International Journal of Business, Human and Social Sciences
ISSN: 2517-9411
Vol:9, No:5, 2015

Development of Researcher Knowledge in Mathematics Education: Towards a Confluence Framework

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***Abstract*—**We present a framework of researcher knowledge development in conducting a study in mathematics education. The key components of the framework are: knowledge germane to conducting a particular study, processes of knowledge accumulation, and catalyzing filters that influence a researcher decision making. The components of the framework originated from a confluence between constructs and theories in Mathematics Education, Higher Education and Sociology. Drawing on a self-reflective interview with a leading researcher in mathematics education, Professor Michèle Artigue, we illustrate how the framework can be utilized in data analysis. Criteria for framework evaluation are discussed.

***Keywords*—**Community of practice, knowledge development, mathematics education research, researcher knowledge.

I. Background

SINCE its establishment in the second half of the previous century, mathematics education (ME) community has invested a considerable effort in situating itself as an independent research field [1]. A significant part of this effort has been put into instilling traditions of research excellence to graduate students. This is done through programs, courses and other activities aimed at developing students' knowledge and proficiency in conducting studies in ME. Experienced scholars from various research areas in ME are engaged in education of prospective researchers [2], [3].

However, there are two important issues that bear mentioning regarding the above situation: First, research on professionalism and expertise repeatedly shows that experts excel in their core practices, but not necessarily in their analysis and communication of these practices [4]. Second, while in the some fields, such as medicine, research knowledge and its development among graduate students are studied systematically [5]; empirical research on this topic does not exist in ME [3].

Reflecting on education of doctoral students, Shulman commented:

> "Our practices in doctoral education are a combination of longstanding traditions, replications of how we ourselves were trained, administrative convenience, and profound inertia. We do not subject our programs to the kinds of experimental, skeptical, adventurous innovations and tests that we claim to value in our scholarly work" [2, p.9].

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Shulman's position is shared by Boaler, Ball and Even, who suggest that "Many of the components of successful research remain implicit and are left to new researchers to glean from finished products" [3, p. 489].

We interpret these positions as an open call for empirical exploration of practicing researchers who, in our case, conduct studies in the field of ME. The insights that emerge from this exploration will be useful for the refinement of the current approaches used in educating prospective ME researchers, and for designing new approaches.

Research is a highly complicated enterprise that involves various types of knowledge [2]. Indeed, a researcher uses her/his previously accumulated knowledge and the knowledge of others in carrying out a study, which in turn, is an act of knowledge accumulation. This paper is a report on an ongoing exploration of researcher knowledge development in conducting studies in ME. The goal of this paper is to present and illustrate a theoretical framework that: (a) characterizes various components of researcher knowledge that is involved in conducting a study in ME; and (b) describes the development of researcher knowledge through carrying out a study in ME.

In Section II we present the considerations underpinning the design of the proposed framework. The literature background for the framework is presented in Section III. Section IV presents the framework. This is followed, in Section V, by illustrations of how the framework can be utilized in data analysis. The aim of the illustrations is to demonstrate how such analysis illuminates various paths of development of researcher knowledge. The illustrations are taken from the self-reflective interview with Professor Michèle Artigue, who is acknowledged as one of the coryphaei in ME [6]. In Section VI we conclude by providing initial evaluation of the framework with respect to the set criteria.

II. Framework Design Considerations

For designing a framework of researcher knowledge in conducting a study in ME, we use the methodology of modified analytic induction [7]. This methodology requires identification of a phenomenon of interest and a descriptive initial hypothesis or a theoretical framework, which often emerges from the literature. In the further stages, the theory is systematically refined based on the analysis of the collected data. In this paper we focus on the emergence of the initial framework based on the selected literature.

When selecting literature for constructing a framework of researcher knowledge development, we draw on theories and constructs mainly coming from Sociology, Higher Education and Mathematics Education. By and large, these bodies of knowledge complement each other in the following way: Many sociological theories are concerned with knowledge development as a participation process in a community of practice, but they rarely attend to the particular specifications of these communities [8]. Higher Education addresses the specifications of scientific communities, but the research discipline is rarely taken into consideration [9]. Mathematics Education is suggestive about students' mathematical knowledge and teachers' pedagogical content knowledge, but it is in its infancy regarding researcher knowledge [3].

Thus, we attempt to address the aforementioned issues by constructing a confluence framework drawn on multiple research fields.

III. Theoretical Background

A. Knowledge

Shulman argues that conducting a study demands a highly complex set of understandings and skills [2]. Boaler and her colleagues conceptualize research as an active process of investigation when knowledge is mobilized into practice [3]. We adopt this separation between knowledge and practices for our framework.

We chose to use the literature on teacher knowledge as a starting point for two reasons: *First*, practicing researchers are the ones to teach graduate students. Consequently, their knowledge is a special case of teacher knowledge. The literature on teacher knowledge, in its turn, continuously acknowledges the role of content. In our case, the content is ME research. *Second*, literature on the development of teacher knowledge highlights the idea of "learning though teaching" [10]. Similarly, our framework acknowledges the development of researcher knowledge as "learning through research".

In the summary of extensive literature on teacher knowledge, Leikin and Zazkis propose to decode knowledge of a mathematics teacher according to three dimensions: *sources*, *forms* and *kinds* [10]. Following Kennedy, sources refer to systematic, craft and prescriptive knowledge. Systematic knowledge has been acquired through courses and reading research papers and professional books. Craft knowledge is developed through experience or practice. Prescriptive knowledge is the one acquired from institutional policies, accountability systems and texts of diverse nature [11].

Following Scheffler, forms of knowledge distinguish between knowing, which has a "propositional and procedural nature", and believing, which is "construable as solely propositional" [12, p. 15]. Scheffler argues that believing is a necessary condition for knowing.

In elaborating the kinds of knowledge, Leikin and Zazkis draw on Shulman's classification [13]. Two of the categories are relevant to our purposes: subject matter knowledge and pedagogical content knowledge. Subject matter knowledge is the knowledge of mathematics, including concept definitions and properties, connections and problem-solving. Pedagogical content knowledge is in the intersection of pedagogy and mathematical subject matter [14]. Pedagogical knowledge consists of broad principles and strategies of classroom management and organization (e.g., group work), educational purposes and values etc. Pedagogical content knowledge is knowing how students approach mathematical tasks, the ability to design tasks fitted to students' learning styles and needs, and knowing of learning setting.

We borrow from Leikin the notion of dimensions of knowledge, and adjust those switching from teacher knowledge to researcher knowledge in ME.

B. Practices

In their self-reflective paper Boaler and her colleagues unpack the competent performance of a researcher in ME by exposing research practices in which "accomplished" researchers engage [3]. By research practices, Boaler and her colleagues refer to specific and recurrent professional activities of a ME researcher that require mobilization of knowledge in different situations. Eventually, they suggest a list of the following practices: reading, formulating a research question, using data carefully to make and ground claims, moving from the particular to the general, considering mathematics, and communicating research findings.

Boaler and her colleagues suggest designing opportunities for prospective researchers to engage in the aforementioned practices during their doctoral studies, as a part of preparation for an academic career in ME. The importance of these practices is unquestionable for conducting a study in ME. However, when focusing on the *doing* component, the scholars do not explicitly attend to the knowledge which is needed for executing these practices and the knowledge that is being developed as the result of their recurrent execution. In addition, the list seems rather unbalanced: For instance, the practice of formulating a research question is much more prescriptive than considering mathematics. It is also relevant only to particular research stages, when reading is fundamental at all research stages. Nevertheless, we use the identified practices from this list to exemplify possible paths of researcher knowledge development.

C. Chance

In an ideal world possessing a broad and deep knowledge combined with a rich repertoire of practices is necessary and sufficient for excelling. However, more pragmatic approaches suggest that chance or luck have an important role in a person's career. Indeed, in the Gagné's and Tannenbaum's models of giftedness and talent, chance is one of the factors responsible for self-realization [15], [16]. Moreover, Gagné argues that chance is embedded in other factors, such as genetics and environment. This is because being born in a particular family or attending a school with a program for talented students is also a matter of chance.

In a study with twenty five mathematicians, Liljedahl found that many of them perceive that chance has a large role in their work, especially in illumination and insight [17]. The researcher distinguishes between *intrinsic* and *extrinsic* chance. Intrinsic chance relates to a successful combination of a mathematician's ideas that result in an insight. Extrinsic chance is all about exposure to the knowledge that is helpful for resolution of the problem that a mathematician is working on. Extrinsic chance is featured in our framework.

D. Community of Practice

Wenger refers to *community of practice* as a group of people who share a concern or a passion, do and learn as they interact regularly [8]. The common characteristics of a community of practice are: a shared domain of interest, which is ME in our case; mutual learning and knowledge sharing, which is research and finding dissemination in our case; and shared resources, that can be conceptualized as a body of knowledge accumulated by the community as a whole. Participation in a community of practice demands awareness of its concepts, facts and structure as well as realization of this knowledge in practice [18]. As such, a researcher's knowledge of the ME community of practice is reflected in our framework.

E. Professional Identity

Professional identity of academics is a complex construct that usually relates to teaching and research activities. On the one hand, it is rooted in the culture of communities of practice in which a researcher participates and it consists of assumptions about what one should know, how professional tasks should be performed, patterns of publication, etc. On the other hand, professional identity reflects personal attributes, such as values, worldviews and perceptions [9].

In our case, the construct of professional identity is in particularly complex because of the variety of communities of practices in which a ME researcher participates. Indeed, a professional identity of a ME researcher reflects many issues, such as mathematics curriculum and teaching methods in school and university, mathematics education curriculum and teaching methods for promoting prospective teachers and researchers, or research methods in ME. Thus, the construct of professional identity is also taken into consideration in our framework.

IV. The Proposed Framework

Our framework of researcher knowledge development in conducting a study in ME consists of three key components: *Germane knowledge*, *Accumulation processes* and *Catalyzing filters*.

A. Germane Knowledge

We associate the knowledge of a researcher with an elastic organism that dynamically changes. For exploring its structure, we focus on a particular element and analyze it in three dimensions: *source*, *kind* and *depth*. *Source* is a modification of Kennedy's categorization, which indicates from what community of practice a particular element of knowledge has originated [11]. We differentiate between three types of sources: *research setting*, *research group* and *public outlets*. Research setting is an environment of the study that was chosen and/or established by the researcher(s) for data collection. Research group is a closely-knit community of practice unified by a common goal - conducting a particular study. Public outlets, such as the World Wide Web, books, research journals and conferences, enable access to knowledge of a particular community of practice. Apparently, it is easier to recall the source of an element in a researcher's knowledge when it is new.

Researcher knowledge contains enormous amount of elements of different kinds. Noting a quote by Alfted Hitchcock that "ideas come from everything", we distinguish between three *kinds* of knowledge, which are in particular relevant for a study in ME: mathematical knowledge, pedagogical knowledge and methodological knowledge. Mathematical knowledge and pedagogical knowledge are adopted from [13], [14]. Methodological knowledge refers to everything related to carrying out a study: from philosophical and epistimological conceptions of research, through research paradigms and approaches, to desings, stages and techniques. Methodological knowledge also includes ethics as an unseparable research component, and technological methods for data collection and analysis.

By *depth*, we refer to a qualitative level of a researcher's understanding of a particular element at the time of depth evaluation. For instance, a researcher may have heard about the Soul conjecture in Riemann geometry, without knowing the details. The depth of this knowledge-element can be quickly increased by searching the web and discovering that the conjecture was proved by Grigori Perelman in 1994.

B. Accumulation Processes

We conceptualize the knowledge development of a researcher as a reorganization of her/his research knowledge, refinement of its elements or extention with the new ones. The development can occur as the result of three types of deeply related processes: *absorption*, *consolidating* and *sharing*.

In *absorption* processes a researcher focuses on particular elements in a research setting, in a research group or in public outlets and interprets them. In this way the elements get to be included in a researcher's knowledge utilized in a paricular study. This can happen when reading professional literature or listening to a conference lecture. In *consolidation* processes a researcher focuses on the relations between various elements of her/his knowledge and looks for connections, simillarities, differences, evidences and contraditions. These processes lead to (re)organization, systematization and refinement, when a researcher's knowledge functions as a self-contained system. In *sharing* a researcher is concerned with communicating her/his knowledge to others, for instance, when writing a paper for a research journal, preparing a presentation for a conference or teaching. Searching ways for sharing knowledge such that it is accessable to others may also lead to clarification and gaining new insights.

C. Catalyzing Filters

Conducting a study can be seen as a continuous process of decision making that has a direct impact on absorbing, consolidating and sharing knowledge: What papers should one chose to read when entering a new field? What ideas should be developed first? In what journals can particular results get published? We put forward three modalities that influence a researcher's decision making: *Norms and standards*, *Proffesional agenda* and *Opportunities*.

Norms and standards are socio-cultural contracts within a particular community of practice that reflect a common understanding regarding absorbing, consolidating and sharing knowledge. It is an aggregate of traditions, rituals, trends and fashions of the community. Some of the norms and standards can be in consensus of various communities of practices in which a researcher participates. For instance, a structure of a standard empirical research paper is a variation on "Introduction – Research goal(s) and question(s) – Theoretical background – Method (ology) – Results – Discussion" format. Some of the norms and standards vary significantly even in relatively close communities of practice. For example, in the call for papers to this conference the authors were instructed to "Do not publish "preliminary" data or results". In the call for papers to the 9th International Conference on Mathematical Giftedness and Creativity reports on research in progress are welcomed. Thus, when participating in a particular community of practice a researcher should be fluent with the specificity of its standards and norms.

Professional agenda is a part of professional identity of a researcher consisting of values and goals in regard to self-capacity and the ability to make a difference in ME. It also involves preferences and beliefs with respect to teaching mathematics and research approaches.

Opportunities refer to a concatenation of circumstances in the career of a researcher. It is a sequence of events that are partially controlled by a researcher and partially depend on chance. Examples of opportunities – that can be seized or missed – are an access to a rich research setting, exposure to a useful theory, or collaboration with a resourceful colleague.

The presented modalities have a dual nature: on the one hand, they prescribe particular decision making and limmit the researcher. On the other hand, an adequate analysis of the opportunities and norms of the community of practice can be exploited by a researcher for promoting her/his professional agenda. Thus, we refer to these modalities as *catalyzing filters*.

Fig. 1 schematically summarizes the proposed framework: The rectangles indicate sources of knowledge. The rings simbolize the catalyzing filters. Stright arrows represent absorption and sharing processes and a round arrow represents consolidation processes.

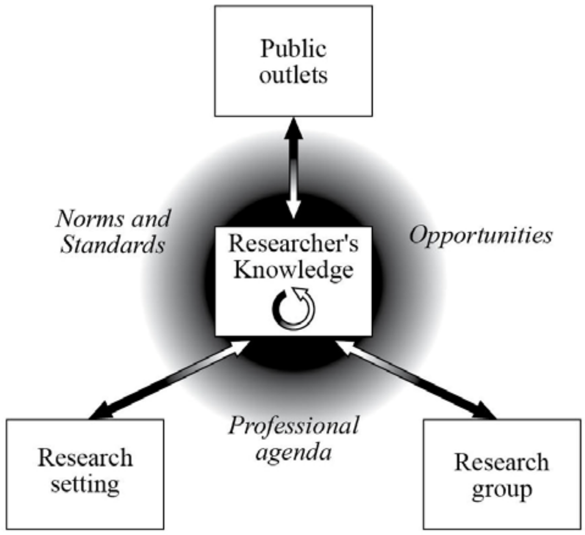

Fig. 1 A proposed framework of researcher knowledge development in conducting a study in ME

V. ILLUSTRATIONS

In the previous section we presented a framework of researcher knowledge development in carrying out a study in ME. In this section we use two illustrations to exemplify how this framework can be used in data analysis. The illustrations refer to the self-reflective interview with Professor Michèle Artigue, who was interviewed by Professor Alexander Karp. The interview is published in a book that comprises eleven interviews with most prominent researchers in ME [6].

Michèle Artigue is a Professor emeritus in the Université de Paris and a leading figure in the ME field. In 2013 she was awarded the Felix Klein Medal by the International Commission on Mathematical Instruction (ICMI) for outstanding lifetime achievements in ME research and development.

In the analysis of the illustrations we were guided by the question: How can the development of Artigue's research knowledge in conducting studies in ME be characterized in terms of the proposed framework?

Illustration I: From Rock Climbing to a New Research Direction

> "[...] I was in contact with Adrien Douady, who was a specialist in dynamic systems. [...D]uring the weekends, we used to climb rocks in the forest of Fontainebleau and Andrien was a member of our group of climbers. He was trying to introduce third year students at the University to the qualitative study of differential equations, and helped me discover this domain. At the IREM, we had very good computer equipment [...]. Adrien and his sister Véronique Gautheron [...] used it for drawing phase portraits and exploring the behavior of dynamic systems. I joined them and with Véronique prepared an exhibition of phase portraits of autonomous systems of order 2 and wrote a book presenting an elementary vision of the qualitative study of differential equations. I began to use it in a course for second year students specializing in biology and earth sciences. Then, with Marc Rogalski, who was creating an experimental section at the University of Lille and his colleagues, I developed a didactical engineering for first year students on the topic. It was implemented during several consecutive years, and systematically investigated. This is how I began to work on the didactic of analysis." [6, p. 18]

Analysis I

In this excerpt Artigue overviews a significant development in her research career that originated from a sequence of opportunities: participation in a rock climbing club, working in a well-equipped university, and networking. Artigue seized these opportunities and exploited them for collaborations and establishing research groups. Her decision to engage in undergraduate mathematics can be explained by a careful reading of the emerging flow in norms and standards of the ME research community. Indeed in those years (early1980s) the research on undergraduate mathematics education started to grow.

In research groups Artigue absorbed a new (for her) mathematical knowledge related to differential equations, consolidated it and shared with ME community through a book. Using the book in her pedagogical practice turned to a rich research setting for herself and for other scholars. Thus, it is an example of how research knowledge is shared with the community and how an opportunity for conducting new exploration is created by a researcher.

Illustration II: From Disagreement with the Ministry of Education to a Funded Research-Project

> "[...] I was asked by the Ministry of Education to join a group that was reflecting on the change that would be necessary if computer algebra systems (CAS) entered the secondary education. [...] I was not at all expert in CAS. [...] I observed their [group] work for about months and then we began working together.
>
> [...]After one year, we wrote a report for the Ministry of Education [basing on empirical data], showing that CAS technology had clear potential for mathematical learning, but that this potential was not easily actualized [...].

For instance, it was commonly claimed that, thanks to technology, students could avoid technical work and concentrate on conceptual and strategic activities, that the learning of algebraic techniques was no longer necessary. This was a big mistake from the instructional point of view. [...W]e tried to promote another vision: a vision based on the assumption that techniques play a crucial role in mathematics conceptualizations and that the relationship between techniques and concepts is really a dialectic relationship.

> [...] The results were not those that the Ministry was expecting but they were interested in the analysis and explanations. A second project, a bigger project, was launched [...] so we could develop our research about these issues, both theoretically and practically." [6, p.20].

Analysis II

This excerpt shows how as the result of extensive increase in the depth of her pedagogical knowledge related to CAS, Artigue succeeded to contribute in an already formed research group. The knowledge that was absorbed and consolidated from the research setting did not correspond to the expectations of the Ministry of Education. Nevertheless the group succeeded in sharing it in a way that fit the norms and standards of the Ministry and in such promoted research agenda of the group. Indeed, additional funding was granted and resulted in new research opportunities.

VI. Summary and Discussion

Dubinsky and McDonald offer a system of six criteria for evaluating a theory (or a framework) in ME from a theoretical and practical perspectives. They suggest that, "Theories in mathematics education can: (i) support prediction, (ii) have explanatory power, (iii) be applicable to a broad range of phenomena, (iv) help organize one's thinking about complex, interrelated phenomena, (v) serve as a tool for analyzing data, and (vi) provide a language for communicating ideas that go beyond superficial descriptions." [19, p. 275, numbering is added]. We choose to discuss our framework of researcher knowledge development in ME in light of these criteria.

The proposed framework is part of our ongoing research project conducted according to modified analytic induction approach [7]. Thus, the prediction power of the framework (criterion (i)) is still questionable and its investigation is a part of our research agenda. Indeed, we expect to refine and/or extend the framework based on the analysis of an extensive data corpus. However, some comments regarding the rest of the criteria can be offered already.

It can be argued that the criteria (ii), (iv) and (vi) are met at least partially as the components of the framework originated from the well-known structures and theories, which have been claimed to have the explanatory power, communicative benefits and usefulness. Moreover, the confluence nature of the framework emphasizes its innovation: From the perspective of Mathematics Education it is concerned with a barely explored population of researchers; from the perspective of Higher Education it puts forward the research discipline.

As an illustration of applicability we used the framework to analyze two episodes from the career of Professor Michèle Artigue. Apparently the analysis of several illustrations is insufficient to decide whether the framework fully meets the criteria (ii)-(vi). However, three remarks can be made in favor of the framework: First, it was used to analyze researcher knowledge development in conducting significantly different studies in ME. This suggests that the framework can meet criterion (iii). Indeed, the framework is designed to capture researcher knowledge development in conducting a particular study in ME. Second, the chosen constructs provided vocabulary to capture, at least in part, the researcher's knowledge development and explain her decision making. This supports the possibility of meeting criteria (ii), (iv) and (vi). Third, in our analysis we used *secondary data* collected by another researcher for another project. The fact that the proposed framework turned to be useful is an argument in favor of its research convenience and its power to capture the central components of the explored phenomenon.

Our concluding remark is directed towards the ME research community. The proposed framework highlights that the transference between (personal) researcher knowledge and the knowledge shared by the community as a whole is filtered through norms and standards, researcher agenda and opportunities. These modalities are rarely discussed in the education of prospective scholars and in the ME community in general. We suggest addressing these vague issues in an explicit form for promoting knowledge development in the community of ME.

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