A New IFO Estimation Scheme for Orthogonal Frequency Division Multiplexing Systems

Keunhong Chae, Seokho Yoon

Abstract—We address a new integer frequency offset (IFO) estimation scheme with an aid of a pilot for orthogonal frequency division multiplexing systems. After correlating each continual pilot with a predetermined scattered pilot, the correlation value is again correlated to alleviate the influence of the timing offset. From numerical results, it is demonstrated that the influence of the timing offset on the IFO estimation is significantly decreased.

Keywords—Estimation, integer frequency offset, OFDM, timing offset.

Fig. 1 Pilot arrangement in a DVB-T system with 2K mode.

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) has attracted much interest as a modulation format in a wide variety of wireless systems such as digital video broadcasting-terrestrial (DVB-T), digital audio broadcasting (DAB), wireless local area networks (WLANs) [1], [2], due to its immunity to multipath fading and high spectral efficiency. Despite the advantages of the OFDM, its performance is very vulnerable to the frequency offset (FO) caused by Doppler shift or oscillator instabilities, and thus, the FO estimation is one of the most important technical issues in OFDM-based wireless systems [3], [4]. The FO normalized to the subcarrier spacing can be divided into integer and fractional parts that bring on a cyclic shift of the OFDM subcarrier indices and intercarrier interference (ICI), respectively [5]. In this paper, we deal with the integer FO (IFO) estimation for OFDM-based DVB-T systems.

Recently, several IFO estimation schemes [6], [7] have been proposed for OFDM-based DVB-T systems. [6] estimates the IFO by using the correlation between the received pilots and reference pilots in the receiver; however, it exhibits poor performance in the multipath channel. In [7], to alleviate the influence of the multipath, the IFO is estimated based on the correlation between each continual pilot (CP) and its most adjacent scattered pilot (SP) in the received OFDM symbol; however, the scheme assumes that the timing synchronization is perfectly achieved before IFO estimation. In practical systems, only coarse timing synchronization is performed before FO estimation, and fine timing synchronization is performed after FO estimation [8]. Thus, the timing offset often remains during the IFO estimation step and affects the IFO estimation performance.

In this paper, we propose a novel pilot-aided IFO estimation scheme robust to the influence of the timing offset for OFDM-based DVB-T systems. By producing the
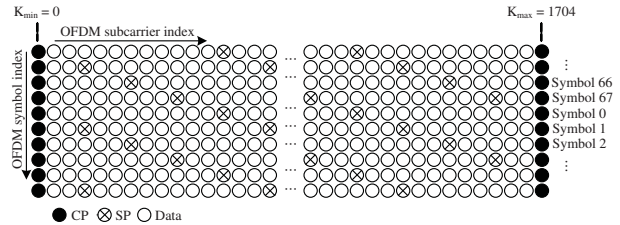
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correlation values between each CP and predetermined SP and re-correlating the correlation values, the proposed scheme reduces the influence of the timing offset on the IFO estimation. Simulation results show that the proposed scheme has more robust and better estimation performance compared with that of the conventional scheme in [7]. The rest of this paper is organized as follows. Section II describes the OFDM-based DVB-T system model. In Section III, we present the conventional IFO estimation scheme, and a novel IFO estimation scheme robust to the influence of the timing offset is proposed in Section IV. Section V demonstrates the IFO estimation performance of the two schemes in the multipath channel. Section VI concludes this paper.

II. System Model

DVB-T systems operate in 2K or 8K mode, of which the former is considered in this paper, where 1705 subcarriers among a total of 2048 subcarriers are used to transmit data and pilots of 45 CPs and 142 or 143 SPs. The pilots are used for frequency and timing synchronization and channel estimation, and have a value of either $+4/3$ or $-4/3$ depending on a pseudo random binary sequence (PRBS). Fig. 1 describes the pilot arrangement in a DVB-T system with 2K mode, where CPs are inserted at the smallest and largest subcarrier indices K_{min} and K_{max} of the active subcarriers, and SPs are periodically inserted every twelve subcarriers in all OFDM symbols and the insertion pattern is repeated every four OFDM symbols [9].

In the transmitter, the OFDM symbol is transmitted with the guard interval inserted at the beginning of the OFDM symbol to prevent the intersymbol interference. After passing through the channel, the n-th sample of the l-th received OFDM symbol in the receiver can be expressed as

$$y_l(n) = x_l(n+\tau)e^{j2\pi\nu(lN_T+n+\tau)/N} + w_l(n),$$
$$\text{for } l = 0,\ 1,\cdots,\ \text{and}\ n = 0,\ 1,\cdots,\ N-1, \quad (1)$$

where τ and ν are the timing and frequency offsets normalized to the OFDM sample and subcarrier spacing, respectively, N_T

is the number of the samples in an OFDM symbol including the guard interval, N is the size of the inverse fast Fourier transform (IFFT), and $w_l(n)$ is the additive white Gaussian noise (AWGN) sample with mean zero and variance $\sigma_w^2 = \mathbf{E}\{|w_l(n)|^2\}$. The signal $x_l(n)$ can be represented as

$$x_l(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_l(k) H_l(k) e^{j2\pi kn/N},$$
$$\text{for } l = 0, 1, \cdots, \text{ and } n = 0, 1, \cdots, N-1, \quad (2)$$

where $X_l(k)$ is a pilot or data transmitted through the k-th subcarrier of the l-th OFDM symbol and $H_l(k)$ is the channel frequency response on the k-th subcarrier of the l-th OFDM symbol.

In this paper, we assume that the fractional FO is precisely measured and compensated before the IFO estimation, then the FFT output corresponding to the k-th subcarrier of the l-th received OFDM symbol is expressed as

$$Y_l(k) = e^{j2\pi \Delta l N_T/N} e^{j2\pi \tau k/N}$$
$$\times H_l(k-\Delta) X_l(k-\Delta) + W_l(k), \quad (3)$$

where $W_l(k)$ is the zero-mean complex AWGN sample in the frequency domain and Δ is the IFO. From (3), we can observe that the IFO and timing offset causes the cyclic shift of the subcarrier indices and the phase rotation in the received OFDM symbol, respectively.

III. CONVENTIONAL SCHEME

The conventional scheme presented in [7] estimates the IFO by using the template $T_m(k)$ expressed as

$$T_m(k) = \frac{Z_m(k')}{Z_m(k)},$$
$$\text{for } k \in C_{cp} \text{ and } m \in \{0, 1, 2, 3\}, \quad (4)$$

where m is the index of the OFDM symbol to distinguish the four different pilot patterns, $Z_m(k)$ denotes the CP with the subcarrier index k in the m-th pilot pattern, and $Z_m(k')$ is the most adjacent SP of $Z_m(k)$, respectively, C_{cp} is the set of the subcarrier indices.

In the conventional scheme, the following metric is first performed for all trial values for the IFO estimation in order to recognize the pilot pattern of the received OFDM symbol.

$$m_0 = \underset{m \in \{0,1,2,3\}}{\arg\max} \left\{ \mathbf{Re}\big(\Psi(f,m)\big) \right\}, \quad \text{for } |f| \le N/2, \quad (5)$$

where $\Psi(f,m) = \sum_{k_m \in C_{cp}} Y_0(k_m + f) Y_0^*(k_m' + f) T_m(k_m)$, f is a trial value, and k_m and k_m' are the indices of the CP and its most adjacent SP in the m-th pilot pattern, respectively. Then, the most reliable α trial values in all trial values are chosen as follows

$$\{f_1, \ldots, f_\alpha\} = \underset{|f| \le N/2}{\arg\max_\alpha} \left\{ \mathbf{Re}\big(\Psi(f, m_0)\big) \right\}, \quad (6)$$

where $\underset{|f| \le N/2}{\arg\max_\alpha}\{b(f)\}$ selects the α largest values among f according to the results of $b(f)$. By exploiting the selected α trial values and correlation values of the pilots in the D

consecutive OFDM symbols, the estimate $\widehat{\Delta}$ of IFO is obtained as follows

$$\Omega(\overline{f}) = \sum_{k_{m_0 \oplus l} \in C_{cp}} \sum_{l=0}^{D-1} Y_l(k_{m_0 \oplus l} + \overline{f})$$
$$\times Y_l^*(k_{m_0 \oplus l}' + \overline{f}) T_{m_0 \oplus l}(k_{m_0 \oplus l}), \quad (7)$$

$$\widehat{\Delta} = \underset{\overline{f} \in \{f_1, f_2, \ldots, f_\alpha\}}{\arg\max_\alpha} \left\{ \mathbf{Re}\big(\Omega(\overline{f})\big) \right\}, \quad (8)$$

where $\overline{f}$ is an element of the set of trial values selected in (6), D is the number of consecutive OFDM symbols used for IFO estimation, and $(m_0 \oplus l)$ is the remainder operation when the sum of the m_0 and l divided by four, respectively. If α is set to be 1, (7) and (8) become meaningless. Due to the narrow subcarrier spacing in the OFDM-based DVB-T systems, it can be assumed that the channel frequency responses of a CP and its most adjacent SPs are the same. Thus, when there exists the timing offset, (7) can be rewritten as

$$\Omega(\overline{f}) = \sum_{k_{m_0 \oplus l} \in C_{cp}} \sum_{l=0}^{D-1} e^{j2\pi \tau (k_{m_0 \oplus l} - k_{m_0 \oplus l}')/N}$$
$$\times \left| H_l(k_{m_0 \oplus l} + \overline{f} - \Delta) \right|^2 X_l(k_{m_0 \oplus l} + \overline{f} - \Delta)$$
$$\times X_l^*(k_{m_0 \oplus l}' + \overline{f} - \Delta) \frac{X^{(m_0 \oplus l)}(k_{m_0 \oplus l}')}{X^{(m_0 \oplus l)}(k_{m_0 \oplus l})}$$
$$+ \widehat{W}_l(k_{m_0 \oplus l}), \quad (9)$$

where $\widehat{W}_l(k_{m_0 \oplus l})$ represents the noise term. As shown in (9), the phase rotation caused by the timing offset affects the estimation of the IFO in the conventional scheme.

IV. PROPOSED SCHEME

In this section, we propose a novel IFO estimation scheme robust to the timing offset. In the proposed scheme, the correlation value between each CP and a predetermined SP is calculated for all CPs, and then, the correlation values are classified into several groups according to the predetermined sample distances between the CP and predetermined SP. Since the correlation values with the same sample distance are equally affected by the timing offset, the influence of the timing offset can be removed by re-correlating the classified correlation values.

If the subcarrier indices of the CP and SP are the same, the pilot on the index is considered as the CP, and the most adjacent SP to the CP is called the predetermined SP. According to the DVB-T standard document, the sample distances between a CP and its predetermined SPs in an OFDM symbol are among ± 3, ± 6, ± 9, and ± 12 [9], and thus, the pilot pattern of the received OFDM symbol is recognized as follows

$$m_0 = \underset{m \in \{0,1,2,3\}}{\arg\max} \left\{ \mathbf{Re}\big(\Lambda(f,m)\big) \right\}, \quad \text{for } |f| \le N/2, \quad (10)$$

where $\Lambda(f,m)$ is given by

$$\Lambda(f,m) = \sum_{g \in G} \sum_{i=1}^{G_n(g)-1} \sum_{j=i+1}^{G_n(g)} Y_0(I_{g,m}(i)+f)$$
$$\times Y_0^*(I_{g,m}(i)+g+f)T_m(I_{g,m}(i))$$
$$\times \Big\{ Y_0(I_{g,m}(j)+f)Y_0^*(I_{g,m}(j)+g+f)$$
$$\times T_m(I_{g,m}(j)) \Big\}^*, \qquad (11)$$

where g is the sample distance between the CP and predetermined SP, G is the set of the sample distances between the CPs and predetermined SPs, and $G_n(g)$ is the number of the CPs with the sample distance g. $I_{g,m}(i)$ is the subcarrier index of the i-th CP included in the group with the sample distance g in the m-th pilot pattern. The template of the proposed scheme is expressed as

$$T_m(I_{g,m}(k)) = \frac{Z_m(I_{g,m}(k)+g)}{Z_m(I_{g,m}(k))},$$
$$\text{for } m \in \{0,1,2,3\}. \qquad (12)$$

After recognizing the pilot pattern of the received OFDM symbol, we select the most reliable α trial values as follows:

$$\{f_1, \ldots, f_\alpha\} = \underset{|f| \leq N/2}{\arg \max_\alpha} \Big\{ \mathbf{Re}\big(\Lambda(f,m_0)\big) \Big\}. \qquad (13)$$

By exploiting the selected α trial values and correlation values of the pilots in the D consecutive OFDM symbols, the proposed scheme estimates the IFO as follows:

$$\Gamma(\overline{f}) = \sum_{l=0}^{D-1} \sum_{g \in G} \sum_{i=1}^{G_n(g)-1} \sum_{j=i+1}^{G_n(g)} Y_l(I_{g,m_0 \oplus l}(i)+\overline{f})$$
$$\times Y_l^*(I_{g,m_0 \oplus l}(i)+g+\overline{f})T_{m_0 \oplus l}(I_{g,m_0 \oplus l}(i))$$
$$\times \Big\{ Y_l(I_{g,m_0 \oplus l}(j)+\overline{f})Y_l^*(I_{g,m_0 \oplus l}(j)+g+\overline{f})$$
$$\times T_{m_0 \oplus l}(I_{g,m_0 \oplus l}(j)) \Big\}^* \qquad (14)$$

and

$$\widehat{\Delta} = \underset{|f| \leq N/2}{\arg \max} \Big\{ \mathbf{Re}\big(\Gamma(\overline{f})\big) \Big\}, \qquad (15)$$

where $\overline{f}$ is an element of the set of trial values selected in (13).

Assuming that the channel frequency responses of the CP and predetermined SP are the same, when there exists a timing offset, we can re-write (14) as

$$\Gamma(\overline{f}) = \sum_{l=0}^{D-1} \sum_{g \in G} \sum_{i=1}^{G_n(g)-1} \sum_{j=i+1}^{G_n(g)} \Big|H_l(A)\Big|^2 X_l(A)$$
$$\times X_l^*(A+g)\frac{X^{(m_0 \oplus l)}(I_{g,m_0 \oplus l}(i)+g)}{X^{(m_0 \oplus l)}(I_{g,m_0 \oplus l}(i))}\Big|H_l(B)\Big|^2$$
$$\times X_l^*(B)X_l(B+g)$$
$$\times \Big\{ \frac{X^{(m_0 \oplus l)}(I_{g,m_0 \oplus l}(j)+g)}{X^{(m_0 \oplus l)}(I_{g,m_0 \oplus l}(j))} \Big\}^*$$
$$+ \widehat{W}_l(I_{g,m_0 \oplus l}(j)), \qquad (16)$$

where $A = I_{g,m_0 \oplus l}(i)+\overline{f}-\Delta$, $B = I_{g,m_0 \oplus l}(j)+\overline{f}-\Delta$, and $\widehat{W}_l(I_{g,m_0 \oplus l}(j))$ represents the noise term. Since the proposed
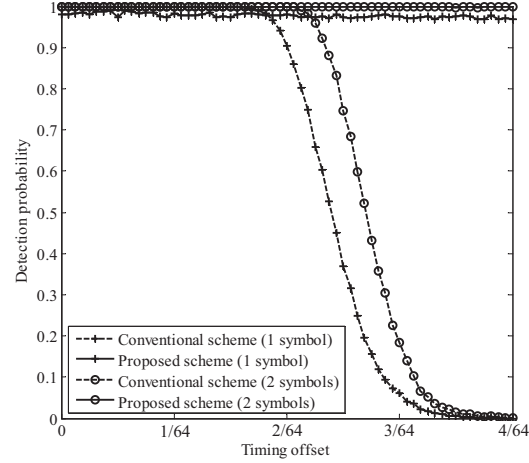

Fig. 2 IFO detection probabilities of the conventional and proposed schemes when the SNR is 5 dB.

scheme is not influenced by the timing offset as shown in (16), it is expected that the estimation performance of the proposed scheme is robust to the timing offset.

V. SIMULATION RESULTS

In this section, the proposed scheme is compared with the conventional scheme in terms of the IFO detection probability in the multipath channel. We consider a DVB-T system with 2K mode and quadrature amplitude modulation. The parameters used in the simulation are as follows: the guard interval size of 128, $D = 1$ and 2, $N = 2048$, $\Delta = 1$, and $\alpha = N/4$. The signal to noise ratio (SNR) is defined as σ_x^2/σ_w^2 where $\sigma_x^2 = \mathbf{E}\{|x_l(n)|^2\}$. We consider a ten-path Rayleigh fading channel with path delays of $0, 10, \cdots, 90$ samples and an exponential power delay profile where the power ratio of the first path to the last path is 20 dB. The phase of each path is assumed to be distributed uniformly in $(-\pi, \pi]$ and the maximum Doppler frequency is set to be 100 Hz.

Fig. 2 shows the IFO detection probabilities of the proposed and conventional schemes as a function of the timing offset normalized to the length of one OFDM symbol when the SNR is 5 dB. As shown in Fig. 2, the IFO detection probability of the conventional scheme is severely degraded as the timing offset is increased, whereas that of the proposed scheme is almost constant regardless of the timing offset value.

Figs. 3-5 show the IFO detection probabilities of the proposed and conventional schemes as a function of the SNR when the timing offset is $10/512$, $15/512$, and $20/512$, respectively. From the figure, it is found that the difference in IFO estimation performance between the proposed and conventional schemes becomes larger as the timing offset is increased.

VI. CONCLUSION

We have proposed a novel IFO estimation scheme robust to the influence of the timing offset based on the correlation between CPs and their own predetermined SPs. The re-correlation of the correlation values has alleviated the
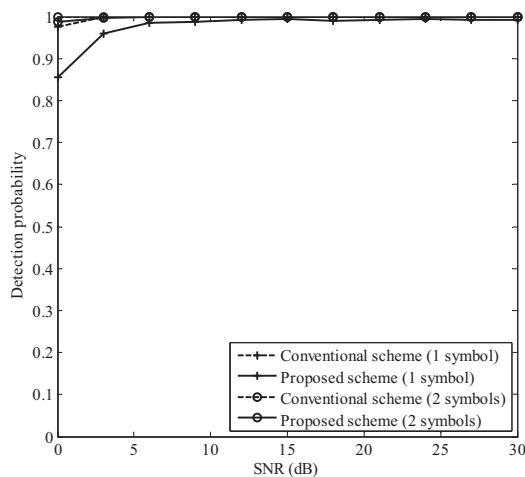
Fig. 3 IFO detection probabilities of the conventional and proposed schemes when the timing offset is 10/512.
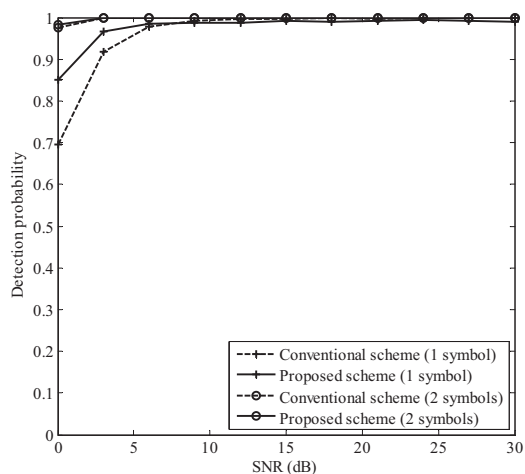

Fig. 4 IFO detection probabilities of the conventional and proposed schemes when the timing offset is 15/512.
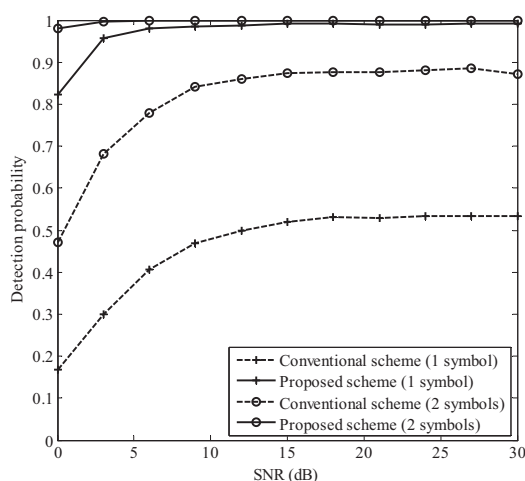

Fig. 5 IFO detection probabilities of the conventional and proposed schemes when the timing offset is 20/512.

influence of the timing offset. From the simulation results, the proposed scheme has been found to be more robust and to have a better estimation performance compared with that of the conventional scheme.

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