International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:9, No:3, 2015

Evaluation of Minimization of Moment Ratio Method by Physical Modeling

Amin Eslami, Jafar Bolouri Bazaz

Abstract—Under active stress conditions, a rigid cantilever retaining wall tends to rotate about a pivot point located within the embedded depth of the wall. For purely granular and cohesive soils, a methodology was previously reported called minimization of moment ratio to determine the location of the pivot point of rotation. The usage of this new methodology is to estimate the rotational stability safety factor. Moreover, the degree of improvement required in a backfill to get a desired safety factor can be estimated by the concept of the shear strength demand. In this article, the accuracy of this method for another type of cantilever walls called Contiguous Bored Pile (CBP) retaining wall is evaluated by using physical modeling technique. Based on observations, the results of moment ratio minimization method are in good agreement with the results of the carried out physical modeling.

***Keywords*—**Cantilever Retaining Wall, Physical Modeling, Minimization of Moment Ratio Method, Pivot Point.

I. Introduction

This work is inspired from the principle of minimization of moment ratio and the concept of shear strength demand proposed by [1]. The usage of this method is to estimate the rotational stability and also the degree of improvement required for the soil in the passive zone to be supported by a rigid cantilever retaining wall. Furthermore, it can be used to design a cantilever retaining wall, by choosing the required *D/H* value (which D is the embedded depth of wall and H is the wall height) for known height of soil to be retained and the angle of internal friction of soil. The location of pivot point can also be found for the chosen *D/H* value [1].

The Rankine earth pressure theory has been used to calculate the earth pressures exerted by the soil on the cantilever wall, in this paper. It is assumed that the active earth pressure is fully developed, as it requires only a small amount of strain for its full mobilization. While the passive earth pressure is assumed to be developed only partially, to an amount required to maintain the wall in equilibrium, as it requires a large amount of strain for its full mobilization [1]

The aim of this paper is to report the attempt to evaluate this method by using the physical modeling technique.

Physical modeling has an old background in Geotechnical engineering. Wen was the first who reported using model piles to study batter and vertical piles [2]. Numerous researchers have used small scale physical modeling and reached valuable results. Matlock and Ripperger worked on lateral loading of piles in cohesive soil using this method [3]. Prakash in his PhD dissertation performed static and cyclic tests to one groups of model piles embedded in sand and concluded that group effect were negligible for spacing greater than 8d (pile diameter) [4]. Davisson and Sally performed lateral load tests on lateral and vertical model piles to develop design criterions for foundations for rocks and dams for U.S. Army Corps of Engineers [5]. Park published a comprehensive study of seismic performance of steel encased concrete piles with focus on the structural behavior of these composite members under lateral loading [6]. And also there are other numerous studies in this field in the literature which their description is out of order for this article.

II. Basic Concepts

A. Earth Pressure Distribution

By Rankine earth pressure theory and limit equilibrium concepts, the earth pressure distribution for a cantilever sheet pile wall in sand is depicted in Fig. 1. For the sandy soil, the forces on the either side of the wall are expressed as [1]:

$$F_{LHS} = \frac{1}{2}K_p\gamma z^2 + \frac{1}{2}K_a\gamma(D+z)(D-z) \tag{1}$$

$$F_{RHS} = \frac{1}{2}K_a\gamma(H+z)^2 + \frac{1}{2}K_p\gamma(2H+D+z)(D-z) \tag{2}$$

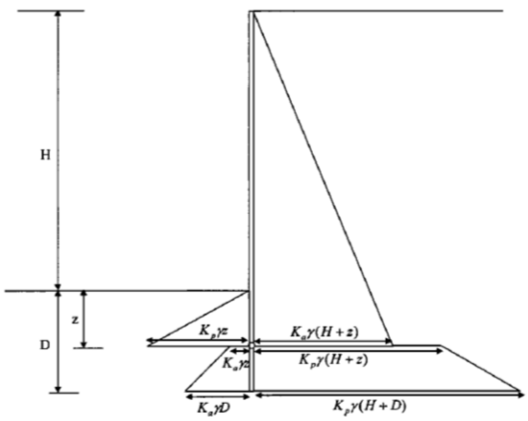


Fig. 1 Active and passive earth pressures on cantilever retaining wall when soil has friction angle of ϕ [1]

The moment due to active forces (M_a) and that due to passive forces (M_r) about the pivot point will be [1]:

$$M_a = K_a\gamma\left[\frac{(H+z)^3}{6} + \frac{z(D-z)^2}{2} + \frac{(D-z)^3}{3}\right] \tag{3}$$

Amin Eslami is with Ferdowsi University of Mashhad, Iran, M.Sc. Student of Civil-Geotechnical Engineering; e-mail: Amin.eslami.1988@gmail.com).

Jafar Bolouri Bazaz is with Ferdowsi University of Mashhad, Iran, Associate Professor of Geotechnical Engineering (e-mail: bolouri@um.ac.ir).

$$\mathrm{M}_r = \mathrm{K}_p\,\gamma\left[\frac{(H+z)(\mathrm{D}-\mathrm{z})^2}{2}+\frac{(D-z)^3}{3}+\frac{z^3}{6}\right] \tag{4}$$

B. Principle of Minimization of Moment Ratio

Since it is assumed that the cantilever retaining wall is rigid, it will rotate about a point at the verge of failure. The point of rotation or the pivot point will be such that the wall will have the least resistance to rotate. At this point, the resisting moment will be least, and at equilibrium, the moment ratio will be equal to unity. The moment ratio (M_r/M_a) is the ratio of the resisting moment to the disturbing moment. The moment ratio gives the resistance to rotation and thus the wall will tend to rotate such that the moment ratio becomes a minimum. Thus, by the minimization of the moment ratio, the location of the pivot point can be estimated either analytically or numerically [1].

C. Concept of Shear Strength Demand

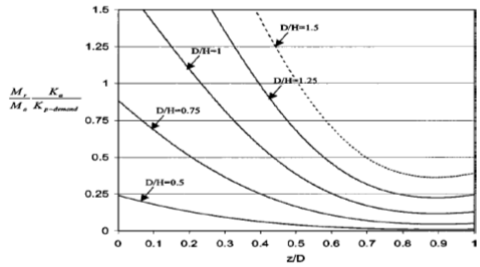
As discussed earlier, it is assumed that the active earth pressure will be fully developed whereas the passive earth pressure is partially mobilized. The passive pressure required to maintain the wall in just-in equilibrium is found in terms of coefficient of passive earth pressure and shear strength parameters (Angle of internal friction, φ and undrained shear strength, c_u). These shear strength values, thus obtained, are termed as "shear strength demand" of the soil. The shear strength demand, hence, indicates the closeness of the wall to failure for a known dimension of wall, retained earth and shear strength of the soil. Shear strength demand will be the shear strength that is required in the passive zone such that the wall is just-in moment equilibrium. If the shear strength demand obtained is greater than the known shear strength of the soil, it will imply that in order to maintain equilibrium, more shear strength of the soil is required, and at the present condition, the retaining wall will fail. The ratio of actual shear strength to shear strength demand, in other words, indicates a factor of safety of the wall against rotation. In the case of wall with cohesionless backfill, the shear strength demand will be found in terms of $K_{p\text{-}demand}$ and thus φ_{demand} (in passive zone), while for wall with cohesive backfill shear strength demand will be expressed in terms of non-dimensional undrained shear strength demand, ($cu_{\text{-}demand}/\gamma H$).[1]

D. The Location of the Pivot Point and Values of Shear Strength Demand

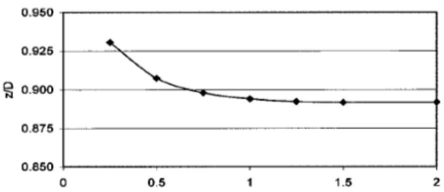
Using the moments obtained for cantilever retaining wall with cohesionless backfill, the moment ratio can be expressed in terms of non-dimensional terms as shown in (5):

$$\frac{Mr}{Ma} = \frac{\mathrm{K}_p\left[\frac{(H+z)(\mathrm{D}-\mathrm{z})^2}{2}+\frac{(D-z)^3}{3}+\frac{z^3}{6}\right]}{\mathrm{K}_a\left[\frac{(H+z)^3}{6}+\frac{z(D-z)^2}{2}+\frac{(D-z)^3}{3}\right]} \tag{5}$$

The moment ratio is expressed non-dimensionally as $(M_r/M_a)(K_a/K_p)$, which should be minimized to obtain the location of pivot point. Fig. 2 (a) depicts a plot of the moment ratio against the location of pivot point (z/D) for various values of D/H ratios, all in their non-dimensional forms. It can be observed that the non-dimensional moment ratio varies with the pivot point, and reaches a minimum at a particular location which offers the least resistance to rotation. Moreover, it is also worth mentioning that the same plot represents the shear strength demand ratio of cohesionless soil ($K_a/K_{p\text{-}demand}$), since it is obtained for the condition that the wall is just-in moment equilibrium i.e. the moment ratio (M_r/M_a) is equal to unity. It can be noticed that the non-dimensional moment ratio and $K_a/K_{p\text{-}demand}$ does not depend on the angle of internal friction, φ, of the soil. The moment ratio (M_r/M_a) is observed to change in magnitude with change in the value of φ, although the location of pivot point will always remain the same [1]:



(a)



(b)

Fig. 2 (a) Minimization of moment ratio for various D/H ratios, (b) Variation of location of pivot point with *D*/*H* ratios [1]

Fig. 2 (b) illustrates the variation of location of pivot point with non-dimensional depth of embedment (*D/H*). It can be seen that as the *D/H* ratio decreases, the location of the pivot point moves towards the toe of the cantilever retaining wall.

The shear strength demand (passive strength of soil required for a wall to be stable) of the soil in the passive zone can be obtained for known values of angle of internal friction of the backfill for various values of *D/H* ratios. The shear strength demand for the case of cohesionless backfill will be expressed in terms of φ_{demand}. A plot of φ_{demand} against φ for various *D/H* ratios is shown in Fig. 3. The shear strength demand, φ_{demand} can be obtained from

$$K_{p-demand} = \frac{M_a}{\gamma\left[\frac{(H+z)(\mathrm{D}-\mathrm{z})^2}{2}+\frac{(D-z)^3}{3}+\frac{z^3}{6}\right]} \tag{6}$$

$$\varphi_{demand} = \sin^{-1}\left(\frac{K_{p-demand}-1}{K_{p-demand}+1}\right) \tag{7}$$

ISSN: 2415-1734
Vol:9, No:3, 2015

Fig. 3 provides the angle of internal friction required in the passive zone for the wall to be in equilibrium. For a particular input condition of sheet pile wall, if the φ_{demand} obtained is greater than the φ of the backfill, it is in unsafe condition. The bold line indicates the condition wherein the sheet pile wall is just-in moment equilibrium and it demarcates the boundary of safe and unsafe regions, with the safe region lying below it. Thus, for known dimensions and angle of internal friction of an existing sheet pile wall, the stability can be checked. This plot can also be used to estimate the degree of improvement required for the soil in passive zone to support the sheet pile wall. Moreover, it can be used to design a sheet pile wall, by choosing the required *D/H* value for known height of soil to be retained and the angle of internal friction of soil [1].

TABLE I
THE MODEL PILE PROPERTIES

Material	**Elasticity Modulus (Gpa)**	**Length (mm)**	**Outer diameter (mm)**	**Inner diameter (mm)**	**Thickness (mm)**	**Pile Length to diameter ratio (L/D)**
Poly Propylene	2	800	32	21.2	5.4	25
Poly Propylene	2	800	40	26.6	6.7	20

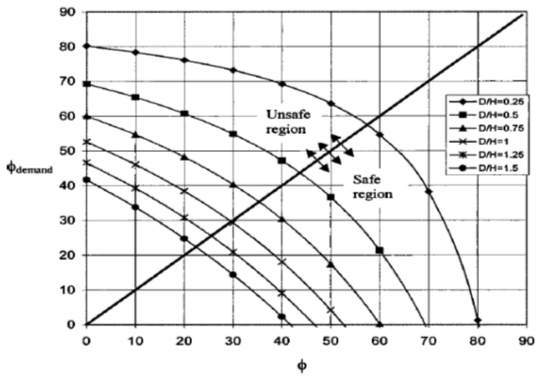


Fig. 3 Variation of ϕ_{demand} with f for various *D*/*H* ratios [1]

III. THE PHYSICAL MODEL DESCRIPTION

The discussed physical model is a steel box with length, width and height of 1.5, 1 and 0.8 m, respectively, filled with Firoozkooh[1] sand in order to model some piles with length of 80 cm and specifications listed in Table I. Also soil properties are listed in Table II.

TABLE II
SOIL PROPERTIES

Soil Type	Dry unit weight (kN/m^3)	Internal friction angle (Degree)	Relative Density (D_r) (%)
Firooz kooh Sand	15	40	57

Fig. 4 shows the modeling system schematically. The test procedure was in such a way that the execution and servicing of a CBP wall in the field could be modeled. In this study, investigation of two modes of free end and fixed end is carried out. In the case of fixed end, a plate with some welded bars for providing fixity in the end of model piles was used. The box was filled after placing the fixed piles. Then the soil in front of piles was excavated gradually in 7 levels of 10 cm to the depth of 70 cm from the top of the box and the lateral deflection of piles were measured simultaneously in each stage of excavation. A wooden raft was also used as the cap beam to make the piles united and balanced. The procedure was the same for the mode of free end piles except that no fixity mechanism was put into practice for end of the piles. Generally, 8 modes of tests were carried out by changing the fixity condition (Free end(R), Fixed end(X)), and changing the wall overall stiffness by testing two values of pile L/D ratios (20,25) and two cases of pile spacing (S1,S2) each of which has 7 stages of excavation (i.e. having a number of 7 ratios of D/H in each set of tests). Table III demonstrates the symbol used for coding the tests.

To measure the lateral deflection of piles during excavation, a number of 4 gauges with accuracy of 0.01 mm were installed to the central pile via some wires with the spacing of 15 cm from top to toe of the pile. Fig. 2 illustrates a view of the physical model.

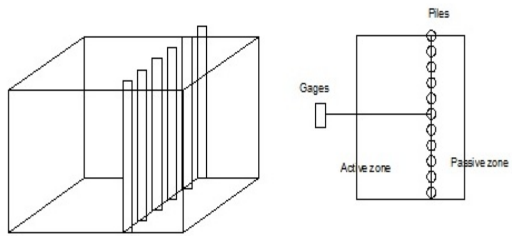


Fig. 4 Scheme and plan of the physical model

Fig. 5 A view of the physical model

[1] A place in the North of Iran

TABLE III
SYMBOL DESCRIPTIONS USED TO CODE THE TESTS

Symbol	Pile spacing		Fixity condition		Length to diameter ratio of piles(L/D)	
	S1	S2	X	R	20	25
meaning	D_p+1 cm	2D_p+1 cm	Fixed end	Free end	$(L/D)_{pile}$=20	$(L/D)_{pile}$=25

* D_p=pile diameter

IV. RESULTS OF PHYSICAL MODELING

Assuming z'=z/D and D'=D/H, (5) can be rewritten as [7]:

$$\frac{Mr}{Ma}\frac{Ka}{Kp}=\frac{\left[\frac{3}{D'}(1+z'D')(1-z')^2+2(1-z')^3+z'^3\right]}{\left[\frac{1}{D'^3}(1+z'D')^3+3z'(1-z')^2+2(1-z')^3\right]} \quad (8)$$

And proposed equation by [1] for obtaining the coefficient of passive earth pressure (6) can be rewritten as [7]:

$$K_{p-demand}=\frac{K_a\left[\frac{1}{D'^3}(1+z'D')^3+3z'(1-z')^2+2(1-z')^3\right]}{\left[\frac{3}{D'}(1+z'D')(1-z')^2+2(1-z')^3+z'^3\right]} \quad (9)$$

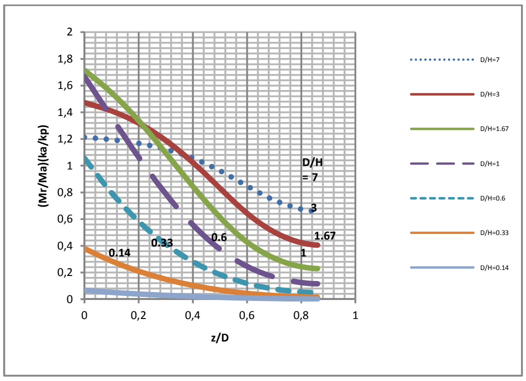
The non-dimensional moment ratio, location of pivot point and variation of ϕ_{demand} with ϕ for various *D*/*H* ratios are replotted in Figs. 6 (a), (b) and 7, respectively, to match the principle of minimization of moment ratio with the physical modeling results.

Fig. 7 can be used to evaluate the accuracy of the estimation of safety factor using the carried out physical modeling in this research. However, taking into account that just one type of sandy soil with internal friction of 40 degrees was used in this research, investigation of just one case would be feasible.

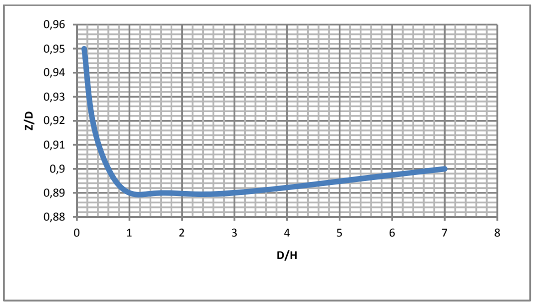
As it can be understood by Fig. 7, for a backfill with internal friction angle of 40 degrees, the minimum required ratio of D/H is 0.6 and the minimum required internal friction is 40 degrees. A calculation of the minimum required cantilever wall embedding depth for the D/H ratios discussed in this research was made based on UK simplified design method [8] to evaluate the accuracy of the estimation. The results of calculation are presented in Table IV.

TABLE IV
RESULTS OF THE EMBEDDING DEPTH CALCULATION USING UK SIMPLIFIED DESIGN METHOD

Problem data	D/H	Stage (%)	Calculated - D	Existing-D	Cal. D/ Exis. D
	7	12.5	0.061	0.7	11.42
Φ=40 °	3	25	0.122	0.6	4.89
	1.6	37.5	0.183	0.5	2.72
γ=15 KN/m^3	1	50	0.245	0.4	1.63
	0.6	**62.5**	**0.306**	**0.3**	***0.97***
	0.3	75	0.367	0.2	0.54
	0.14	87.5	0.428	0.1	0.23



(a)



(b)

Fig. 6 Re-plotting Fig. 2 in order to match with the existing physical model: (a) Minimization of moment ratio for various D/H ratios (b) Variation of location of pivot point with *D*/*H* ratio

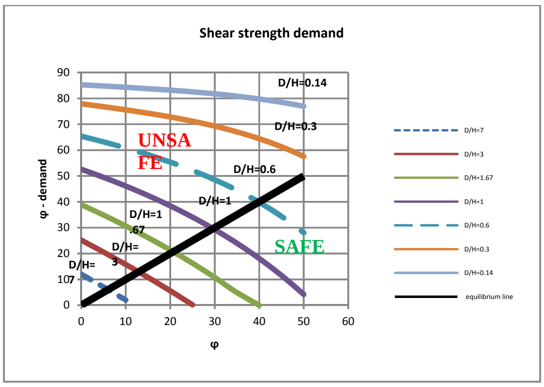


Fig. 7 Replotting Fig. 3 in order to match with the physical model. Variation of ϕ_{demand} with ϕ for various *D*/*H* ratios

Comparing the required embedded depth (calculated by UK simplified method) with the existing embedded depth in each stage, it could be understood that in the excavation stage of 62.5 % (equivalent to D/H=0.6) the ratio of existing embedded depth to the calculated embedded depth falls under unity and therefore falls into the hazardousness zone.

Verifying the point that the D/H ratio of 0.6 is the verge of critical situation can be done by comparing the increment ratios of horizontal displacements in each stage of excavation.

Table V provides the maximum horizontal displacements of the wall measured in each stage of excavation in the carried out physical modeling. Using this table and calculating the increment ratio of displacement per stages, the verge of critical situation can be distinguished.

TABLE V
THE MAXIMUM HORIZONTAL DISPLACEMENTS FOR EACH STAGE OF EXCAVATION

Stage (%)	12.5	25	37.5	50	*62.5*	75	87.5
D/H	**7**	**3**	**1.67**	**1**	***0.6***	**0.33**	**0.14**
Test series	**Maximum horizontal Displacement (0.01 mm)**						
S1-X-25	0	15	180	400	1200	2300	4200
S1-R-25	0	0	90	300	1100	2500	5000
S2-X-25	0	25	230	550	1900	2900	5400
S2-R-25	0	0	210	350	1500	3100	6000
S1-X-20	0	5	85	150	580	1250	2800
S1-R-20	0	0	65	155	600	1400	3100
S2-X-20	0	10	120	350	1100	2430	4550
S2-R-20	0	0	110	330	1300	2500	5500

Table VI presents the increment ratio of displacements in each stage compared to the third stage of excavation.(37.5% or D/H=1.67). It should be noted that it would be better to take the first stage of excavation as the benchmark stage for making such comparison, but due to having zero values in the first and second stages which could result in math errors, the third stage was taken as the benchmark, instead.

TABLE VI
INCREMENT RATIO OF DISPLACEMENTS IN EACH STAGE RELATIVE TO THE THIRD STAGE

Stage (%)	12.5	25	37.5	50	***62.5***	75	87.5
D/H	7	3	1.67	1	***0.6***	0.33	0.14
Test series	**Increase percentage of displacement in each stage of excavation in comparison to 3rd stage(D/H=1.67)(%)**						
S1-X-25	N/A	N/A	0	122	**566**	1177	2233
S1-R-25	N/A	N/A	0	233	**1122**	2677	5455
S2-X-25	N/A	N/A	0	139	**726**	1160	2247
S2-R-25	N/A	N/A	0	66	**614**	1376	2757
S1-X-20	N/A	N/A	0	76	**582**	1370	3194
S1-R-20	N/A	N/A	0	138	**823**	2053	4669
S2-X-20	N/A	N/A	0	191	**816**	1925	3691
S2-R-20	N/A	N/A	0	200	**1081**	2172	4900

Having investigated the increment ratio of horizontal displacements in Table VI, It is revealed that a sudden increase in the displacements can be distinguished after the stage of 62.5 % (D/H=0.6) which was already estimated by the minimization of moment ratio method.

So it can be concluded that the minimization moment ratio method can be put into practice for estimation of the rotational stability of the CBP cantilever walls with accepted accuracy. Also it can be deducted that although this method is initially proposed for cantilever sheet pile walls, it is applicable on other types of cantilever walls such as CBP retaining wall.

V. CONCLUSION

In this paper, a new method for analyzing the rotational stability of cantilever retaining walls called minimization of moment ratio was evaluated using physical modeling technique. The results show that this method is able to predict the stability situation of the wall with acceptable accuracy and in the fastest way possible.

REFERENCES

[1] Madabhushi, S.P.G. and Chandrasekaran, V.S. (2005) “Rotation of Cantilever Sheet Pile Walls” Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 131, 202-212.
[2] Wen, R. (1955). “Model Studies of Laterally Loaded Pile Foundations,” Proc. 34th Annual Meeting, Highway Research Board, pp. 140-152.
[3] Matlock, H. and Ripperger, E. (1957). “Theoretical Analysis and Laboratory Studies of Laterally Loaded Model Pile Segments,” Rpt. To Shell Oil Co., Engineering Science Consultants, July.
[4] Prakash, S. (1962). “Behavior of Pile Groups Subjected to Lateral Loads,” Ph.D. Dissertation, Univ. of Illinois.
[5] Davisson, M. and Salley, J. (1970). “Model Study of Laterally Loaded Piles,” J. Soil Mechanics and Foundation Div., ASCE, 96(5), 1605-1627.
[6] Park, R. (1987), “Seismic Performance of Steel-Encased Concrete Piles,” Rpt. 87/5, Dept. of Civil Eng., University of Canterbury.
[7] N. Kumar, A. Dey,(2013), Rotational Stability of Rigid Cantilever Sheet Pile Walls ,North East Students Geo-Congress on Advances in Geotechnical Engineering (NES Geo-Congress 2013), Guwahati
[8] Padfield, C. J., and Mair, R. J. (1984). Design of retaining walls embedded in stiff clays. CIRIA Report 104, London, Constr. Industry Res. and Information Assoc. (CIRIA), London, England.