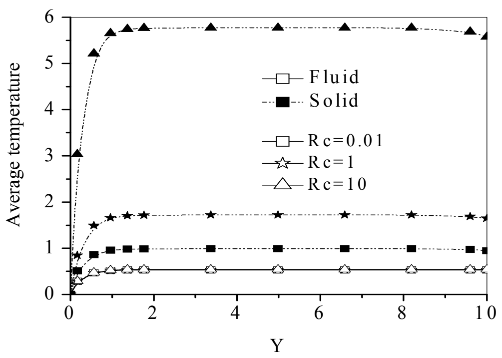
The average temperatures for both solid and fluid phases are defined as:

$$\theta_{fm} = \frac{1}{\frac{1}{2}V_m}\int_0^{\frac{1}{2}} V\theta_f \, dx \; ; \qquad \theta_{sm} = \frac{1}{\frac{1}{2}}\int_0^{\frac{1}{2}} \theta_s \, dx \tag{11}$$

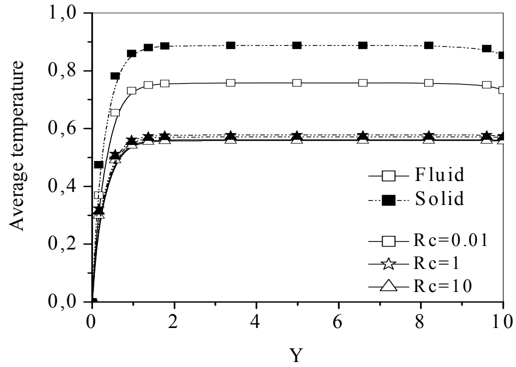
where $V_m$ is the mean velocity:

$$V_m = \frac{1}{\frac{1}{2}}\int_0^{\frac{1}{2}} V dX \tag{12}$$

Figs. 3 and 4 show the effect of thermal conductivity ratio on the average temperature profiles of the two phases, solid and fluid, with a considerable increase in energy generation (q '''= 10 and q''' = 100 respectively).

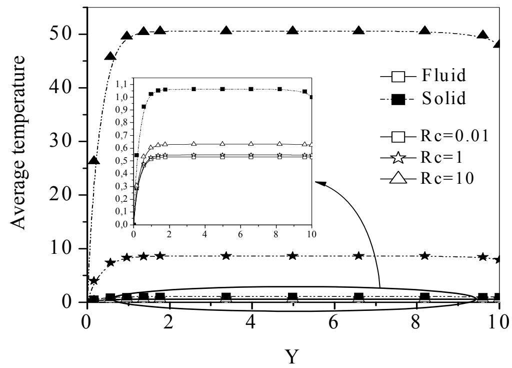



(a)

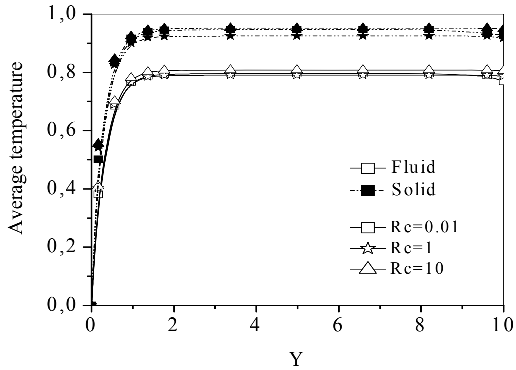



(b)

Fig. 3 Effect of thermal conductivity ratio on the average temperature profiles for fluid and solid phases for: q'''=10, Re=10, Da=$10^{-4}$, Ra=$10^4$ (a) Bi=0 .01, (b) Bi=10




(a)




(b)

Fig. 4 Effect of thermal conductivity ratio on the average temperature profiles for fluid and solid phases for: q'''=100, Re=10, Da=$10^{-4}$, Ra=$10^4$ (a) Bi=0 .01, (b) Bi=10

For a small Biot number (Figs. 3 (a) and 4 (a)), we note that the temperature of the fluid phase changes slightly for the full range of Rc values, and remains below the temperature of the wall despite the high power generation in the solid, unlike the temperature of the solid phase which varies with the change of Rc and reaches very high values for important thermal conductivity ratios, which increases the difference between the solid and fluid temperature. This behavior is due both to the high energy generation in the solid, and also to the low internal thermal communication between the two phases.

When the Biot number increases (Figs. 3 (b), and 4 (b)), we can observe that the average temperature of the solid phase decreases to values below the temperature of the wall in spite of a high energy generation, we also note that the average temperature profiles of the fluid phase vary with the change in Rc and the gap between the two temperatures is decreased and becomes negligible for high thermal conductivity ratio values. This can be explained by the strong internal heat transfer between the two phases.

It is also noticed that the increase in the local Biot number and the thermal conductivity ratio, diminishes the energy generation effect that increases the temperature of the solid phase towards high values, favoring the overheating of this phase.

## V. Conclusion

The mixed convection in a vertical channel filled with a reactive porous medium and whose walls are maintained at a constant temperature has been numerically analyzed.

The results show that local thermal fields are highly dependent on the combined influence of different effects and parameters involved, such as, in particular, the interstitial convective transfer, reflected in the Biot number, the thermal conductivity ratio values and the relative importance of heat generation in the solid phase. Thus, for low values of the interstitial Biot number, the gap between the two solid and fluid temperatures is important. By cons, increasing the interstitial Biot number will decrease the difference between the two temperatures. This difference also becomes negligible for high values of conductivity ratio, while low values of this ratio tend to reduce the increase effect in the Bi value. Increasing energy generation significantly affects the solid phase temperature particularly for low values of the Biot number.

It appears, therefore, that the LTE (local thermal equilibrium) model application may be incorrect in some situations where the differences in transfers at the two phases can be significant.

## Nomenclature

$Rc$ Thermal conductivity ratio
Re Reynolds number
$Da$ Darcy number
$h_{sf}$ Thermal interstitial coefficient
Pr Prandtl number
$Ra$ Rayleigh number
$\mathrm{Re}_p$ Particle Reynolds number
$Bi$ Biot interstitial number
$q'''$ Volumetric heat generation
$Q$ Heat generation per unit volume
$\alpha$ Thermal diffusivity
ν Kinematic viscosity
$\theta$ Dimensionless temperature

*Subscripts*

$_s$ Solid
$_f$ fluid

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