

New Approach for Minimizing Wavelength Fragmentation in Wavelength-Routed WDM Networks

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Abstract—Wavelength Division Multiplexing (WDM) is the dominant transport technology used in numerous high capacity backbone networks, based on optical infrastructures. Given the importance of costs (CapEx and OpEx) associated to these networks, resource management is becoming increasingly important, especially how the optical circuits, called “lightpaths”, are routed throughout the network. This requires the use of efficient algorithms which provide routing strategies with the lowest cost. We focus on the lightpath routing and wavelength assignment problem, known as the RWA problem, while optimizing wavelength fragmentation over the network. Wavelength fragmentation poses a serious challenge for network operators since it leads to the misuse of the wavelength spectrum, and then to the refusal of new lightpath requests. In this paper, we first establish a new Integer Linear Program (ILP) for the problem based on a node-link formulation. This formulation is based on a multilayer approach where the original network is decomposed into several network layers, each corresponding to a wavelength. Furthermore, we propose an efficient heuristic for the problem based on a greedy algorithm followed by a post-treatment procedure. The obtained results show that the optimal solution is often reached. We also compare our results with those of other RWA heuristic methods.

Keywords—WDM, lightpath, RWA, wavelength fragmentation, optimization, linear programming, heuristic.

I. INTRODUCTION

TRANSPORT technologies such as Synchronous Digital Hierarchy (SDH/SONET) and Asynchronous Transfer Mode (ATM) cannot cope with the high bandwidth requirements of some services (e.g., HDTV or video conferencing) because they do not realize the full potential of the optical medium. The transmission speed of these technologies does not exceed few tens of Gbps. In contrast, the *Wavelength Division Multiplexing* (WDM) technique can achieve link capacities in the order of Tbps in optical transport networks by allowing multiple channels to be operated in a single fiber simultaneously at different wavelengths. WDM-based networks are also known as *wavelength-routed networks* [1], [2] since they use wavelength routing techniques. Wavelength routing allows the same wavelength to be reused in spatially disjoint segments of the network.

Practical limitations of optical devices and of the WDM technology restrict the number of available wavelengths per fiber link (80 wavelengths in the case of most operational DWDM networks). As a consequence, only a limited number of optical connections can be established in a WDM transport

network. Given a set of connection requests, the general objective of the *routing and wavelength assignment* (RWA) problem is to minimize the number of used wavelengths to establish connections [3]–[5]. Each connection request must be given a route in the network between its source and destination nodes and one wavelength (or several wavelengths if wavelength converters are used in intermediate nodes). Such an all-optical path is referred to as a lightpath [1], [6]. There are two types of wavelength conversion: O-E-O conversion and all optical conversion. The first one ensures that incoming optical signals are first mapped into electrical signals and then electrical signals are later mapped into outgoing optical signals. This optoelectronic conversion not only is very expensive but also has limited performance [7], [8]. The second conversion is less expensive than optoelectronic conversion but also less efficient and rarely used in practice [8]. In addition to that, wavelength conversion introduces additional processing and control delay. Therefore, WDM network operators often try to avoid the use of wavelength converters. We then propose to solve the RWA problem without considering the use of converters.

In RWA problem, the establishment of lightpaths is subject to the following constraints: (a) the wavelength must be consistent for the entire path (assuming that no wavelength converters are allowed or available), and (b) two lightpaths can share the same optical link, provided a different wavelength is used. The former constraint is referred to as the wavelength continuity constraint, whereas the latter one is referred to as the distinct wavelength constraint.

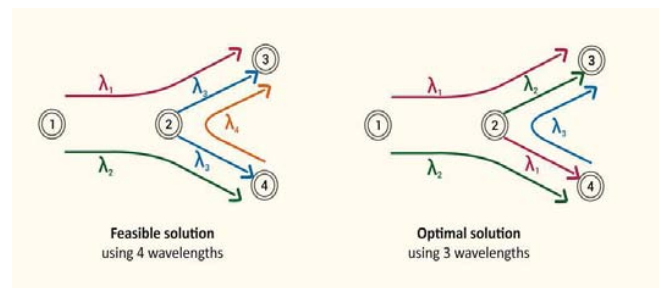


Fig. 1. RWA problem instance where the objective is to Minimize the number of used wavelengths.

As mentioned above, the objective of the RWA problem is to minimize the number of used wavelengths. This is achieved by reusing already-in-use wavelengths as much as possible before

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activating a new wavelength. This strategy allows to avoid wavelength fragmentation in the network. Fig. 1 illustrates two different solutions of an RWA problem instance. In this example, lightpath requests are represented by arrows and are routed over the network using a set of wavelengths. The first case shows an instance of wavelength fragmentation problem, where the reuse of resources (i.e. wavelengths) is not optimized: an additional wavelength λ_4 is used although it could have been avoided.

In this paper, we address the RWA problem and propose a new approach to minimize wavelength fragmentation, and thus minimizing future demands rejection or blocking. Our contribution consists in developing a new ILP formulation of the problem that takes into account a reduced number of variables and constraints compared to known standard formulations. Based on a decomposition approach, we also propose a new two-phase heuristic method where the first phase provides an initial solution by solving iteratively a hierarchy of simple sub-problems, and the second phase attempts to improve the initial solution by finding alternate routing decisions.

The rest of this paper is organized as follows. Section II gives an overview of related works. Section III gives an integer linear programming (ILP) formulation of the RWA problem. This section also describes another ILP formulation, which forms the basis of the heuristic method described in Section IV. Section V is devoted to experimental results where we evaluate the performance of our proposed method and compare it with other existing ones. Finally, Section VI concludes this paper and synthesizes the key points of this work.

II. RELATED WORK

The RWA problem in WDM networks has been addressed under two settings. The first setting, which is the one considered in our study, is known as *Static Lightpath Establishment* (SLE). Under this setting, the set of lightpath requests is known in advance. This setting pertains to the planning phase of the network and gives rise to an NP-hard optimization problem [6] that has to be solved using offline algorithms. In the second setting, known as *Dynamic Lightpath Establishment* (DLE), lightpath requests arrive sequentially at random times and have to be routed in the network using an online algorithm [9]. In this literature survey, we focus on the static setting. Offline algorithms proposed in previous works can be broadly classified in three classes:

A. Joint ILP-Based Algorithms

These algorithms jointly solve the routing and wavelength assignment problems. Jaumard and Meyer introduced in [4] different RWA ILP formulations (path-based, edge-based and arc-based formulations). Several objectives have been considered such as minimizing the blocking rate, minimizing the number of used wavelengths assuming that all connection requests can be accepted, minimizing the maximum number of used wavelengths per fiber link, etc. Since proposed ILPs are intractable, relaxed linear programs have been used to get

bounds on the optimal value that can be achieved [10], [11]. Iterative fixing and Rounding techniques have also been used to provide an integer solution from relaxed problems [12]. In [13], the authors prove that their new path-based formulation achieves a decrease of up to two orders of magnitude in running time compared to existing formulations. Link selection techniques were considered in recent work [5] to reduce the size of the link-based formulation in terms of both the number of variables and the number of constraints.

B. Decomposition-Based Algorithms

These algorithms solve the routing and wavelength assignment problems individually and sequentially. The authors in [11], [14], [15] propose a two-step decomposition technique. The first step uses integer programming to assign paths to the demands while minimizing the maximal number of demands routed over a link. The second step is expressed as a graph coloring problem. The final solution is an approximate solution of the original complete RWA problem that is often not optimal. Authors in [11], [15] formulated the routing sub-problem as a continuous multicommodity flow problem and applied a randomized rounding technique to provide an integer solution. The work in [3] synthesized a lot of known approaches for solving the routing subproblem such as fixed routing, fixed-alternate routing and adaptive routing. Ten dynamic wavelength assignment heuristic methods were also discussed. A performance comparison between an RWA ILP-based algorithm and a decomposition-based algorithm was made in [11].

C. Heuristic Methods

These Heuristic methods are dedicated to solve the joint RWA problem. In [16]–[19], many heuristic methods were presented and evaluated. We summarize and describe some of the most known heuristic methods used to solve the joint RWA problem. In the heuristic method known as *Shortest First Fixed Path* (SFFP) [17], the objective is to maximize the network throughput (i.e. number of established lightpaths). The method begins by finding the shortest path for all the given node-pairs using Dijkstra's algorithm. The resulting set of shortest paths is sorted in non-decreasing order based on path lengths and wavelengths are assigned to lightpaths in that order. The method known as *Longest First Fixed Path* (LFFP) or *Longest-path First* [6], [16] is similar but assigns wavelengths to lightpaths in the reverse order (i.e., the longest lightpath first). A comparison with the SFFP algorithm is made in [17] based on the number of established lightpaths. In [16], the RWA problem is formulated as a routing problem where the link cost is determined based on the load of each link. The authors then propose the *Heaviest Path Load Deviation* (HPLD) heuristic method. The HPLD algorithm attempts to re-route some lightpaths that pass through the most loaded link in order to minimize the number of wavelengths. In the same context, other heuristics are proposed such as the *Longest First Alternate Path* (LFAP) algorithm [16], [18] and the *Minimum Number of Hops* (MNH) algorithm [16], [20]. These algorithms are compared with the previously described ones

in [16]. The author of [19] proposes new heuristic methods based on bin packing algorithms and compare them with an efficient existing algorithm.

III. ILP-BASED APPROACH

We consider an optical transport network represented by a directed graph $G := (V, E)$, where V is a set of nodes corresponding to the WDM switches and E is the set of optical fibers between nodes. We denote by $\mathcal{W} = \{1, \dots, W\}$ the set of wavelengths (or colours) that can be assigned to connection requests. We are given a set \mathcal{K} of connection requests that have to be routed in the network. For each lightpath $k \in \mathcal{K}$, we let s_k be its source node and t_k be its destination node. Each lightpath request must be given a route and a wavelength while respecting the wavelength continuity and distinct wavelength constraints.

A. ILP Formulation of the RWA Problem

Our idea consists in defining a layer of the network, as a graph $G_w = (V_w, E_w)$, for each colour $w \in \mathcal{W}$. Each node of V_w is obtained by duplicating the corresponding node of V , whereas each edge of E_w is obtained by duplicating the corresponding directed edge in E . We thus have as many network layers as there are possible colours. This multilayer approach is illustrated in Fig. 2 in the case of two colours. Each network layer is considered as a separate network where each link has capacity 1, so that only a single lightpath can be routed on it. Then, the problem can be formulated as that of routing each lightpath on a single path in one and only one network layer.

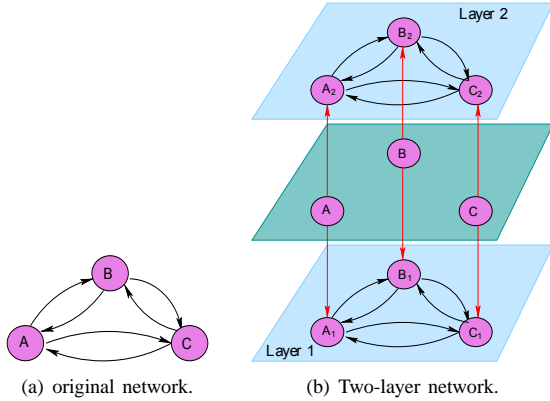


Fig. 2. The original network and the two layers obtained by duplicating nodes and links in the case of two colours.

For all lightpaths $k \in \mathcal{K}$ and all colours $w \in \mathcal{W}$, we define the binary decision variable y_k^w as follows: y_k^w is 1 if lightpath k is assigned colour w , and 0 otherwise. Similarly, we define the binary decision variable x_k^e as follows: x_k^e is 1 if lightpath k is routed on link e , and 0 otherwise. Finally, the binary decision variable u_w has value 1 if at least one lightpath is assigned the colour w , and 0 otherwise. With these definitions, the RWA problem can be formulated as an integer-linear program:

$$\text{minimize } w_{max} \quad (\text{RWA})$$

s. t.

$$w u_w \leq w_{max}, \quad \forall w, \quad (1)$$

$$\sum_{k \in \mathcal{K}} y_k^w \leq K u_w, \quad \forall w, \quad (2)$$

$$\sum_{k \in \mathcal{K}} x_k^e \leq 1, \quad \forall e \in E_w, \forall w, \quad (3)$$

$$\sum_{e \in I_w(n)} x_k^e - \sum_{e \in O_w(n)} x_k^e = h_{k,n}(y_k^w), \quad \forall n, k, w \quad (4)$$

$$x_k^e \leq y_k^w, \quad \forall e \in E_w, \forall w, k, \quad (5)$$

$$\sum_{w \in \mathcal{W}} y_k^w = 1, \quad \forall k, \quad (6)$$

$$x_k^e, y_k^w, u_w \in \{0, 1\}, \quad \forall e \in E_w, \forall w, k, \quad (7)$$

$$w_{max} \geq 0, \quad (8)$$

where $h_{k,n}(y_k^w)$ is $-y_k^w$ if $n = s_k$, y_k^w if $n = t_k$, and 0 otherwise. Equations (1) define w_{max} as the number of used wavelengths. Equations (2) express that wavelength w is used if at least one lightpath is routed in layer w . Equations (3) are capacity constraints stating that at most one lightpath can be routed through each link of layer w . In (4), $I_w(n)$ (resp. $O_w(n)$) represents the set of directed edges of E_w that are incoming (resp. outgoing) at node $n \in V_w$. These constraints are the usual flow balance equations. Equations (5) state that if lightpath k is routed over link $e \in E_w$, then it is routed in layer w , while (6) express that each lightpath has to be routed in one and only one layer. Finally, (7)-(8) give the ranges of the variables. We emphasize that an optimal solution of (RWA) is feasible for the original problem if and only if $w_{max} \leq W$.

We note that the RWA problem is known to be NP-hard [21]. As a consequence, the above ILP formulation can be solved only for small problem instances. We describe in the following section another ILP formulation which forms the basis for the proposed algorithm.

B. Another ILP Formulation

Consider the following integer linear program:

$$\text{minimize } \sum_{w \in \mathcal{W}} K^{w-1} \sum_{k \in \mathcal{K}} y_k^w \quad (\text{EQ-RWA})$$

s. t.

$$\sum_{w \in \mathcal{W}} y_k^w = 1, \quad \forall k, \quad (9)$$

$$x_k^e \leq y_k^w, \quad \forall e \in E_w, \quad \forall w, k, \quad (10)$$

$$\sum_{e \in I_w(n)} x_k^e - \sum_{e \in O_w(n)} x_k^e = h_{k,n}(y_k^w), \quad \forall n, k, w \quad (11)$$

$$\sum_{k \in \mathcal{K}} x_k^e \leq 1, \quad \forall e \in E_w, \quad \forall w, \quad (12)$$

$$x_k^e, y_k^w \in \{0, 1\}, \quad \forall e \in E_w, \quad \forall w, k, \quad (13)$$

where the variables and the constraints have the same interpretation than in problem (RWA).

In the above ILP, the coefficient K^{w-1} represents the cost of routing one lightpath in network layer w . These routing costs increase exponentially from one layer to the next one, so that opening a new network layer cannot be optimal if the current one is not saturated. As a simple example, consider a network with 2 available wavelengths ($W = 2$) and assume that 10 lightpaths have to be routed ($|\mathcal{K}| = 10$). In that case, the costs associated to the two network layers are respectively $c_1 = 1$ and $c_2 = 10$. Consider a first solution routing 5 lightpaths in the first layer ($w = 1$) and the other ones in the second layer ($w = 2$). The value of the objective function is then $1 \cdot 5 + 10 \cdot 5 = 55$. Assume now that another feasible solution routes 6 lightpaths in the first layer and the remaining ones in the second layer. The cost of the second solution is $1 \cdot 6 + 4 \cdot 10 = 46$, showing that it is clearly better than the first one. This simple example illustrates the fact that formulation (EQ-RWA) maximize the utilization of a wavelength in the network before activating a new one. This perfectly matches the criterion we seek to optimize: wavelength fragmentation.

From a theoretical point of view, the study of problem (EQ-RWA) is motivated by Theorem 1 below [22].

Theorem 1: Let (\mathbf{x}, \mathbf{y}) be any optimal solution of problem (EQ-RWA). Then $(\mathbf{x}, \mathbf{y}, \mathbf{u}, w_{max})$ is an optimal solution of problem (RWA), where

$$u_w = \min \left(1, \sum_{k \in \mathcal{K}} y_k^w \right), \quad w = 1, \dots, W, \quad (14)$$

and $w_{max} = \max_{w \in \mathcal{W}} (w u_w)$.

According to Theorem 1, we can readily obtain an optimal solution to the original problem from an optimal solution

of problem (EQ-RWA). We note that the converse is not necessarily true. Although (EQ-RWA) has significantly less variables and constraints than the original problem, large problem instances are still intractable and so heuristic methods must be used instead of integer linear programming. The interest of formulation (EQ-RWA) is precisely to suggest an efficient method for (approximately) solving the original problem, as discussed below.

IV. HEURISTIC METHOD

In this section, we describe the heuristic method proposed to solve the RWA problem. This method is based on the ILP formulation (EQ-RWA) discussed in Section III-B. The method works in two phases. In the first phase, we solve a hierarchy of "small" integer-linear programs corresponding to Maximum Edge-Disjoint Paths (MEDP) problems. The second phase is a post-processing phase in which the method seeks to reduce the number of used wavelengths by rerouting all the lightpaths assigned to a network layer.

A. First Phase

In view of formulation (EQ-RWA), a natural greedy approach to solve problem RWA is first to route as many as possible lightpaths in the first network layer, then to route as many as possible of the remaining lightpaths in the second layer, etc.

Given a network layer w , the aim is to route the maximum amount of flows (i.e. lightpaths) in that network layer. The problem to be solved therefore corresponds to a Maximum Multicommodity Flow (MMF) problem [23], [24]. More precisely, since each link in a network layer can accommodate at most one lightpath, the problem to be solved falls within the scope of a particular case of the MMF problem, known as Maximum Edge-Disjoint Paths (MEDP) problem [25]. In the MEDP problem, given a set of requests, a feasible solution is given by a subset of requests (representing the accepted requests) and an assignment of edge-disjoint paths to all requests in that subset. The objective is to maximize the cardinality of this subset. The MEDP problem can be formulated as an integer linear program. Several heuristic methods have also been proposed to solve it, such as the Shortest-Path-First (SPF) algorithm suggested by Kolliopoulos and Stein [25], [26].

As described in Algorithm 1, the proposed method works in rounds. At each round, it considers a network layer and maximizes the number of remaining lightpaths routed in this layer by solving a MEDP problem. The algorithm stops when there are no more requests.

At round w , the problem solved by the algorithm is as follows:

$$\text{maximize } \sum_{k \in \mathcal{K}} y_k^w \quad (\text{MEDP-}w)$$

s. t.

$$\sum_{k \in \mathcal{K}} x_k^e \leq 1, \forall e \in E_w, \quad (15)$$

$$\sum_{e \in I_w(n)} x_k^e - \sum_{e \in O_w(n)} x_k^e = h_{k,n}(y_k^w), \forall n, k \quad (16)$$

$$x_k^e, y_k^w \in \{0, 1\}, \forall e \in E_w, \forall k, \quad (17)$$

where \mathcal{K} represents the set of lightpaths that have not been routed in layers $1, 2, \dots, w-1$.

Algorithm 1 First phase of the algorithm to solve (EQ-RWA)

Require: $G = (V, E)$, \mathcal{K} and \mathcal{W}

- 1: $w \leftarrow 1$
 - 2: **while** $\mathcal{K} \neq \emptyset$ **do**
 - 3: Solve problem (MEDP- w) for network layer w .
 - 4: $\mathcal{K}_w \leftarrow \{k \in \mathcal{K} : y_k^w = 1\}$
 - 5: $\mathcal{K} \leftarrow \mathcal{K} \setminus \mathcal{K}_w$
 - 6: $w \leftarrow w + 1$
 - 7: **end while**
-

B. Second Phase

The first phase of the algorithm seeks to maximize the number of lightpaths routed in a network layer before proceeding to the next one. The transition to the next layer is done once the current one is saturated. Therefore, it is clear that no lightpath $k \in \mathcal{K}_w$ can be rerouted in a layer $w' < w$. However, it may happen that a lightpath $k \in \mathcal{K}_w$ can be rerouted in a layer $w' > w$, since layers $w+1, w+2, \dots$ may be considered as underused. Define $L_R = \{w \in \mathcal{W} : \sum_{k \in \mathcal{K}} y_k^w \geq 1\}$ as the set of used wavelengths obtained after the first phase of the algorithm.

The second phase of the algorithm is a post-processing procedure based on a successive approximation algorithm. In this phase, we attempt to decrease the number of used network layers $|L_R|$ by trying to reroute all lightpaths of a given layer $w < |L_R|$ in layers $w+1, w+2, \dots, |L_R|$. In other words, for each layer w , we seek to find a new routing strategy for the lightpaths $k \in \mathcal{K}_w$ such that wavelength w is no longer used. The second phase is described in Algorithm 2.

At iteration w , we check if each lightpath $k \in \mathcal{K}_w$ can be rerouted over the shortest path in one of layers $w+1, \dots, |L_R|$ (lines 5 to 10 in Algorithm 2). If at least one of w -layer lightpaths can not be rerouted, we keep the initial configuration of layers $\lambda \in [w \dots |L_R|]$ (lines 11 to 14 in Algorithm 2)

and we move to the next iteration $w+1$. However, in the case of possible rerouting of all lightpaths $k \in \mathcal{K}_w$, we admit the new configurations \mathcal{K}_λ of layers $\lambda \in [w \dots |L_R|]$ where \mathcal{K}_w becomes an empty set, and we success in decreasing the number of used wavelengths $|L_R|$ (lines 16 to 19 in Algorithm 2). The algorithm stops when all layers $w \in L_R$ are treated.

Algorithm 2 Post-Processing procedure

Require: L_R, \mathcal{K}_w for all $w \in L_R$

- 1: $\mathcal{K}'_w \leftarrow \mathcal{K}_w \quad \forall w \in L_R$
 - 2: $D \leftarrow \emptyset$
 - 3: **for** $w \in L_R$ **do**
 - 4: **for** $k \in \mathcal{K}_w$ **do**
 - 5: **for** $\lambda = w+1 \dots |L_R|$ **do**
 - 6: **if** Find Shortest Path over λ -layer **then**
 - 7: $\mathcal{K}'_\lambda \leftarrow \mathcal{K}'_\lambda \cup \{k\}, \mathcal{K}'_w \leftarrow \mathcal{K}'_w \setminus \{k\}$
 - 8: **break**
 - 9: **end if**
 - 10: **end for**
 - 11: **if** $k \in \mathcal{K}'_w$ **then**
 - 12: $\mathcal{K}'_\lambda \leftarrow \mathcal{K}'_\lambda \quad \forall \lambda \in [w \dots |L_R|]$
 - 13: **break**
 - 14: **end if**
 - 15: **end for**
 - 16: **if** $\mathcal{K}'_w = \emptyset$ **then**
 - 17: $\mathcal{K}'_\lambda \leftarrow \mathcal{K}'_\lambda \quad \forall \lambda \in [w \dots |L_R|]$
 - 18: $D \leftarrow D \cup \{w\}$
 - 19: **end if**
 - 20: **end for**
 - 21: $L_R \leftarrow L_R \setminus D$
-

V. NUMERICAL RESULTS

Several experiments were performed in order to evaluate the effectiveness of the proposed method to solve the RWA problem. The aim of these experiments is to analyze the performance of our algorithms in terms of optimality gap and computing times and compare the proposed methods. The GUROBI solver [27] is used for solving integer linear programs. All experiments are done on an i5-2500 processor running at 3.3 GHz with 6 MB of cache and 4 GB of RAM.

We first evaluate the performance of our two-phase heuristic method when the hierarchy of MEDP problems is solved with the GUROBI solver. Then, we evaluate its performance when the MEDP problems are (approximately) solved using the SPF algorithm. Finally, we compare our results with those obtained by other existing RWA algorithms like SFFP and LFFP.

A. Performances of the Two-Phase Heuristic Method with Integer Programming

1) Error On The Optimal Number Of Wavelengths: Since the exact solution of the RWA problem cannot be computed for large problem instances, we considered a simplistic topology with 10 nodes, 26 fibers and 40 available wavelengths per fiber. Twenty sets of connections requests were randomly generated for this topology. For each problem instance, we computed

the minimum number of used wavelengths by solving problem (EQ-RWA) with the GUROBI solver. Fig. 3 shows the relative error of our two-phase method for each of the twenty instances. In each instance, all lightpath requests are routed. As can be seen, our method does not obtain an optimal solution for only 5 of the 20 instances. The average relative error is about 1, 15%.

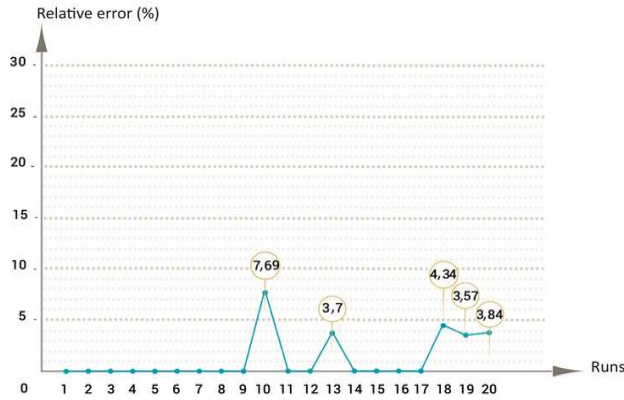


Fig. 3. Relative error of the two-phase algorithm for a network topology with 10 nodes, 26 fiber links and 40 wavelengths per fiber link.

We also evaluated our two-phase method on 4 other topologies. For each of these topologies, we randomly generated 20 sets of lightpath requests, thereby obtaining 80 problem instances. Table I describes the topologies and gives the average relative error of our algorithm for each of them. We note that, in all studied problem instances, all lightpath requests are routed as the number of available wavelengths ($|W| = 40$) is sufficiently large (as is the case of the planning phase where it is supposed that network resources are properly dimensioned). These results clearly show the efficiency of the two-phase method. We also note that for several instances the second phase succeeded in enhancing the solution obtained at the end of the first phase.

TABLE I
AVERAGE RELATIVE ERROR OF THE TWO-PHASE METHOD

Topology Size	Average Relative Error over 20 random instances (%)
$ V = 10, E = 26, W = 40$	1.15
$ V = 14, E = 42, W = 40$	2.03
$ V = 18, E = 60, W = 40$	1.89
$ V = 24, E = 82, W = 40$	2.73

2) Computation Times: We evaluated the computing times of our algorithm on 8 topologies of different sizes. For each topology, we generated 10 random sets of lightpaths. Table II shows the average computing times of our algorithm for each of the topologies. For comparison purposes, it also gives the average computing times of the GUROBI solver for solving problem (EQ-RWA). These results show that our method is able to solve real-size instances in reasonable computing times. Regarding memory consumption, our two-phase heuristic method consumes about 280 MB of RAM to solve the largest topology in Table II, whereas GUROBI requires more than 3 GB of RAM for the same topology.

TABLE II
AVERAGE COMPUTING TIMES OF THE TWO-PHASE METHOD

Topology Size	(EQ-RWA)	two-phase algorithm
$ V = 5, E = 12, W = 20$	0.6s	0.26s
$ V = 8, E = 20, W = 20$	58s	2.1s
$ V = 10, E = 26, W = 30$	2m12s	6.3s
$ V = 15, E = 40, W = 40$	9m16s	18.4s
$ V = 20, E = 56, W = 40$	15m58s	50s
$ V = 30, E = 86, W = 60$	43m22s	3m58s
$ V = 40, E = 122, W = 60$	>2h	6m07s
$ V = 50, E = 158, W = 80$	>3h	10m29s

B. Using SPF Algorithm to Reduce Computing Times

For very large topology sizes (e.g., 100 nodes), the computing times of our two-phase method can become important. This is primarily due to the use of integer programming for solving a hierarchy of MEDP problems. As already mentioned, these problems can be approximately solved using the Shortest-Path-First (SPF) algorithm which is known to often provides close-to-optimal solutions. The use of the SPF algorithm instead of integer programming in the first phase of our algorithm enables to drastically reduce the computing times. For instance, we were able to obtain a feasible solution for a large problem instance with 50 nodes, 158 fibers and 80 wavelengths per fiber in only 5 seconds. This reduction of computing times is of course at the expense of an increase in the error, but this increase seems to be moderate. For instance, on the simplistic topology with 10 nodes, 26 fibers and 40 available wavelengths per fiber used to produce the results in Fig. 3, the average error over the 20 instances was only 3.57% (instead of 1.15% with integer programming). We conclude that using the SPF algorithm in our two-phase heuristic method yields a very interesting tradeoff between accuracy and computing times for large instances.

C. Comparison between Our Two-Phase Heuristic Method and Other Known Algorithms

We now compare our two-phase algorithm with two existing algorithms: SFFP [17] and LFFP [6]. The first one aims to maximize the number of established lightpaths given a set of available wavelengths, whereas the second one attempts to minimize the number of used wavelengths given a set of lightpath requests (which exactly matches with our studied problem). We evaluate the performance of the three algorithms by comparing their errors on the optimal number of used wavelengths (calculated by (EQ-RWA) exact algorithm). We consider the same set of runs (20 runs) studied in the case of Fig. 3 for a network topology with 10 nodes, 26 fiber links and 40 wavelengths per fiber link. We illustrate in Table III the obtained results for the three algorithms. We also give for each run the number of lightpath requests to route in the network.

Results presented in Table III clearly show that the two-phase algorithm always provides better solutions than SFFP and LFFP algorithms. In this case, the average of relative errors (among the 20 runs) is about 7, 25% for LFFP, 17, 6% for SFFP and 1, 15% for our two-phase algorithm. It is expected that SFFP doesn't give good results since it was mainly conceived to maximize the network throughput. Other

TABLE III
ERRORS ON THE OPTIMAL NUMBER OF USED WAVELENGTHS.

Run	Number of lightpaths	two-phase algorithm (%)	LFPP (%)	SFFP (%)
1	89	0,0	7,69	15,38
2	86	0,0	7,14	7,14
3	104	0,0	6,25	12,5
4	97	0,0	7,14	21,42
5	80	0,0	7,69	15,38
6	81	0,0	0,0	7,14
7	82	0,0	0,0	6,25
8	98	0,0	6,66	20,0
9	83	0,0	7,14	14,28
10	92	7,69	7,69	15,38
11	169	0,0	8,0	24,0
12	174	0,0	14,28	14,28
13	187	3,7	7,4	25,92
14	158	0,0	4,54	22,72
15	174	0,0	11,53	19,23
16	168	0,0	7,4	18,51
17	172	0,0	11,11	22,22
18	165	4,34	8,69	26,08
19	169	3,57	7,14	25,0
20	177	3,84	7,69	19,23

results on other network topologies confirm these observations and always show a benefit of our two-phase method.

VI. CONCLUSION

This paper proposes a new approach for minimizing wavelength fragmentation in WDM optical transport networks. We have proposed a new ILP formulation of the RWA problem with a number of variables and constraints significantly reduced with respect to standard formulations. In addition, we have proposed a two-phase heuristic method. Numerical results show that the proposed algorithm yields close-to-optimal solutions in reasonable computing times. Computing times can even be reduced at the expense of a small increase in the error by using the SPF algorithm instead of interger programming. We have also shown that our two-phase algorithm outperforms SFFP and LFPP algorithms.

Future works will address several issues such as the re-optimization of already routed lightpaths. Service providers and operators should be interested in this proposition as it increases the capability of their networks to accomodate more future lightpath requests and then increasing their revenue.

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