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Analysis of Filtering in Stochastic Systems on Continuous-Time Memory Observations in the Presence of Anomalous Noises

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Abstract**—**For optimal unbiased filter as mean-square and in the case of functioning anomalous noises in the observation memory channel, we have proved insensitivity of filter to inaccurate knowledge of the anomalous noise intensity matrix and its equivalence to truncated filter plotted only by non anomalous components of an observation vector.

Keywords**—**Mathematical expectation, filtration, anomalous noise, memory.

I. Introduction

THE article considers the topical estimations of multidimensional dynamic systems, the behaviour of which is described through stochastic differential equations. In this paper some filter properties, the synthesis of which was carried out in [1] are investigated. Problems of casual processes estimation, as well as problems detecting anomalous noises are of both theoretical and practical interest. To measure channel models with memory unit, the multiplicity of which is based on suggested algorithm synthesis, is optimal as mean square does not exclude the filter - interpolation of the performance analysis of estimated algorithm.

II. Analysis of Sensitivity

Filter sensitivity determined by theorem in [1], to inaccurate knowledge of matrix of anomalous noise intensity is investigated using the technique [2], [3]. Suppose $\Theta^*(t)$is correct, i.e. $Q(t)$ - the noise intensity matrix used in the filter, and $\tilde{\mu}_r^0(\tau,t)$ - a real estimation error $\tilde{\mu}_r(\tau,t)$. The equation for $\tilde{\mu}_r(\tau,t)$follows from [1]

$$\dot{\tilde{\mu}}_r(\tau,t)=\tilde{F}(t)\tilde{\mu}_r(\tau,t)+\tilde{K}(t)[z_r(t)-H(t)\tilde{\mu}_r(\tau,t)], \quad (1)$$

where $z_r(t)$ - real observations with real matrix of intensity $\Theta^*(t)$ for $f(t)$,

$$\tilde{K}(t)=K(t)\tilde{Y}(t)=\begin{bmatrix}\tilde{K}_0(t)\\ \tilde{K}_1(t)\end{bmatrix}=\begin{bmatrix}K_0(t)\tilde{Y}(t)\\ K_1(t)\tilde{Y}(t)\end{bmatrix}=\begin{bmatrix}\tilde{H}_0^T(t)\tilde{R}^{-1}(t)\tilde{Y}(t)\\ \tilde{H}_1^T(t)\tilde{R}^{-1}(t)\tilde{Y}(t)\end{bmatrix}, \quad (2)$$

$$\tilde{R}(t)=R(t)+C\Theta(t)C^T, \quad (3)$$

where $\tilde{H}_0(t)$ and $\tilde{H}_1(t)$ are determined by (9), (10) in [1]. Process $\tilde{x}(\tau,t)=[x(t)\mid x(\tau)]$, as it follows from (1), (13), (15) in [1], is determined by

$$\dot{\tilde{x}}(\tau,t)=\tilde{F}(t)\tilde{x}(\tau,t)+\tilde{\omega}(t). \quad (4)$$

As

$$z_r(t)=H(t)\tilde{x}(\tau,t)+\tilde{\upsilon}(t), \quad (5)$$

where

$$\tilde{\upsilon}(t)=\upsilon(t)+Cf(t), \quad (6)$$

Then from (1), (4) it follows that an error $\tilde{\mu}_r^0(\tau,t)$ of real estimate $\tilde{\mu}_r(\tau,t)$ is determined by

$$\dot{\tilde{\mu}}_r^0(\tau,t)=\overline{F}_0(t)\tilde{\mu}_r^0(\tau,t)+\tilde{\omega}(t)-\tilde{K}(t)\tilde{\upsilon}(t), \quad (7)$$

where $\overline{F}_0(t)=\tilde{F}(t)-\tilde{K}(t)H(t)$. Then, similarly (29) in [1], the solution (5) is written as

$$\tilde{\mu}_r^0(\tau,t)=\overline{\Phi}(t,t_0)\tilde{\mu}_r^0(\tau,t_0)+\int_0^t\overline{\Phi}(t,t_0)\left[\tilde{\omega}(\sigma)-\tilde{K}(\sigma)\tilde{\upsilon}(\sigma)\right]d\sigma. \quad (8)$$

Hence, taking independence into account $x_{0,}$ $\omega(t)$, $\upsilon(t)$, $f(t)$ it follows that the matrix of second moment of real estimation error $\tilde{\Gamma}_r(\tau,t)=M\left\{\tilde{\mu}_r^0(\tau,t)\left(\tilde{\mu}_r^0(\tau,t)\right)^T\right\}$ is expressed by

$$\begin{aligned}\tilde{\Gamma}_r(\tau,t)&=\overline{\Phi}(t,t_0)M\left\{\tilde{\mu}_r^0(\tau,t_0)\left(\tilde{\mu}_r^0(\tau,t_0)\right)^T\right\}\overline{\Phi}^T(t,t_0)+\\&+\int_0^t\int\overline{\Phi}(t,\sigma)\left[M\left\{\tilde{\omega}(\sigma)\tilde{\omega}^T(\xi)\right\}+\tilde{K}(\sigma)M\left\{\tilde{\upsilon}(\sigma)\tilde{\upsilon}^T(\xi)\right\}\tilde{K}^T(\xi)\right]d\sigma d\xi.\end{aligned} \quad (9)$$

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According to [1]

$$M\{\tilde{\omega}(\sigma)\tilde{\omega}^T(\xi)\} = Q(\sigma)\delta(\sigma-\xi). \quad (10)$$

As $\upsilon(t)$ and $f(t)$ are independent in the problem formula and correct value of intensity matrix $f(t)$ is equal to $\Theta^*(t)$, then according to [1]

$$M\{\tilde{\upsilon}(\sigma)\tilde{\upsilon}^T(\xi)\} = R^*(\sigma)\delta(\sigma-\xi) + Cf_0(\sigma)f_0^T(\xi)C^T, \quad (11)$$

where $R^*(\sigma) = R(\sigma) + C\Theta^T(\sigma)C^T$. As $\tilde{K}(t) = K(t)\tilde{Y}(t)$, then after substitution (10), (11) in (9) taking unbiasedness property $\tilde{Y}(t)C = 0$ and properties of δ - Dirac function into account we obtain:

$$\tilde{\Gamma}_r(\tau,t) = \overline{\Phi}(t,t_0)\Gamma_r^0\overline{\Phi}^T(t,t_0) + \int_0^t \overline{\Phi}(t,\sigma)Q^*(\sigma)\overline{\Phi}^T(t,\sigma)d\sigma, \quad (12)$$

where

$$Q^*(t) = \tilde{Q}(t) + \tilde{K}(\sigma)R^*(t)\tilde{K}^T(\sigma). \quad (13)$$

Differentiating (12) over t results in

$$\dot{\tilde{\Gamma}}_r(\tau,t) = \overline{F}_0(t)\tilde{\Gamma}_r(\tau,t) + \tilde{\Gamma}_r(\tau,t)\overline{F}_0(t) + \tilde{K}(t)R^*(t)\tilde{K}^T(t) + Q(t). \quad (14)$$

Let us introduce and consider the sensitivity function

$$\Psi_{ij}(t) = \left.\frac{\partial\tilde{\Gamma}_r(\tau,t)}{\partial\Theta_{ij}^*}\right|_{\Theta_{ij}^*=\Theta_{ij}} \quad (15)$$

of second moment matrix $\tilde{\Gamma}_r(\tau,t)$ of real estimation error to (i,j)- matrix element of anomalous noise intensity. Then from (12) and (13) the following equation for $\Psi_{ij}(t)$ is

$$\dot{\Psi}_{ij}(t) = \tilde{K}(t)CI_{ij}C^T\tilde{K}^T(t),\ \Psi_{ij}(0) = O, \quad (16)$$

where I_{ij} Boolean $(r\times r)$ - matrix, which unit is situated at (i,j) while other elements are null. As $\tilde{K}(t) = K(t)\tilde{Y}(t)$, and taking unbiased property into account $\tilde{Y}(t)C = 0$ we obtain $\psi_{ij}(t) = 0$ for all $i = \overline{1;r}$, $j = \overline{1;r}$. Thus, we can state the following:

Theorem 1. Optimal unbiased filter as the mean-square synthesized in [1], is insensitive to the inaccurate knowledge of matrix of anomalous noise intensity.

Let us investigate the Bayesian unbiased filter with memory in terms of the sensitivity property relative to inaccurate knowledge of matrix of anomalous noise intensity. The filter is determined by the equation in [1]

$$\dot{\tilde{\mu}}(\tau,t) = \tilde{F}(t)\tilde{\mu}(\tau,t) + K(t)[z(t) - H(t)\tilde{\mu}(\tau,t) - Cf_0(t)]. \quad (17)$$

Then the real estimation for it is $\tilde{\mu}_r(\tau,t)$ and the error $\tilde{\mu}_r^0(\tau,t)$ will be determined by

$$\dot{\tilde{\mu}}(\tau,t) = \tilde{F}(t)\tilde{\mu}_r(\tau,t) + K(t)[z_r(t) - H(t)\tilde{\mu}_r(\tau,t) - Cf_0(t)], \quad (18)$$

$$\dot{\tilde{\mu}}_r^0(\tau,t) = \tilde{\tilde{F}}(t)\tilde{\mu}_r^0(\tau,t) + \tilde{\omega}(t) - K(t)[\upsilon(t) + C\tilde{f}(t)], \quad (19)$$

where $\tilde{\tilde{F}}(t) = \tilde{F}(t) - K(t)H(t), \tilde{f}(t) = f(t) - f_0(t)$, obtained similarly to (1), (7). In the case of introducing a transfer matrix $\tilde{\Phi}(t,\sigma)$, corresponding to matrix results in the following equation for $\tilde{\Gamma}_r(\tau,t)$ Bayesian filter with memory

$$\dot{\tilde{\Gamma}}_r(\tau,t) = \tilde{\tilde{F}}(t)\tilde{\Gamma}_r(\tau,t) + \tilde{\Gamma}_r(\tau,t)\tilde{\tilde{F}}^T(t) + K(t)R^*(t)K^T(t) + \tilde{Q}(t), \quad (20)$$

The derivation of this equation is similar to the derivation of (14). The equation for a sensitivity function (15) according to (20) is written as

$$\dot{\Psi}_{ij}(t) = K(t)CI_{ij}C^TK^T(t), \Psi_{ij}(0) = 0. \quad (21)$$

Thus,

$$\Psi_{ij}(t) = \int_0^t K(\tau)CI_{ij}C^TK^T(\tau)d\tau, \quad (22)$$

i.e. Bayesian filter with memory is insensitive to inaccurate knowledge of matrix of anomalous noise intensity.

III. Filter Structure

Suppose $i_1, i_2, ..., i_r$ - component numbers of the vector of observation $z(t)$, where the components $f_1(t), f_2(t), ..., f_r(t)$ of anomalous noises function $f(t)$. Suppose $\bar{z}(t)$ - is a vector of size $(l-r)$, which is obtained from observation vector $z(t)$ excluding anomalous components $z_{i_1}(t), z_{i_2}(t), ..., z_{i_r}(t)$. Suppose $\overline{H}_0(t), \overline{H}_1(t)$ are matrices of size $[(l-r)\times n]$, and $\overline{R}(t)$ is a matrix of size $[(l-r)\times(l-r)]$, which can be obtained from matrices $H_0(t)$, $H_1(t)$ excluding rows and correspondingly excluding rows and columns with numbers $i_1, i_2, ..., i_r$. Thus the observed process is of $\bar{z}(t)$ size of $(l-r)$ form

$$\bar{z}(t) = \overline{H}_0(t)x(t) + \overline{H}_1(t)x(\tau) + \bar{\upsilon}(t) = \overline{H}(t)\tilde{x}(\tau,t) + \bar{\upsilon}(t), \quad (23)$$

where $\overline{H}(t)=\left[\overline{H}_0(t) \,\vdots\, \overline{H}_1(t)\right]$, and $\overline{\upsilon}(t)$ is vector of size $(l-r)$, which is obtained from vector of regular noises excluding components with numbers $i_1, i_2, ..., i_r$, and will be free from anomalous noises. Process $\overline{z}(t)$ will be called the truncated vector of observations, and Bayesian optimal filter as mean-square, plotted by $\overline{z}(t)$, will be called the truncated filter.

Statement. The truncated filter is determined by

$$\dot{\mu}(t)=F(t)\overline{\mu}(t)+\tilde{\tilde{\overline{H}}}{}_0^T(t)\overline{R}^{-1}(t)\tilde{\tilde{\overline{z}}}(t), \tag{24}$$

$$\dot{\mu}(\tau,t)=\tilde{\tilde{\overline{H}}}{}_1^T(t)\overline{R}^{-1}(t)\tilde{\tilde{\overline{z}}}(t), \tag{25}$$

$$\dot{\overline{\Gamma}}(t)=F(t)\overline{\Gamma}(t)+\overline{\Gamma}(t)F^T(t)+Q(t)-\tilde{\tilde{\overline{H}}}{}_0^T(t)\overline{R}^{-1}(t)\tilde{\tilde{\overline{H}}}{}_0(t), \tag{26}$$

$$\dot{\overline{\Gamma}}_{11}(\tau,t)=-\tilde{\tilde{\overline{H}}}{}_1^T(t)\overline{R}^{-1}(t)\tilde{\tilde{\overline{H}}}{}_1(t), \tag{27}$$

$$\dot{\overline{\Gamma}}_{01}(\tau,t)=F(t)\overline{\Gamma}_{01}(\tau,t)-\tilde{\tilde{\overline{H}}}{}_0^T(t)\overline{R}^{-1}(t)\tilde{\tilde{\overline{H}}}{}_1(t), \tag{28}$$

where

$$\begin{aligned}\tilde{\tilde{\overline{z}}}(t)&=\overline{z}(t)-[\overline{H}_0(t)\overline{\mu}(t)+\overline{H}_1(t)\overline{\mu}(\tau,t)]=\\&=\overline{z}(t)-H(t)\tilde{\tilde{\overline{\mu}}}(\tau,t),\end{aligned} \tag{29}$$

$$\tilde{\tilde{\overline{H}}}{}_0(t)=\overline{H}_0(t)\overline{\Gamma}(t)+\overline{H}_1(t)\overline{\Gamma}_{01}^T(\tau,t), \tag{30}$$

$$\tilde{\tilde{\overline{H}}}{}_1(t)=\overline{H}_0(t)\overline{\Gamma}_{01}(\tau,t)+\overline{H}_1(t)\overline{\Gamma}_{11}^1(\tau,t). \tag{31}$$

This follows from the statement [1].

Theorem 2. The filter determined by the theorem in [1], and the truncated filter are equivalent.

Proof. It is evident that the truncated filter and the filter, determined in [1] are written as

$$\frac{d\tilde{\tilde{\overline{\mu}}}(\tau,t)}{dt}=\tilde{F}(t)\tilde{\tilde{\overline{\mu}}}(\tau,t)+\overline{K}(t)\tilde{\tilde{\overline{z}}}(t), \tag{32}$$

$$\begin{aligned}\frac{d\tilde{\tilde{\overline{\Gamma}}}(\tau,t)}{dt}&=\tilde{F}(t)\tilde{\overline{\Gamma}}(\tau,t)+\tilde{\overline{\Gamma}}(\tau,t)\tilde{F}^T(t)-\\&-\tilde{\overline{\Gamma}}(\tau,t)\overline{H}^T(t)\overline{R}^{-1}(t)\overline{H}(t)\tilde{\tilde{\overline{\Gamma}}}(\tau,t),\end{aligned} \tag{33}$$

$$\frac{d\hat{\mu}(\tau,t)}{dt}=\tilde{F}(t)\tilde{\mu}(\tau,t)+\tilde{K}(t)\tilde{z}(t), \tag{34}$$

$$\begin{aligned}\frac{d\tilde{\Gamma}(\tau,t)}{dt}&=\tilde{F}(t)\tilde{\Gamma}(\tau,t)+\tilde{\Gamma}(\tau,t)\tilde{F}^T(t)-\\&-\tilde{\Gamma}(\tau,t)H^T(t)\tilde{R}^{-1}(t)\tilde{Y}(t)H(t)\tilde{\Gamma}(\tau,t)+\tilde{Q}(t),\end{aligned} \tag{35}$$

where

$$\overline{K}(t)=\tilde{\tilde{\overline{\Gamma}}}(\tau,t)\overline{H}(t)\overline{R}^{-1}(t), \tag{36}$$

$$\tilde{K}(t)=K(t)\tilde{Y}(t), \quad K(t)=\tilde{\Gamma}(\tau,t)H^T(t)\tilde{R}^{-1}(t). \tag{37}$$

From (32)-(35) it follows that, the proof of this theorem can be reduced to proving relations

$$\overline{K}(t)\tilde{\tilde{\overline{z}}}(t)=\tilde{K}(t)\tilde{z}(t), \tag{38}$$

$$\overline{H}^T(t)\overline{R}^{-1}(t)\overline{H}(t)=H^T(t)\tilde{R}^{-1}(t)\tilde{Y}(t)H(t). \tag{39}$$

We first prove (39). Let us derive and consider Boolean $[(l-r)\times l]$ - matrix E, which is obtained from the identity matrix of size $(l\times l)$ excluding the rows with numbers $i_1, i_2, ..., i_r$. As $\overline{H}(t)=EH(t)$, $\overline{R}(t)=ER(t)E^T$, the proof of (39) can be reduced to

$$E^T\left[ER(t)E^T\right]^{-1}E=\tilde{R}^{-1}(t)\tilde{Y}(t). \tag{40}$$

As $\tilde{R}(t)=R(t)+C\Theta(t)C^T$ [1], then [4]

$$\tilde{R}^{-1}(t)=R^{-1}(t)-R^{-1}(t)C\left[\Theta^{-1}(t)+C^TR^{-1}(t)C\right]^{-1}C^TR^{-1}(t). \tag{41}$$

Multiplying both sides of (42) by C^T on the left and by C on the right, and then removing the right side according (41), we have

$$C^T\tilde{R}^{-1}(t)C=\left[\Theta(t)+\left[C^TR^{-1}(t)C\right]^{-1}\right]^{-1}. \tag{42}$$

From (42) it follows

$$\Theta(t)=\left[C^T\tilde{R}^{-1}(t)C\right]^{-1}-\left[C^TR^{-1}(t)C\right]^{-1}. \tag{43}$$

Multiplying both sides of (41) by C^T on the left and by C on the right and taking $\tilde{R}(t)=R(t)+C\Theta(t)C^T$ into account the result is

$$\tilde{R}(t)=R(t)+C\left[C^T\tilde{R}^{-1}(t)C\right]^{-1}C^T-C\left[C^TR^{-1}(t)C\right]^{-1}C^T. \tag{44}$$

Let us rewrite (44) as

$$\tilde{R}(t)-C\left[C^T\tilde{R}^{-1}(t)C\right]^{-1}C^T=R(t)-C\left[C^TR^{-1}(t)C\right]^{-1}C^T. \tag{45}$$

From (45), with regard to $\tilde{R}(t)\tilde{R}^{-1}(t)=I_l$, $R(t)R^{-1}(t)=I_l$ it follows

$$\begin{aligned}&\tilde{R}(t)\left[\tilde{R}^{-1}(t)-\tilde{R}^{-1}(t)C\left[C^T\tilde{R}^{-1}(t)C\right]^{-1}C^T\tilde{R}^{-1}(t)\right]\tilde{R}(t)=\\&=R(t)\left[R^{-1}(t)-R^{-1}(t)C\left[C^TR^{-1}(t)C\right]^{-1}C^TR^{-1}(t)\right]R(t).\end{aligned} \tag{46}$$

Suppose $\widetilde{\Psi}(t)$ - left side of (44). Using for $\widetilde{R}(t)$, as factors in a square bracket on the left and on the right in (46), (11) from [1], we have

$$\widetilde{\Psi}(t)=R(t)\left[\widetilde{R}^{-1}(t)-\widetilde{R}^{-1}(t)C\left[C^T\widetilde{R}^{-1}(t)C\right]^{-1}C^T\widetilde{R}^{-1}(t)\right]R(t), \quad (47)$$

Thus, from (46) it follows that

$$\begin{aligned}&\widetilde{R}^{-1}(t)-\widetilde{R}^{-1}(t)C\left[C^T\widetilde{R}^{-1}(t)C\right]^{-1}C^T\widetilde{R}^{-1}(t)=\\&=R^{-1}(t)-R^{-1}(t)C\left[C^TR^{-1}(t)C\right]^{-1}C^TR^{-1}(t).\end{aligned} \quad (48)$$

Using (48) in (40), taking into account (46), $\widetilde{Y}(t)=\left[I_l-CY(t)\right]$ from [1], we get the proof of (39) which can be reduced to the proof of identity

$$R(t)E^T\left[ER(t)E^T\right]^{-1}E+C\left[C^TR^{-1}(t)C\right]^{-1}C^TR^{-1}(t)=I_l. \quad (49)$$

Let us denote,

$$\begin{aligned}R(t)E^T\left[ER(t)E^T\right]^{-1}E=A_1,\\C\left[C^TR^{-1}(t)C\right]^{-1}C^TR^{-1}(t)=A_2\end{aligned} \quad (50)$$

According to the plotting of matrices C and E we get that $EC=O.$ using this property shows that for matrices A_1 and A_2 follows

$$A_1A_2=O, \quad A_2A_1=O. \quad (51)$$

For the ranks of arbitrary matrices A and B the properties are valid [4]

$$rk[AB]=rk\left[A^+AB\right]=rk\left[ABB^+\right] \quad (52)$$

Taking into account that invertible matrix are $D^+=D^{-1}$, we have a consistent application (52) to A_1 and A_2 .

$$rk[A_1]=rk\left[E^T\left[ER(t)E^T\right]^{-1}EE^+\right], \quad (53)$$

$$rk[A_2]=rk\left[C^+C\left[C^TR^{-1}(t)C\right]^{-1}C^T\right], \quad (54)$$

Since by construction E - matrix is with linearly independent rows and C - matrixis with linearly independent columns, then [5]

$$EE^+=I_{l-r}, \quad C^+C=I_r. \quad (55)$$

Using (55) and (52) in (53) and (54), we have

$$\begin{aligned}rk[A_1]=rk\left[E^T\right]=l-r,\\rk[A_2]=rk\left[C^T\right]=r.\end{aligned} \quad (56)$$

Hence

$$rk[A_1]+rk[A_2]=l. \quad (57)$$

From (48) we get that $A_1^2=A_1$, $A_2^2=A_2$, i.e. matrices A_1 and A_2 which are projection matrices [6]. Since the projection matrices fulfil the conditions of (51) and (56) have the property $A_1+A_2=I_l$, then taking into account (50) it proves (49), and also (39).

Let us prove (38). Since having proved (39), we have proved the equality $\widetilde{\widetilde{\Gamma}}(\tau,t)=\widetilde{\Gamma}(\tau,t)$ .From (36)-(38) it follows that, the proof of (38) is equivalent to the proof of the following relation

$$\overline{H}^T(t)\overline{R}^{-1}(t)\widetilde{\widetilde{z}}(t)=H^T(t)\widetilde{R}^{-1}(t)\widetilde{Y}(t)\widetilde{z}(t). \quad (58)$$

As

$$\widetilde{\widetilde{z}}(t)=E\widetilde{z}(t),\ \overline{H}(t)=EH(t),\ \overline{R}(t)=ER(t)E^T(t). \quad (59)$$

From (57) it follows that, the proof of (38) can be reduced to the proof of (40), which completes the proof of the Theorem.

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