

# Equalization Algorithms for MIMO System

Said Elkassimi, Said Safi, B. Manout

**Abstract**—In recent years, multi-antenna techniques are being considered as a potential solution to increase the flow of future wireless communication systems. The objective of this article is to study the emission and reception system MIMO (Multiple Input Multiple Output), and present the different reception decoding techniques. First we will present the least complex technical, linear receivers such as the zero forcing equalizer (ZF) and minimum mean squared error (MMSE). Then a nonlinear technique called ordered successive cancellation of interferences (OSIC) and the optimal detector based on the maximum likelihood criterion (ML), finally, we simulate the associated decoding algorithms for MIMO system such as ZF, MMSE, OSIC and ML, thus a comparison of performance of these algorithms in MIMO context.

**Keywords**—Multiple Input Multiple Outputs (MIMO), ZF, MMSE, Ordered Interference Successive Cancellation (OSIC), ML, Interference Successive Cancellation (SIC).

## I. INTRODUCTION

COMMUNICATION systems comprise three fundamental elements: transmitter, channel, and receiver. When signals are transmitted through a communications system, they are obstructed by some distortions that are mainly inter-symbol interference (ISI) and noise [20]. The transmitted signal is distorted by ISI which is caused by multipath effect in a band limited (frequency selective) time dispersive channels and is the cause of bit errors on the receiver side [19]. ISI considers the main factor as negatively affecting the fast transmission of data over wireless channels. To eliminate or minimize these distortions, equalizers are employed in these systems. Equalization is the method of compensating for, eliminating or reducing the amplitude and phase distortion introduced by the transmission medium in communications systems. In a general meaning, the term equalization refers to any signal processing operation which minimizes ISI. Equalization techniques can be categorized into linear or nonlinear techniques depending on the way the output of an adaptive equalizer is used for subsequent control of the equalizer [18]. The decision making device of the receiver processes the equalizer's output and determines the value of the digital data bit being received before applying a slicing or thresholding operation (a nonlinear operation) to determine the value of the reconstructed message data. If this data is not used in the feedback path for the adapting of the equalizer, it is

a linear type of equalization, but on the other hand, if the decision making device feeds the reconstructed data back in order to alter the equalizer's subsequent outputs, the equalization is nonlinear [17]. MIMO transmissions were designed in 1996 by Bell Lab [10] to increase the channel capacity. This type of transmission uses multiple transmit antennas and multiple reception antennas in order to convey a message. At the reception many decoding techniques receptor [7] have been proposed, such as zero-forcing equalizer or maximum likelihood decoder (ML) [6]... This paper aims to familiarize themselves with MIMO and reception decoding techniques associated with them. The simulations of decoding algorithms with MIMO system [3] are presented.

## II. MIMO SYSTEM MODEL

In the MIMO system that employs multiple antennas in the transmitter and/or receiver, the correlation between transmit and receive antenna is an important aspect of the MIMO channel [4]. We consider a MIMO system [1] along with  $N_t$  transmitters and  $N_r$  receivers (Fig. 1). It is assumed that the transmission channel is frequency selective and non-selective in time, so coefficient  $h_{mn}$  represents the path between the  $m^{\text{eme}}$  transmitting antenna and the  $n^{\text{eme}}$  receiving antenna [8].

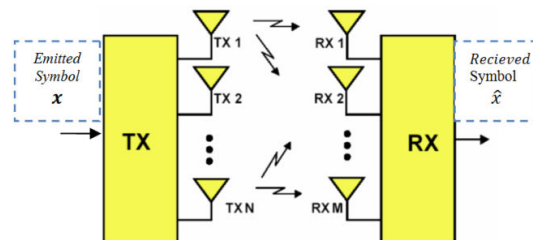


Fig. 1 MIMO transmission system

We call  $x_m(k)$  the transmitted signal on the  $m^{\text{eme}}$  antenna at time  $k$ , the received symbol on the  $n^{\text{eme}}$  antenna [9] is then:

$$y_n(k) = \sum_{m=1}^{N_t} h_{nm} x_m(k) + b_n(k) \quad (1)$$

where  $b_n$  is an additive white noise. Considering all of the signals received simultaneously, the refer to (1) can be written in matrix form dimension  $N_r \times N_t$ :

$$y(k) = Hx(k) + b(k) \quad (2)$$

with

$$H = \begin{pmatrix} h_{11} & \dots & h_{1N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r1} & \dots & h_{N_rN_t} \end{pmatrix}, y = (y_1 \dots y_{N_r})^T \\ x = (x_1 \dots x_{N_t})^T, b = (b_1 \dots b_{N_r})^T$$

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The MIMO channel capacity is now given by a sum of the capacities of the virtual SISO channels, that is,

$$C = \sum_{i=1}^r C_i(\gamma_i) = \sum_{i=1}^r \log_2 \left( 1 + \frac{E_x \gamma_i}{N_T N_0} \lambda_i \right) \quad (3)$$

The capacity in (3) can be maximized by solving the following power allocation problem:

$$C = \max_{\{\gamma_i\}} \sum_{i=1}^r \log_2 \left( 1 + \frac{E_x \gamma_i}{N_T N_0} \lambda_i \right) \quad (4)$$

subject to  $\sum_{i=1}^r \gamma_i = N_T$ . It can be shown that a solution to the optimization problem in (4) is given as:

$$\gamma_i^{opt} = \left( \mu - \frac{N_T N_0}{E_x \lambda_i} \right)^+, \quad i = 1, \dots, r \quad (5)$$

$$\sum_{i=1}^r \gamma_i^{opt} = N_T \quad (6)$$

where  $\mu$  is a constant,  $E_x$  is the energy of the transmitted signals, and  $N_0$  is the power spectral density of the additive noise  $\{|b_i\}_{i=1}^{N_R}$ .

### III. DECODING ALGORITHMS ASSOCIATED WITH MIMO TECHNIQUES RECEPTION [5]

This section presents the different decoding techniques in linear reception such as zero forcing equalizer (ZF) and minimum mean squared error (MMSE), and nonlinear regards ordered successive interferences cancellation (SIC) and the maximum likelihood (ML) [2].

#### A. Zero-Forcing Equalizer (ZF)

Zero Forcing refers to a technique of linear equalization algorithm used in the world of telecommunications that involves inverse of the frequency response of a particular channel [21]. The zero forcing equalizer is a detection technique by matrix inversion. This technique consists in applying to the received vector an equalization matrix  $W$ . ZF criterion was proposed to eliminate the SIC of the output of the equalizer. The matrix  $W$  is equal to the pseudo-inverse of the channel matrix [13] which is written by:

$$W = (H^H H)^{-1} H^H \quad (7)$$

#### B. Minimum Mean Square Error Equalizer (MMSE)

In telecommunication, a Minimum Mean Square Error (MMSE) estimator is an estimator which follows an estimation method, through which it minimizes the mean square error for the fitted values of various dependent variables. The method MMSE more closely refers to the estimation of a quadratic cost function in Bayesian setting. The thinking procedure behind this Bayesian approach is to estimate stems from various practical conditions where we sometimes have some major information about the parameters that are required to be estimated. MMSE receiver holds back both interference as well as noise components, but as far as the ZF receiver is a concern, it only eliminates the interference or the noise. From this, we can conclude that the Mean Square

Error (MSE) is minimized. To overcome the drawback of noise enhancement of ZF, the concept of MMSE is introduced. So, we can say that MMSE is pretentious to ZF in the presence of noise and interference. Now the Linear Minimum Mean Square Estimator for the MIMO System is [21].

$$\hat{x} = P_d H^H (H^H + \sigma_n^2)^{-1} y \quad (8)$$

where  $P_d$  is the power of each diagonal element, and  $\sigma_n^2$  is the power of noise component. The MMSE equalizer is based on minimizing of the mean squared error:

$$\hat{W} = \arg \min_W E[\|W^H y - x\|^2] \quad (9)$$

The optimal equalization matrix is then given by:

$$W = H^H \left( \frac{\sigma_b^2}{\sigma_x^2} I_{N_r} + H H^H \right)^{-1} \quad (10)$$

Avec  $E[bb^H] = \sigma_b^2 N_{N_r}$  et  $[xx^H] = \sigma_x^2 I_{N_t}$ . In the presence of noise, the matrix to be inverted is always defined positive and therefore invertible.

#### C. Ordered Interference Successive Cancellations (OSIC) of Equalizer [12]

ZF and MMSE equalizers are not always satisfactory: The first is sensitive to noise and the second does not remove any ISI. Thus, Golden and Foschini proposed in [5], [11] an algorithm based on a ZF or MMSE criterion decision feedback [15] to decode the BLAST codes [7]. The principle of this algorithm called OSIC is: the contribution of the symbol  $x$  which has just been detected is subtracted from the received vector, which yields a vector containing less interference. The transmitted symbol on the data path having the strongest power is decoded first. After decoded, its contribution is canceled on the received vector; the operation is repeated for all transmitted symbols. The transmitted symbol on the data path having the strongest power is first decoded. After decoded, its contribution is canceled on the received vector; the operation is repeated until all the symbols transmitted. The following algorithm:

$$\begin{aligned} & \textbf{Initialization} \\ & i = 1 \\ & G_1 = H^\dagger \\ & k_1 = \arg \min_j \|(G_1)_j\|^2 \\ & \textbf{Iterative Loop} \\ & W_{ki} = (G_i)_{ki} \\ & r_{ki} = w_{ki}^T r_i \\ & \hat{x}_{ki} = Q(y_{ki}) \\ & y_{i+1} = y_i - \hat{x}_{ki} (H)_{ki} \\ & G_{i+1} = (H_{ki}^-)^\dagger \\ & k_{i+1} = \arg \min_{j \notin \{k_1, \dots, k_i\}} \|(G_{i+1})_j\|^2 \\ & i = i + 1 \end{aligned}$$

This algorithm uses the following notations:

- $(G_i)_j$  is the  $j^{\text{eme}}$  line from  $G_i$ .
- The  $k_i$  represent the symbols of the detection order.
- $Q$  symbolizes the quantization process.

➤  $H_{ki}$  is the channel matrix  $H$  cancel the contributions of the  $k_i$  first transmitter.

Like all decision feedback equalizers [16], OSIC has the disadvantage of propagating errors.

#### D. Maximum Likelihood (ML) Equalizer

Here, we develop the maximum likelihood equalization procedure for isolated word recognition [2]. However, it is general enough and can be used for continuous speech recognition employing sub-word units [24]. We use here the cepstral coefficients derived through linear prediction analysis as recognition features [22]. Let the input utterance to be recognized be represented by a sequence of observation (cepstral) vectors,  $y = \{y_1, y_2, \dots, y_T\}$ , where  $T$  is the number of frames in the input utterance. Since this utterance is spoken under adverse conditions, it is distorted. Our aim here is to clean up this distortion. For this, we transform each vector of this utterance such that the likelihood function is minimized. Let  $F_\eta$  denote the transformation (parameterized in terms of a parameter vector  $\eta$ ), and  $x = \{x_1, x_2, \dots, x_T\}$  the observation sequence after transformation. Then:

$$x_t = F_\eta(y_t), \text{ for } 1 \leq t \leq T \quad (11)$$

Our goal is to find this transformation such that it maximizes the likelihood function under the HMM framework [23]. For finding this transformation, consider a continuous density HMM  $\lambda = [N, \pi, A, B]$ , where  $N$  = the number of states in the model,  $\pi = \{\pi_i, 1 \leq i \leq N\}$ , the initial state probability vector ( $\pi_i$  is the probability that the model is in state  $i$  initially),  $A = \{a_{ij}, 1 \leq i, j \leq N\}$ , the transition matrix of underlying Markov chain ( $a_{ij}$  is the probability of transition from state  $i$  to state  $j$ ) [24], and  $B = \{b_j(x_t), 1 \leq j \leq N\}$ , the output probability matrix. Here  $b_j(x_t)$  is the probability of outputting the vector  $x_t$  when the model is in state  $j$ . In our study, we represent  $b_j(x_t)$  as a mixture of  $M$  normal probability density functions (PDFs), i.e.,

$$b_j(x_t) = \sum_{k=1}^M C_{jk} N(x_t, \mu_{jk}, \sigma_{jk}) = \sum_{k=1}^M \frac{C_{jk}}{[(2\pi)^d / 2 \prod_{i=1}^d \sigma_{jki}]} \quad (12)$$

where  $d$  is the number of features in an observation (cepstral) vector,  $C_{jk}$  is the mixture weight of  $k$ -th mixture of  $j$ -th state, and  $\mu_{jk}$  and  $\sigma_{jk}$  are the mean and standard deviation vectors, respectively, of  $j$ -th state and  $k$ -th mixture. Note that we use here only diagonal covariance matrices (i.e., we assume off-diagonal elements to be zero). The transformation  $F_\eta$  is estimated by the maximum likelihood formulation in two steps: segmentation and maximization. In the segmentation stage, the model  $\lambda$  is assumed to be known and the Viterbi algorithm is used to segment the observation sequence into states. Let the state sequence be given by:

$$q_1^T = \{q_1, q_2, \dots, q_T\}$$

In the maximization stage, the transformation is obtained by maximizing the likelihood function which is expressed as the

probability of the sequence  $x$  given the model and state sequence and is written as:

$$P(x|q_1^T, \lambda) = \prod_{t=1}^T b_{q_t}(x_t) \quad (13)$$

Let us denote  $q_t$  by  $j$ . Then, the log-likelihood of the sequence  $x$  is:

$$L(x|q_1^T, \lambda) = \log(P(x|q_1^T, \lambda)) \quad (14)$$

$$L(x|q_1^T, \lambda) = \sum_{t=1}^T \log(b_j(x_t)) \quad (15)$$

By substituting the value of  $x_t$  from (11) into (15), we get the likelihood function in terms of the transformation  $F_\eta$ . In order to solve for this transformation, we consider two cases. In case 1, we assume that the functional form of the transformation is known and, it is represented in terms of a few parameters. For example, we know that the additive noise introduces multiplicative distortion in the cepstral vector. This means that  $x_t = \alpha y_t$ , where  $\alpha$  is a constant for a given utterance whose value depends on the amount of additive noise distortion. Similarly, we know that the channel mismatch distortion becomes additive in the cepstral domain. This means that  $x_t = y_t - B$ , where  $B$  is the bias vector which remains constant for the input utterance. The parameters of this transformation can be computed by minimizing the likelihood function (15). In case 2, we do not know the functional form of the transformation. In this case, we use a multilayer perceptron to approximate this transformation. Note that the multilayer perceptron can provide nonlinear transformation. The connection weights of the multilayer perceptron can be estimated by the back-propagation algorithm using the likelihood function (15) as the cost function. Note that in this procedure we compute the transformation for each input utterance we recognize. This may be computationally expensive in some applications. For this, we suggest an alternate way where we provide a small amount of calibration speech to the recognizer before its use to compute the transformation for a given adverse environment. Once we have learnt this transformation, we can use the recognizer with this transformation as long as our environmental condition does not change. In the present paper, we assume this transformation to be linear and it is characterized by an additive bias vector  $B$  in the cepstral domain; i.e.,

$$x_t = y_t - B \quad (16)$$

The bias  $B$  is obtained by maximizing the following log-likelihood function:

$$L(y|q_1^T, \lambda) = \sum_{t=1}^T \log[\sum_{m=1}^M C_{jm} N(y_t - B, \mu_{jm}, \sigma_{jm})] \quad (17)$$

where  $j$  denotes state  $q_t$ , and observation probability from this state is given by the weighted sum of  $M$  Gaussian mixtures. The  $k$ -th component of bias vector  $B$ , obtained from this maximization, is given by:

$$B_k = \frac{\sum_{t=1}^T \sum_{m=1}^M \{ \gamma_{jmt} (\gamma_{tk} - \mu_{jmk}) / \sigma_{jmk}^2 \}}{\sum_{t=1}^T \sum_{m=1}^M \{ \gamma_{jmt} / \sigma_{jmk}^2 \}} \quad (18)$$

where

$$\gamma_{jmt} = \frac{C_{jm}^N (\gamma_{t, \mu_{jm}, \sigma_{jm}})}{\sum_{m=1}^M C_{j\mu}^N (\gamma_{t, \mu_{j\mu}, \sigma_{j\mu}})} \quad (19)$$

Thus, the ML equalizer is optimal in terms of BER [14]. This method consists of comparing all signals can be received with the actual received signals; to select the most likely of them we use the following equation:

$$\hat{x} = \operatorname{argmin} \|y - Hx\|^2 \quad (20)$$

The complexity of this algorithm grows exponentially with the number of antennas, used in transmission, and the number of states  $M$  of the modulation [9]. In fact, the receiver has to compare  $M^{Nt}$  possible solutions with the vector of received signals.

#### IV. SIMULATION RESULTS

##### A. MIMO System Performance

In the simulation we test the MIMO system performance with respect to the number of antennas by calculate the ergodic capacity according to SNR for a MIMO system, SIMO, MISO and SISO system, with application of Rayleigh channel [14].

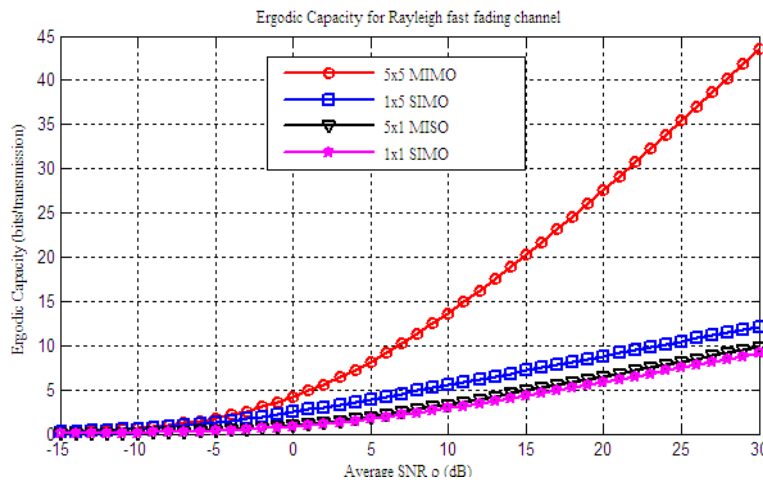


Fig. 2 The capacity of the MIMO system antennas of 5 x 5 in function of the SNR values

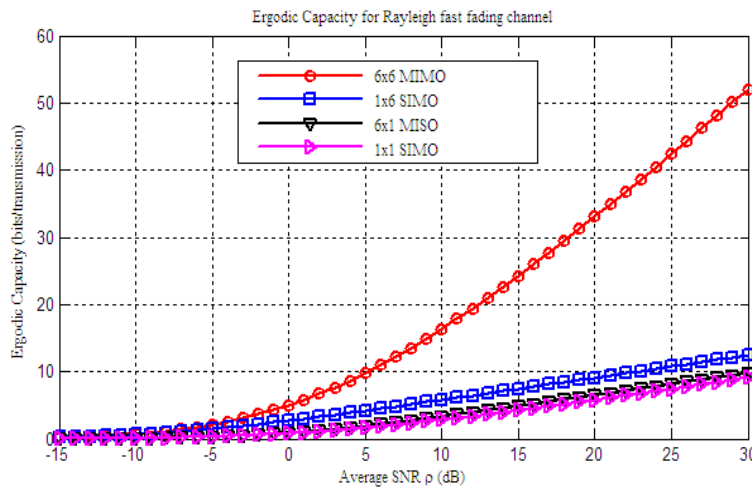


Fig. 3 The capacity of the MIMO system antennas of 6 x 6 in function of the SNR values

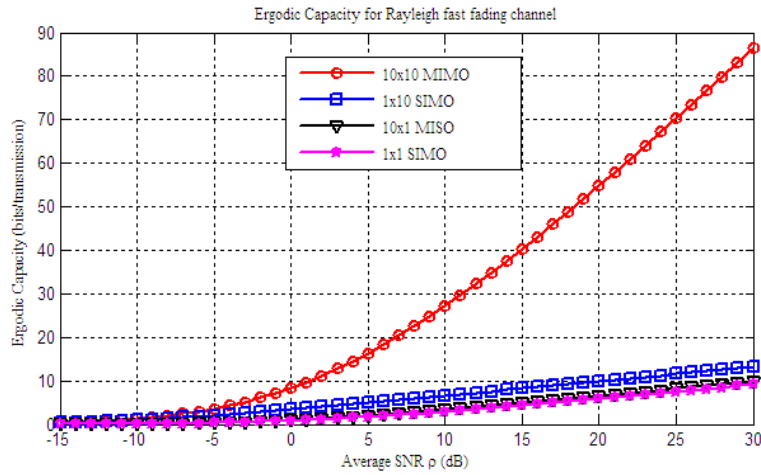


Fig. 4 The capacity of the MIMO system antennas of 10 x 10 in function of the SNR values

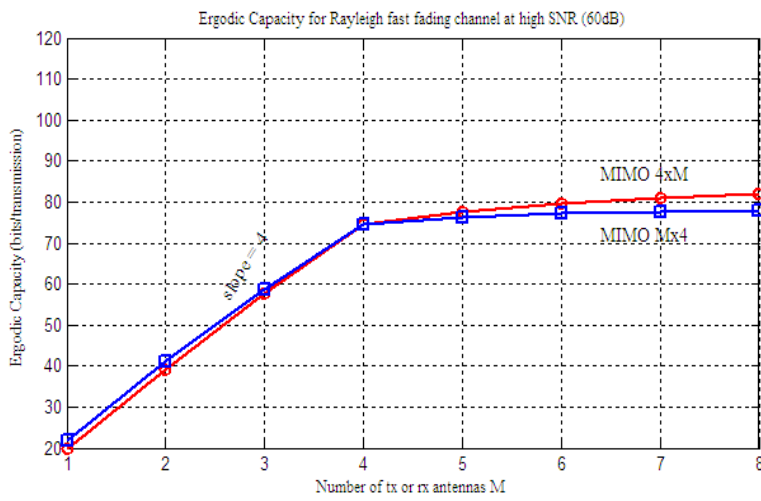


Fig. 5 The capacity of the MIMO system according to the number of antennas with SNR = 60 dB

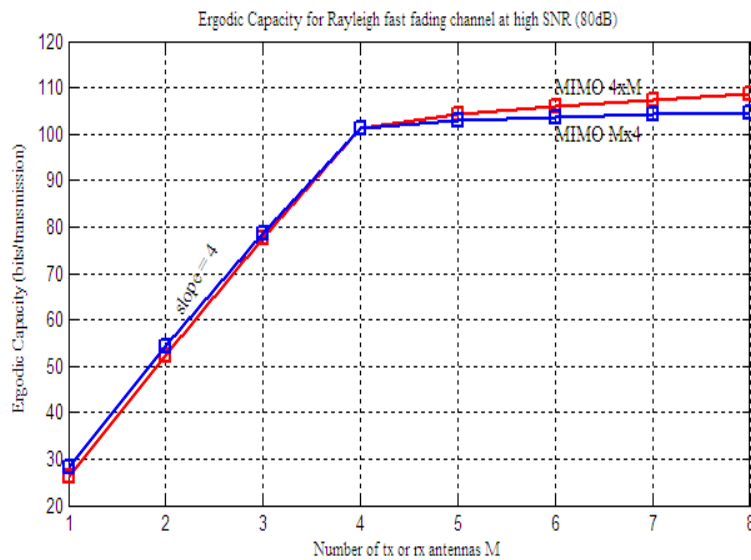


Fig. 6 The capacity of the MIMO system according to the number of antennas with SNR = 80 dB

From the result, we note that MIMO capacity increases if the numbers antenna is important, with and without noise (Figs. 2-4), so the MIMO system is most power of full compared to other systems (SISO, SIMO, MISO). Thus, we see that MIMO capacity is very high if the number of antennas is important to the receiver level, with the value of SNR is greater (Figs. 5 and 6).

*B. In Receive Equalization Algorithms*

With the equalization algorithms, we will calculate the BER for BPSK modulation using the Rayleigh channel with the MIMO system. The equalization is performed using the ZF, MMSE, OSIC and ML

- MIMO\_ZF ALGORITHM: The simulation results shown in figure (Fig. 7) demonstrate that the MIMO\_ZF algorithm equalization is very powerful to equalize the channel if

the number of reception antennas is important, so it reduces the number of errored bit (BER) [14].

- MIMO\_MMSE ALGORITHM: From Fig. 8, we are noted that the BER values are decreased if the number of antennas is increased in reception.
- MIMO\_OSIC ALGORITHM: The MIMO\_OSIC algorithm plays an important role in the 2x5 MIMO system relative to the SISO and SIMO systems (1x1 and 1x2), to equalize the channel (Fig. 9).
- MIMO\_ML ALGORITHM: From Fig. 10, we see that the number of errored bit is important in the SISO and the SIMO system (1x1, 1x2), compared with the MIMO (2x5), then this algorithm gives good results equalization in the case of number antennas is important in reception MIMO system.

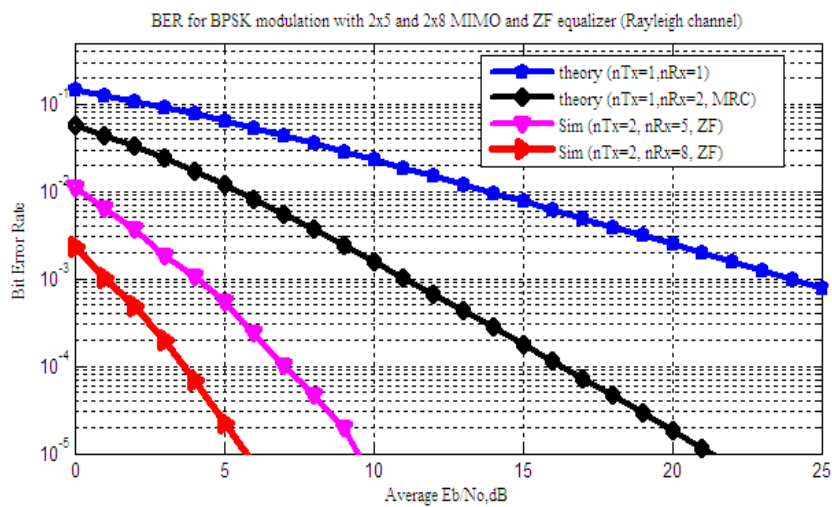


Fig. 7 Performance of the MIMO\_ZF algorithm

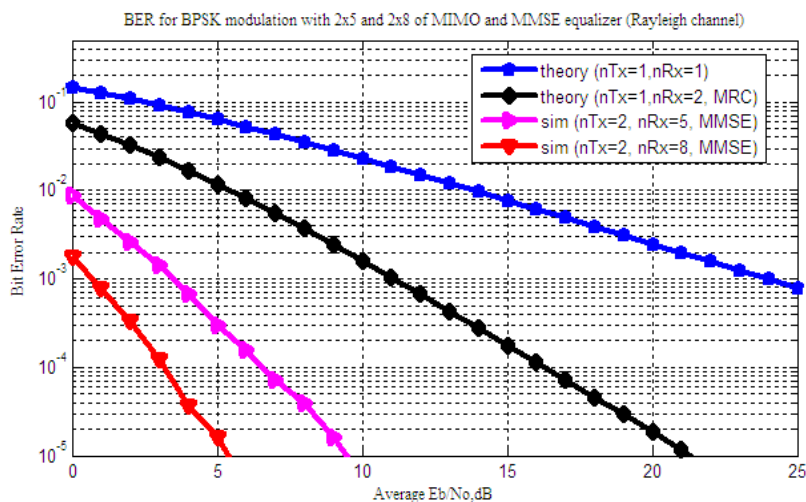


Fig. 8 Performance of the MIMO\_MMSE algorithm

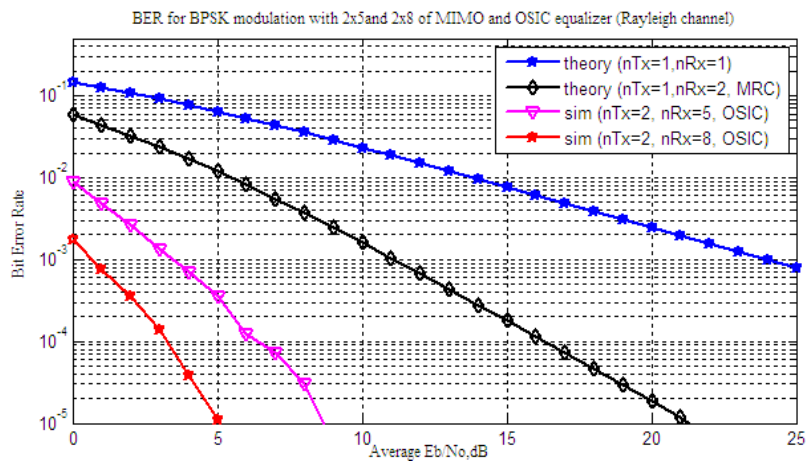


Fig. 9 Performance of the MIMO\_OSIC algorithm

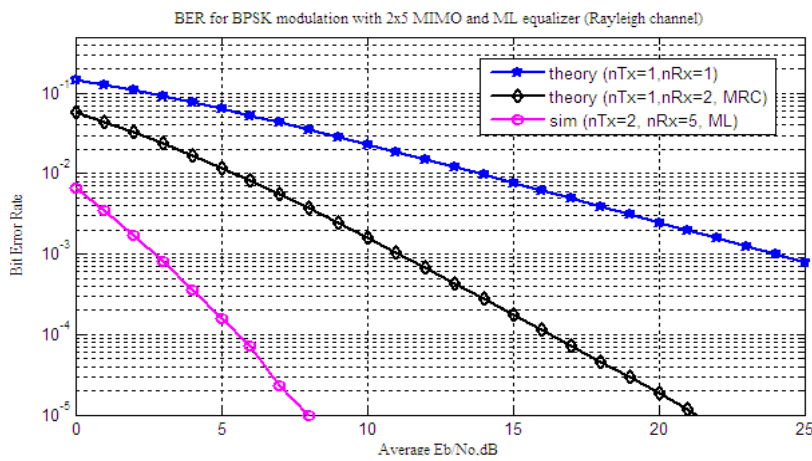


Fig. 10 Performance of the MIMO\_ML algorithm

C. Comparison Results

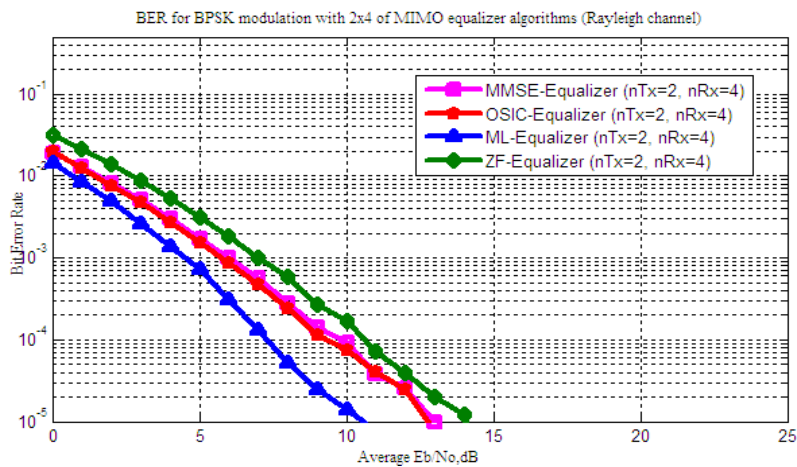


Fig. 11 Comparison of Performance algorithms equalization with MIMO (2x4)

From the simulation result equalization algorithms decrease errored bits that are transmitted to the channel (Fig. 11). reception in MIMO (2x4), we see that the MIMO\_ML equalization algorithm is stronger than other algorithms to

## V.CONCLUSION

In this paper we have studied the equalization MIMO system, the equalizations at reception is performed using the: zero-forcing (ZF), minimum mean square error (MMSE), ordinate successive interference cancellation (OSIC) and the maximum likelihood (ML) Equalizer. The simulation results show that the MIMO system has a large transmission capacity than SISO, SIMO and MISO system, or the MIMO capacity is very high if the number of antennas is important to the receiver level, with the value of SNR is greater. The equalization algorithms give good results if the number of antennas is important in MIMO system reception. Thus ML equalization algorithm is very powerful than MMSE, OSIC and ZF, for the MIMO equalization system.

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