

A New Reliability Allocation Method Based On Fuzzy Numbers

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Abstract—Reliability allocation is quite important during early design and development stages for a system to apportion its specified reliability goal to subsystems. This paper improves the reliability fuzzy allocation method, and gives concrete processes on determining the factor and sub-factor sets, weight sets, judgment set, and multi-stage fuzzy evaluation. To determine the weight of factor and sub-factor sets, the modified trapezoidal numbers are proposed to reduce errors caused by subjective factors. To decrease the fuzziness in fuzzy division, an approximation method based on linear programming is employed. To compute the explicit values of fuzzy numbers, centroid method of defuzzification is considered. An example is provided to illustrate the application of the proposed reliability allocation method based on fuzzy arithmetic.

Keywords—Reliability allocation, fuzzy arithmetic, allocation weight.

I. INTRODUCTION

RELIABILITY allocation is of great importance in the early design and development stages to set reasonable reliability goals for subsystems or components. Nowadays, as products are getting more and more complex with frequent faults and multiple faults, reliability statistical data will differ greatly with different statistical approaches adopted, and vary from version to version. Thus, reliability data might be incomplete or inaccurate for reliability allocation. In addition, many factors can influence products' reliability, so it is essential to introduce the concept of fuzziness.

Researchers have been developing a lot of reliability allocation methods over the past several decades considering available information of the subsystems. Conventional reliability allocation methods based on system behavior and performance, and relative information have used allocation weights include Aeronautical Radio Inc. (ARINC) method [1], Advisory Group on Reliability of Electronic Equipment (AGREE) method [2], the feasibility of objectives method [1], Karmiol method [3], the integrated factors method[4], the comprehensive method [5], the maximal entropy ordered weighted averaging method [6]. Recently, [7] proposes a new reliability allocation approach based on the subsystem failure severity and its relative frequency. Yadav [8] proposes a modified criticality measure for allocating system level reliability improvement goal to subsystems, and [10] proposes

the fuzzy arithmetic based method. The result of the reliability allocation depends on the weighting method used.

This paper improves the reliability fuzzy allocation method, and gives concrete processes on determining the factor and sub-factor sets, the weight sets, judgment set, and multi-stage fuzzy comprehensive evaluation.

II. FACTORS BASED ON CONVENTIONAL ALLOCATION METHOD

If no definitive information on the given system can be obtained, we can use the equal apportionment technique. However, more reasonable methods were suggested when additional information is available. In this paper, various factors such as design and manufacture, system properties, and use and maintenance are considered. Design and manufacture include two sub-factors, namely cost (Co) and state of the art (S). System properties involve three comprehensive sub-factors: complexity (K), technical difficulty (D), and operating time (O). Use and maintenance concern criticality (Cr) and maintainability (M). Evaluation of these sub-factors are multiplied or divided to give each subsystem a score.

As no specific information about the system can be acquired in the early stage of design and development, and we can assume the system consisting of n subsystems is in series. So the relationship of apportioned reliability between system and sub systems is given below:

$$R = \prod_{i=1}^n R_i. \quad (1)$$

where R is the system reliability and R_i is the reliability target assigned to the i th subsystem.

Let w_i be the weight adopted. The relationship is rewritten as follows:

$$R_i = R^{w_i}, \quad i = 1, 2, \dots, n. \quad (2)$$

Assume that all failure rates of subsystems are constants, the failure rate allocated to the i th subsystem is given by:

$$\lambda_i = w_i \lambda \quad (3)$$

where λ and λ_i are failure rates corresponding to R and R_i .

The weight w_i can be expressed with proportionality factor ϖ_i as,

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$$w_i = \frac{\varpi_i}{\sum_{i=1}^n \varpi_i} \quad (4)$$

$$U_1 = \{u_{11}, u_{12}\} \quad (9)$$

where u_{11}, u_{12} represent cost and state of the art.

$$U_2 = \{u_{21}, u_{22}, u_{23}\} \quad (10)$$

where u_{21}, u_{22}, u_{23} represent complexity, technical difficulty and operating time.

$$U_3 = \{u_{31}, u_{32}\} \quad (11)$$

where u_{31}, u_{32} represent criticality and maintainability.

The equations above should meet conditions:

$$\begin{cases} \bigcup_{i=1}^m U_i = U \\ U_i \cap U_j = \emptyset \quad i \neq j \end{cases} \quad (12)$$

For single factor fuzzy evaluation, evaluation set compromises all the possible evaluation results. However, as to multi-stage fuzzy evaluation method, evaluation set is made up of all the subsystems. Evaluation set is expressed by:

$$S = \{s_1, s_2, s_3, \dots, s_n\} \quad (13)$$

B. The Factor and Sub-Factor Weight Set

The factor weight shows the effect degree of various factors on system's reliability and reflecting status in the process of decision-making. Different factors usually have different impact on reliability, therefore every factor and sub-factor should be endowed with corresponding weight values. That is to say, we need to establish the factor and sub-factor weight sets.

Let a_i be the i th weight, and then the factor weight set is given by:

$$A = (a_1, a_2, a_3) \quad (14)$$

where

$$\sum_{i=1}^3 a_i = 1, a_i > 0.$$

Similarly, let A_1, A_2, A_3 be the three sub-factor weight sets respectively.

At present, there are two ways to determine the weight: subjective and objective weighting methods. The latter usually need a lot of reliability allocation data from similar products, so it is usually inappropriate to in engineering application. The subjective evaluation method is conducted based on experience of experts in the related field. Although it has strong subjectivity, it is suitable for the reliability allocation during the early design stage because of the lack of reliability data [11]. This paper adopts modified trapezoidal fuzzy numbers to evaluate weights.

The cost usually increases much in order to improve reliability, so it is advisable to allocate a relatively lower value of reliability for a costlier subsystem. Hence, $\varpi_1 \propto Co$. with high state of the art, it is possible to achieve high reliability for components. Therefore, $\varpi_1 \propto 1/S$.

Hence, for design and manufacture (U_1), the proportionality factor (ϖ_1) can be determined by:

$$\varpi_1 = \frac{Co}{S} \quad (5)$$

The more complex the subsystem is, the more likely it is going to break down. Thus, the failure rate is allocated proportional to the subsystem's complexity, namely $\varpi_2 \propto K$. When a subsystem need high technical difficulty, it is quite tough to further improve its reliability. So it is logical to allocate relatively low reliability. Hence, $\varpi_2 \propto D$. to make sure the whole system operates normally, subsystems with long operating time should be allocated high reliability. Hence, $\varpi_2 \propto 1/O$.

Hence, for system properties (U_2), the proportionality factor (ϖ_2) can be determined by:

$$\varpi_2 = \frac{K \times D}{O} \quad (6)$$

Obviously, critical subsystem should be given a high reliability target. So $\varpi_3 \propto 1/Cr$. The subsystem with high maintainability is easier to be periodically maintained or repaired. Therefore it permitted to allocate low reliability. So $\varpi_3 \propto M$.

Hence, for use and maintenance (U_3), the proportionality factor (ϖ_3) can be determined by:

$$\varpi_3 = \frac{M}{Cr} \quad (7)$$

III. PROPOSED ALLOCATION METHOD

A. Factor Set, Sub-Factor Set and Evaluation Set

Factor set is a set containing all the factors which can affect system reliability [10]. This paper considers three factors: design and manufacture (U_1), system properties (U_2), and use and maintenance (U_3), the factor set (U) is given by:

$$U = \{U_1, U_2, U_3\} \quad (8)$$

Then factors U_i ($i = 1, 2, 3$) can be subdivided into sub-factors, namely sub-factor set:

Trapezoidal fuzzy numbers are parameterized by (a, b, c, d) , and the membership functions of these numbers are defined by:

$$\mu_{\tilde{U}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ \frac{d-x}{d-c}, & c \leq x \leq d, \\ 0, & \text{otherwise,} \end{cases} \quad (15)$$

where $\mu_{\tilde{U}}(x)$ is degree of membership of element x in fuzzy set \tilde{U} .

Multiplication of trapezoidal fuzzy numbers [12] is given by:

$$\tilde{A} \times \tilde{B} = (a_1, a_2, a_3, a_4) \times (b_1, b_2, b_3, b_4) = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4) \quad (16)$$

Fuzzy division can introduce linear programming, therefore, Sriramdas et al. proposed a method for the fuzzy division of trapezoidal fuzzy numbers in 2014 [9].

Let, \tilde{P} and \tilde{Q} be two trapezoidal fuzzy numbers parameterized by $(l_1, \alpha_1, \beta_1, r_1)$, and $(l_2, \alpha_2, \beta_2, r_2)$, where l_1 and l_2 , β_1 and β_2 , α_1 and α_2 , and r_1 and r_2 denotes left end points, left center points, right center points, and right end points, respectively. The result of fuzzy numbers is given by:

$$\tilde{X} = \frac{\tilde{P}}{\tilde{Q}} \quad (17)$$

where $\tilde{X} = (x_l, x_\alpha, x_\beta, x_r)$ is a fuzzy number.

The constraints can be written as:

$$\begin{cases} x_l \geq \frac{\alpha_1}{\alpha_2} - \frac{\alpha_1 - l_1}{\alpha_2 - l_2} \\ x_r \leq \frac{\beta_1}{\beta_2} + \frac{r_1 - \beta_1}{r_2 - \beta_2} \end{cases} \begin{cases} x_l < \frac{\alpha_1}{\beta_2} \\ x_r > \frac{\alpha_2}{\beta_1} \end{cases} \quad (18)$$

The objective function is expressed as:

$$\text{Max } f(x) = x_r - x_l \quad (19)$$

The most standard defuzzification is to compute the centroid, which means the center of gravity of the curve describing a fuzzy variable. For a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$, we can use (20) to solve problem of defuzzification [13]. The centroid is expressed as:

$$C_{\tilde{A}} = \frac{c^2 + d^2 + cd - a^2 - b^2 - ab}{3(c+d-a-b)} \quad (20)$$

C. Factor Rating Set and Membership Function

As we do in FMEA, we cannot describe factors in an completely precise manner. Therefore fuzzy language has been introduced to evaluate allocation factors. The terms and their corresponding fuzzy numbers in Table I explain the linguistic evaluating method, and the terms are regarded as trapezoidal fuzzy numbers because of experts' uncertainty in the evaluation.

TABLE I
FUZZY RATING FOR RELIABILITY ALLOCATION FACTORS

Factors/scale	Design and manufacture(U1)			System properties(U2)		Use and maintenance(U3)	
	Cost (Co)	State of art (S)	Complexity (K)	Technical difficulty(D)	Operating time(O)	Criticality (Cr)	Maintenance (M)
(7,8,9,10)	Very high	Very high	Very high	Very high	Very high	Very high	Very high
(5,6,7,8)	High	High	High	High	High	High	High
(3,4,5,6)	Medium	Medium	Medium	Medium	Medium	Medium	Medium
(1,2,3,4)	Low	Low	Low	Low	Low	Low	Low

Each grade has a corresponding membership degree, which can be regarded as the standard value of each grade, and the value is determined by the factors' various effect on evaluation scores.

The corresponding membership of all levels compose a score set, which can be written as

$$V = (v_1, v_2, v_3, v_4) = (0.25, 0.5, 0.75, 1) \quad (21)$$

In the formula above, the greater the corresponding membership degree is, the lower reliability the unit has. In other words, the failure rate, and the evaluation score is high, which means it is hard to maintain a high reliability[14].

D. Multi-Stage Fuzzy Evaluation

Firstly, all the sub-factors of each factor should be single factor fuzzy evaluated by synthesizing impacts of their ratings on products. The expert gives membership scores for four

evaluating levels, including low, middle, high, very high. The marking interval is between 0 and 1. Therefore, the rating matrix for the i th factor is given by:

$$R_i = [r_{ij}]_{j \times 4} \quad (22)$$

Combined with weight sets of sub-factors A_i ($i = 1, 2, 3$), the one-stage fuzzy evaluation for all the sub-factors of the i th factor is given by:

$$B_i = A_i \cdot R_i \quad (23)$$

As one-stage evaluation reflects the impact on scores, it can act as the rating matrix for multi-stage fuzzy evaluation, namely

$$R = [B_1 \ B_2 \ B_3]^T \tag{24}$$

$$\lambda = -\frac{\ln R}{t} \tag{27}$$

With allocating weight of factors A , the transformation of multi-stage fuzzy evaluation is expressed as:

$$B = A \cdot R \tag{25}$$

where B represents the evaluating result, and it reflects the membership of each subsystem for reliability requirements after considering all the factors that might affect the reliability^[15].

The comprehensive evaluation scores for all the subsystems are as:

$$\omega = B \cdot V^T \tag{26}$$

IV. CASE EXAMPLE

In this section, we illustrate the application of the proposed reliability allocation method based on fuzzy arithmetic with a multi-function display system (MFDS). The MFDS is made up of power supply module, display control module, video processing module and screen control module. All the four subsystems are connected in series. According to the reliability goal, the MFDS need operate 1000 h with a probability of $R_0 = 0.906$. In order to apply the above methods expediently; failure rate should be obtained according total reliability goal:

So we can get the total required failure rate is $\lambda_0 = 9.8716 \times 10^{-5}$. Then, we take power supply module as an example to show the whole process of the method. The same procedure can be easily adapted for other subsystems.

An expert team consisting of three members uses the linguistic terms defined in Table I to judge allocation factors. Table II shows the experts' determination of the seven sub-factors for power supply module.

The three team members are assumed to have different importance because of their different domain knowledge and expertise. Therefore, the three experts E1, E2, and E3 are assigned with relative weights of 50%, 20% and 30%, respectively, and we use weighted summation to get the fuzzy numbers of all the sub-factors for power supply module, as shown in Table III.

TABLE II
ALLOCATION INFORMATION FOR POWER SUPPLY MODULE

Experts	Allocation factors						
	Co	S	K	D	O	Cr	M
E1 (50%)	H	M	VH	M	H	H	M
E2 (20%)	VH	H	H	M	H	H	H
E3 (30%)	H	H	VH	L	H	M	H

TABLE III
AGGREGATED FUZZY EVOLUTION INFORMATION FOR POWER SUPPLY MODULE

Factors	Co	S	K	D	O	Cr	M
Fuzzy numbers	(5.4,6.4,7.4,8.4)	(4,5,6,7)	(6.6,7.6,8.6,9.6)	(2.4,3.4,4.4,5.4)	(5,6,7,8)	(6.4,7.4,8.4,9.4)	(4,5,6,7)

With MATLAB, we defuzzify the trapezoidal fuzzy numbers according to (20). Tables IV-VI describe the weight ages of sub-factors, and sub-factor weight sets can be easily acquired.

TABLE IV
EVALUATION RESULT OF SUB-FACTORS OF U_1

Factors	Fuzzy numbers	Defuzzified	Weightage(A1)
Co	(5.4,6.4,7.4,8.4)	6.9	0.5565
S	(4,5,6,7)	5.5	0.4435

The sub-factor weight set of U_1 is $A_1 = (0.5565, 0.4435)$.

TABLE V
EVALUATION RESULT OF SUB-FACTORS OF U_2

Factors	Fuzzy numbers	Defuzzified	Weightage(A2)
K	(6.6,7.6,8.6,9.6)	8.1	0.4378
D	(2.4,3.4,4.4,5.4)	3.9	0.2108
O	(5,6,7,8)	6.5	0.3514

The sub-factor weight set of U_2 is $A_2 = (0.4378, 0.2108, 0.3514)$.

The sub-factor weight set of U_3 is $A_3 = (0.5896, 0.4104)$.

TABLE VI
EVALUATION RESULT OF SUB-FACTORS OF U_3

Factors	Fuzzy numbers	Defuzzified	Weightage(A3)
Cr	(6.4,7.4,8.4,9.4)	7.9	0.5896
M	(4,5,6,7)	5.5	0.4104

TABLE VII
EVALUATION RESULT OF THE TOTAL PROPORTIONALITY FACTOR

Factors	Fuzzy proportionality factor	Defuzzified	Weightage(A)
U_1	(0.28,1.0667,1.48,2.233)	1.4342	0.1708
U_2	(0.3,6914,6.3067,19.4057)	5.7563	0.6856
U_3	(0.05952,0.8108,1.7143)	1.2059	0.1436

The factor fuzzy evaluation in (5)-(8) can be calculated using (16)-(19), and Table VII shows the weightages of three factors, so the factor weight set is expressed as:

$$A = (0.1708, 0.6856, 0.1436)$$

Let each subsystem be the factors in the evaluation set $V = \{v_1, v_2, v_3, v_4\}$, and v_1, v_2, v_3, v_4 respectively denote power supply module, display control module, video processing module and screen control module.

After a collection of the three experts' judgment, the rating

matrixes of sub-factor set U_1, U_2, U_3 are given by:

$$R_1 = \begin{bmatrix} 0.1 & 0.1 & 0.6 & 0.4 \\ 0.1 & 0.4 & 0.6 & 0.1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0.1 & 0.1 & 0.4 & 0.7 \\ 0.3 & 0.6 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.9 & 0.1 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.1 & 0.3 & 0.7 & 0.1 \\ 0.1 & 0.5 & 0.5 & 0.1 \end{bmatrix}$$

The evaluating results of three sub-factor sets is obtained from (23)

$$B_1 = (0.1000 \quad 0.2331 \quad 0.6000 \quad 0.2670)$$

$$B_2 = (0.1422 \quad 0.2054 \quad 0.5125 \quad 0.3637)$$

$$B_3 = (0.1000 \quad 0.3821 \quad 0.6179 \quad 0.1000)$$

Therefore, according to (25), the evaluating result is expressed as:

$$B = [0.1289 \quad 0.2355 \quad 0.5426 \quad 0.3086]$$

From (26), we obtain the comprehensive evaluation score of power supply module is $\varpi_1 = 0.8655$. Similarly, scores of the other three subsystems are $\varpi_2 = 0.9033$, $\varpi_3 = 0.8253$, $\varpi_4 = 0.7433$.

Since the proportional factors are acquired, the weight age value of each subsystem is calculated using (4), and the target of reliability and failure rate value for each subsystem are evaluated by (2) and (3) respectively. Therefore, the final allocation results are provided in Table VIII.

TABLE VIII
RESULT OF PROPOSED ALLOCATION METHOD

Subsystem	Power supply	Display control	Video processing	Screen control	Total
$\lambda(\times 10^{-5}/h)$	2.5600	2.6718	2.4411	2.1986	9.8716
R(1000h)	0.9747	0.9736	0.9759	0.9783	0.9060

According to the allocation results given in Table VIII, the reliability target assigned to each subsystem is consistent with its technology, structure and features. Therefore, the proposed reliability allocation method is reasonable and feasible in the application of MFDS.

V. CONCLUSION

In this paper, we introduce fuzzy arithmetic to reliability allocation to avoid giving factors explicit values. A modified reliability fuzzy allocation method is proposed and the processes on determining the factor and sub-factor sets, the weight sets, judgment set, and multi-stage fuzzy evaluation are illustrated clearly. We use the modified trapezoidal numbers to determine the weight of factor and sub-factor sets. As experts have different background and expertise, different importance is given to experts, making evaluation from different perspectives more accurate. Besides, linear programming based fuzzy division and centroid method of defuzzification are considered.

This method may also be used to allocate reliability based on possible environment stresses to which the product might be exposed, and it may open a new horizon for reliability allocation.

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