

Computation of Induction Current in a Set of Dendrites

Sudhakar Tripathi, R. B. Mishra

Abstract—In this paper, the cable model of dendrites have been considered. The dendrites are cylindrical cables of various segments having variable length and reducing radius from start point at synapse and end points. For a particular event signal being received by a neuron in response only some dendrite are active at a particular instance. Initial current signals with different current flows in dendrite are assumed. Due to overlapping and coupling of active dendrite, they induce currents in the dendrite segments of each other at a particular instance. But how these currents are induced in the various segments of active dendrites due to coupling between these dendrites, It is not presented in the literature. Here the paper presents a model for induced currents in active dendrite segments due to mutual coupling at the starting instance of an activity in dendrite. The model is as discussed further.

Keywords—Currents, dendrites, induction, simulation.

I. INTRODUCTION

THIS paper presents a generalized computational model of dendrites induction current considering cable model as reference model. Cable theory for dendrites was developed in 1959 by W. Rall precisely for the purpose of current signaling [1].

We present a mathematical model that describes the flow of electric current in morphologically and physiologically realistic dendritic trees that receive synaptic inputs at various sites and times [2].

Cable theory for dendrites, complemented by the compartmental modeling approach [3], played an essential role in the estimation of dendritic parameters [4], [5].

Due to current signaling in some prime dendrites the currents are induced in other dendrites due to coupling [6]-[8]. A mathematical model has been developed which collapses a dendritic neuron of complex geometry into a single electronically tapering equivalent cable by Roman R. Poznanski [9].

In this paper we have considered the induction of currents in dendrites due to overlapping and crossing each other in complex manner.

In Section II we have presented the detailed method of computation. In Section III we have presented result and its analysis and finally the work is concluded in Section IV.

II. METHODS

Following are the steps of the model:

The reference model is cable model of dendrites.

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1. A total number of N dendrites are considered to be active at a particular instance of synaptic current impulse (initial current through dendrites). Dendrites are $X_1, X_2, X_3 \dots X_N$.
2. Dendrites are considered in the form of cable (cylindrical) with finite length (L_i) having finite number of segments(n_i) with reducing radius(diameter) and variable lengths.
3. For X_i dendrite, L_i =Length of i th dendrite n_i = total number of Segments in X_i . $l_i(1), l_i(2), l_i(3), \dots, l_i(n_i)$ are lengths of segments of X_i respectively where $L_i = \sum(l_i)$ for $i=1 \dots n_i$.
4. Each dendrite crosses each other at segments represented by $x_p(i,j)$, segment at which X_i crosses X_j , although in general it is not necessary that they cut each other in same segment. But in this paper and simulation, it is considered to be same.
5. θ_{ij} represents the crossing angle between X_i and X_j that may vary from 0 to $\pi/2$ radians (0 to 90 degrees).
6. The perpendicular distance between segment of a dendrite to another dendrite is represented by $d_{ij}(j,k)$, which is perpendicular distance between j th segment of dendrite X_i and Dendrite X_k .
7. The perpendicular distance in crossing segment of Dendrites are taken to be $d_{ij}(j,k)$.
8. Each dendrites start radius(at soma joint) is represented by $r_s(i)$ for X_i dendrite and end segment radius is represented by $r_e(i)$.
9. Radius of segments of dendrite X_i is represented by $r_i(1), r_i(2), r_i(3), \dots, r_i(n_i)$.
10. Each dendrites radius from start to end is assumed to be reducing according to length.
11. The radius of segments of X_i is calculated as per

$$r_i(j) = r_s(i) - [(r_s(i) - r_e(i)) / L_i] * \sum l_i(1 \text{ to } j)$$

12. Surface area of segments of dendrite X_i is denoted by A_{ij} i.e. surface area of j th segment of dendrite X_i . It is calculated as per:

$$A_{ij} = 2 * \pi * r_i(j) * l_i(j)$$

13. As all the active dendrites at a particular instance gets initialized with input current in it through synapse, they induce currents in each other's segment according to mutual coupling coefficient. Mutual coupling coefficient is calculated as per following formula in between different segments of dendrite.

$$M(i, m, j, n) = [A_i(m) / A_j(n)] * p$$

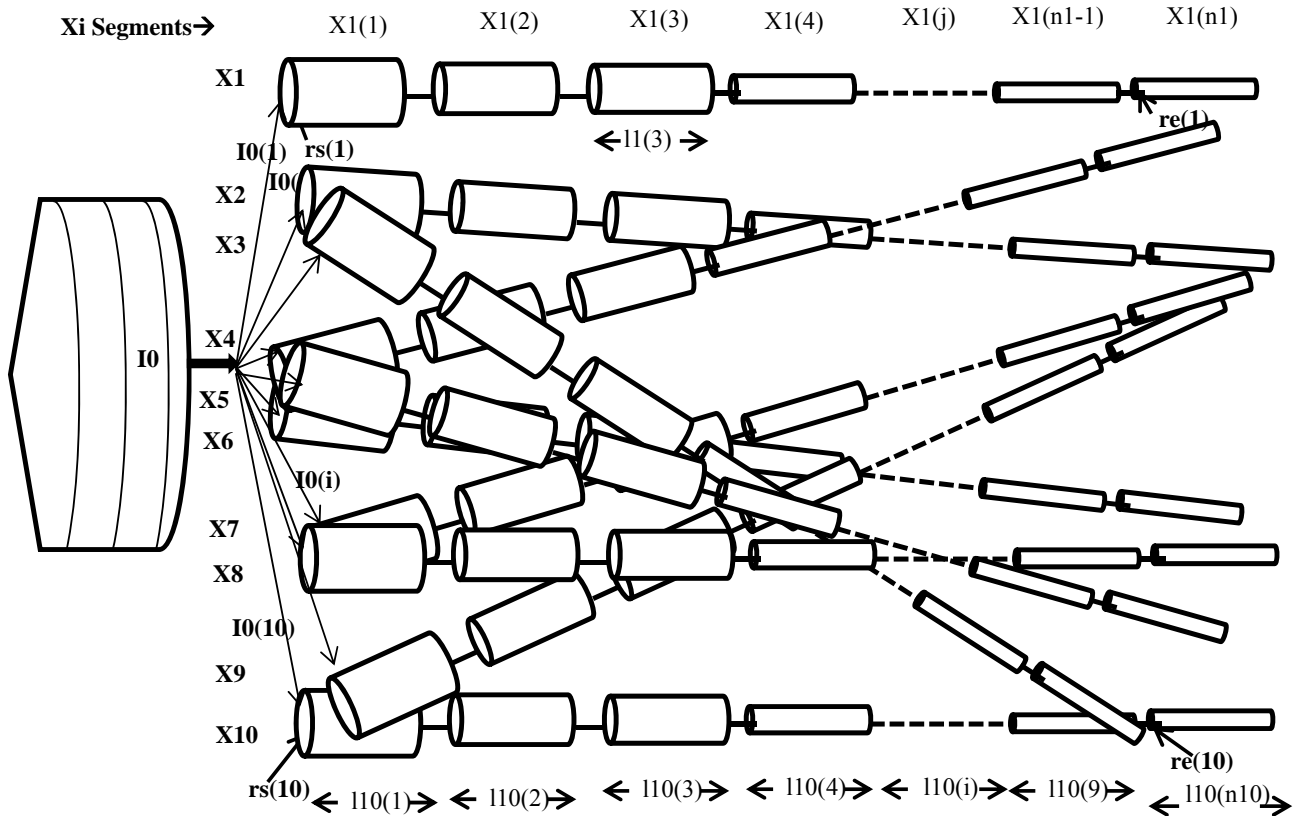


Fig. 1 Cable model of dendrites

$M(i, m, j, n)$ = Mutual Coupling coefficient between m th segment of X_i and n th segment of X_j . p = permeability of soma/dendrite medium.

14. The initial currents in dendrites are represented as $I_0(1)$, $I_0(2)$, $I_0(N)$ in dendrites respectively.

15. The current induced in n th segment of Dendrite X_j , due to m th segment of dendrite X_i is represented by

$$I_{ji}(n, m) = M(i, m, j, n) * I_0(m) * (1 / (d_j(n, m) / \cos(((i - x_p(j, i)) * \theta_c * (\pi / 180))))))$$

where θ_c = angle reduction constant in successive segments from perpendicular crossover point segments of dendrites (in this paper it is to be taken 3).

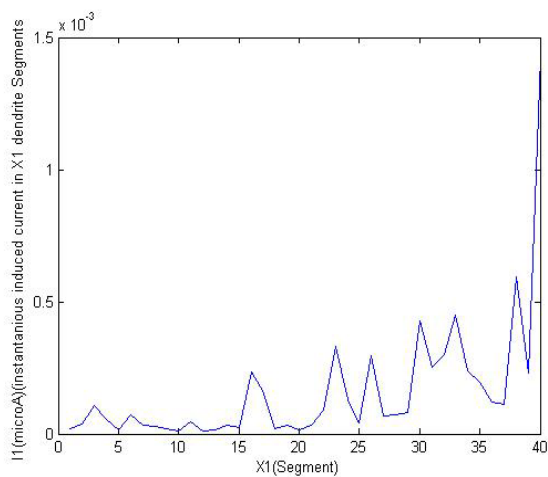
III. RESULTS AND ANALYSIS

Simulations of model for induced currents in segments of active dendrite due to mutual coupling, the parameters [10] are used as per given in Table I using model formulation. MATLAB 13b is used to implement the model.

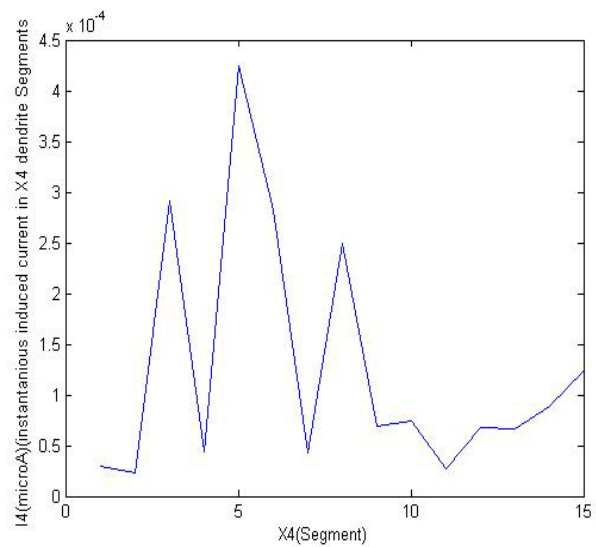
In Figs. 2 (a)-(j), it is shown the currents induced in various segments of dendrites due to other active dendrite segments. It is clear that induced current depends on mutual coupling coefficient, radius, length, permeability, crossing point and crossing angle of dendrites at an instance of event activation in successive neurons.

TABLE I
MODEL PARAMETERS

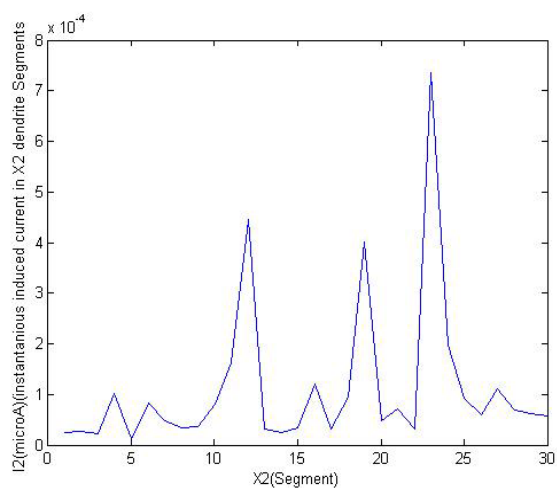
Parameter	Value	Description
N	10	Number of active dendrite at an instance i.e X_i
L_i	450-2350 μm	Total length of i th dendrite i.e. X_i
n_i	10-60	Number of segments in dendrite i i.e. X_i
l_{ij}	10-70 μm	Length of j th segment of dendrite i, i.e. X_i
$x_p(i, j)$	1-9	Crossing point of dendrite i and j, i.e. X_i and X_j
θ_{ij}	0	Angle between dendrite i and j, i.e. X_i and X_j
$d_i(j, k)$	0.0001-0.01 μm	Perpendicular distance between j th segment of dendrite i and k, i.e. X_i and X_k
$r_s(i)$	4-15 μm	Start point radius of dendrite i, i.e. X_i
$r_e(i)$	1-6 μm	end point radius of dendrite i, i.e. X_i
p	0.0000001	permeability of soma/dendrite medium
$I_0(i)$	0.001-0.02 μA	Initial input current in active dendrite i, i.e. X_i
θ_c	3°	angle reduction constant in successive segments



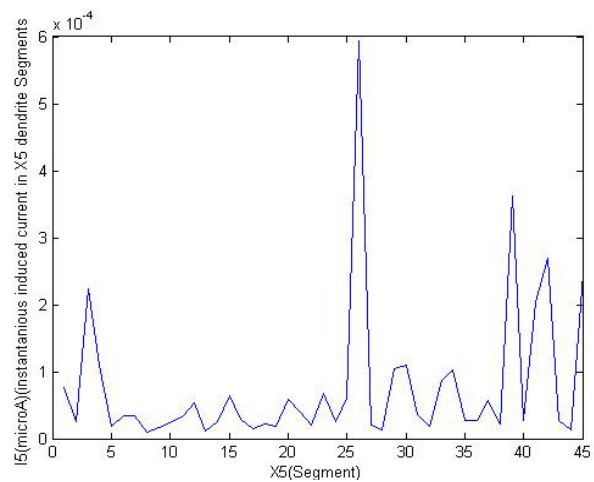
(a)



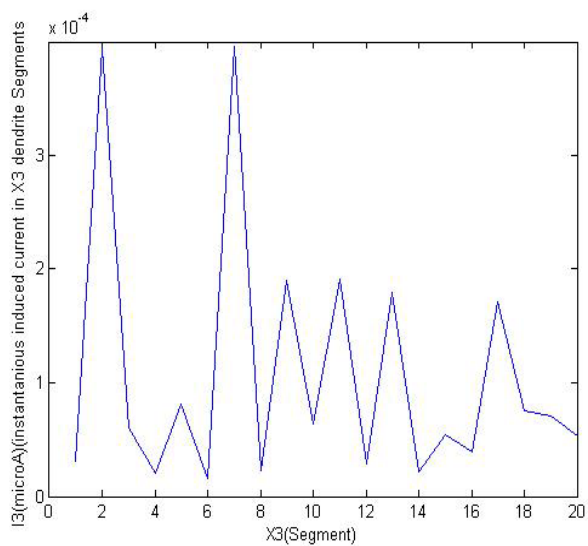
(d)



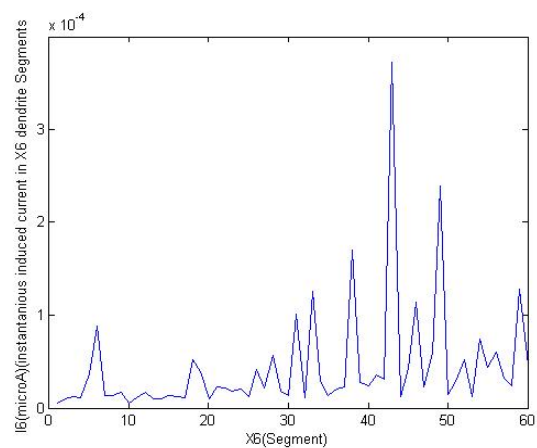
(b)



(e)



(c)



(f)

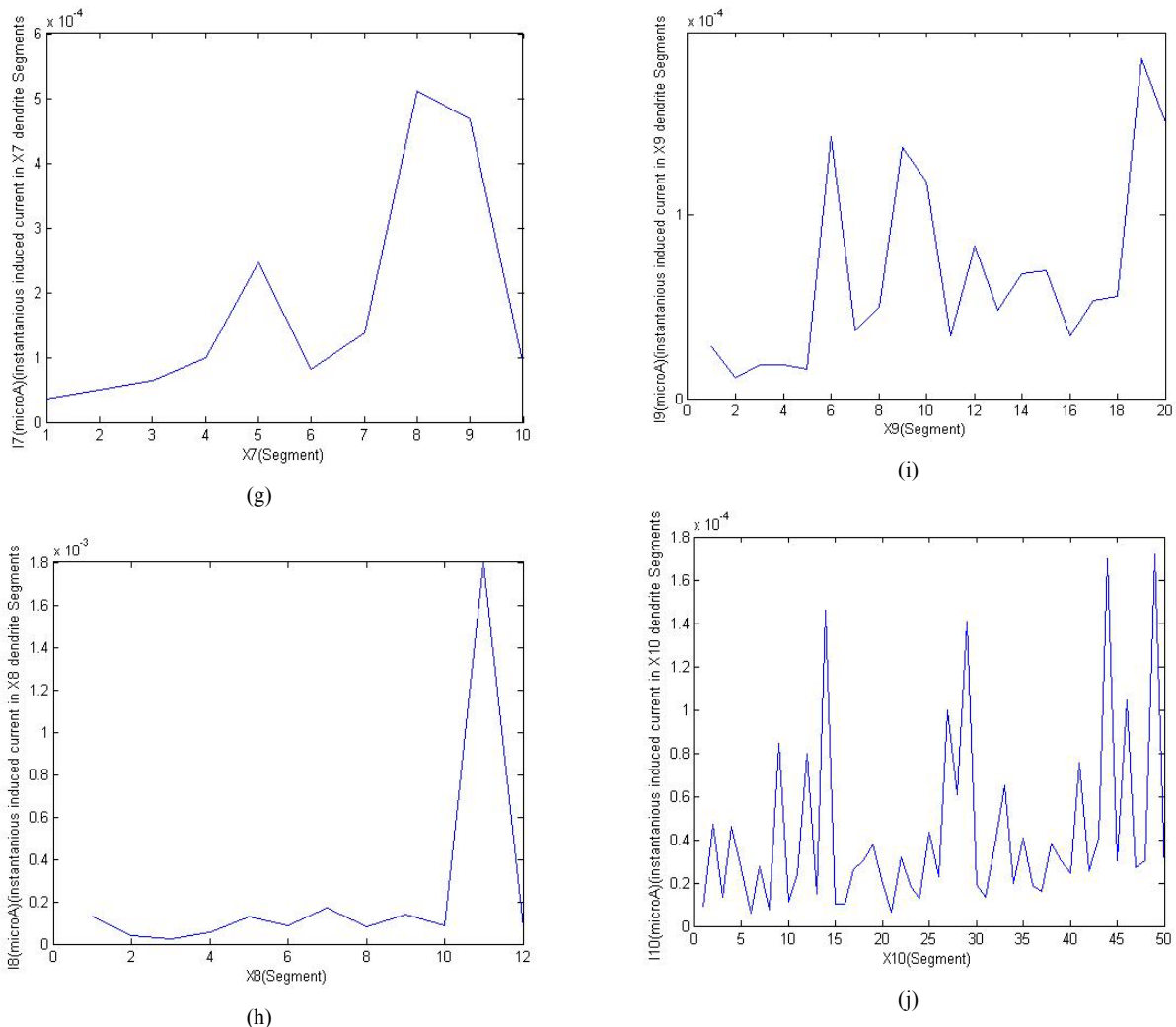


Fig. 2 (a) I1(j) induced current in the segments of dendrite X1 at active instance (b) I2(j) induced current in the segments of dendrite X1 at active instance (c) I3(j) induced current in the segments of dendrite X1 at active instance (d) I4(j) induced current in the segments of dendrite X1 at active instance (e) I5(j) induced current in the segments of dendrite X1 at active instance (f) I6(j) induced current in the segments of dendrite X1 at active instance (g) I7(j) induced current in the segments of dendrite X1 at active instance (h) I8(j) induced current in the segments of dendrite X1 at active instance (i) I9(j) induced current in the segments of dendrite X1 at active instance (j) I10(j) induced current in the segments of dendrite X1 at active instance

In Fig. 3, the combined results of current induction in dendrite segments are shown. Initial currents at the instance of event activity are $I_0(1) = .002$, $I_0(2) = .003$, $I_0(3) = .001$, $I_0(4) = .006$, $I_0(5) = .008$, $I_0(6) = .009$, $I_0(7) = .010$, $I_0(8) = .006$, $I_0(9) = .001$, $I_0(10) = .020$ in dendrite X1, X2, X3, X4, X5, X6, X7, X8, X9, X10 respectively. The Perpendicular distance between j^{th} segments of dendrite i and k, i.e. X_i and X_k has been taken for calculation of distance of various segments according to crossing angle and point between two dendrites. The number of segments in different dendrites is different and they are crossing each other at different points (Fig. 1). The segments falling after crossing point causes positive induction in current and that before the crossing point causes negative induction.

As the crossing points of dendrites have been taken towards starting end of dendrites between 1 and 9, the induced current in a particular segment of a dendrite is sum of currents induced due to all dendrites segment causing a positive increase at an instance. In Fig. 3, the legend shows the various colour lines and current induction in dendrites e.1 I1 to I10. I1 has the highest induction in current of 0.0018 micro Ampere for segment 12 of X1. The second highest induction in current of 0.0014 micro Ampere for segment 42 of X6. The induction in the segments having lower crossing point, distance and angle is higher and that for higher crossing point, distance and angle is lower.

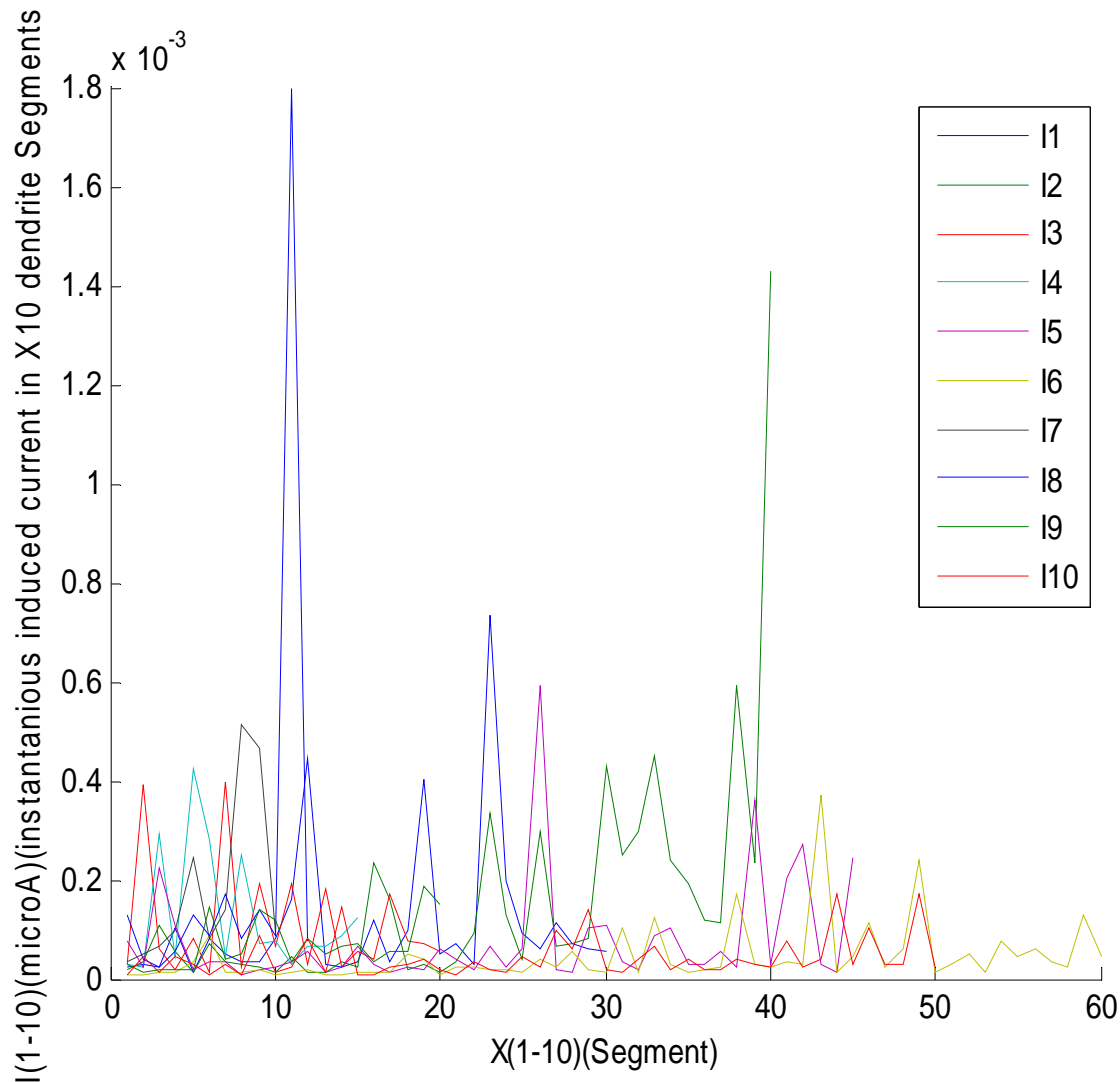


Fig. 3 Currents induced in all active dendrites due to mutual coupling at active instance in all segments

IV. CONCLUSION

In this paper a computational model for current through active dendrites corresponding to a particular event activity have been presented. When an event is active in neural system of brain, the activity signal passes through various neuronal cascades. Not all dendrites of neurons in successive cascades are active. Only some dendrites are active and current signal of activity initializes in only those dendrite. As dendrites crosses over each other with mutual coupling it causes more instantaneous induction of current in the segments of dendrites. The model presented in this paper simulates the behavior of this natural phenomenon with certain assumption states in the model.

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