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Effect of Different BER Performance Comparison of MAP and ML Detection

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***Abstract*—**In this paper, we regard as a coded transmission over a frequency-selective channel. We plan to study analytically the convergence of the turbo-detector using a maximum a posteriori (MAP) equalizer and a MAP decoder. We demonstrate that the densities of the maximum likelihood (ML) exchanged during the iterations are e-symmetric and output-symmetric. Under the Gaussian approximation, this property allows to execute a one-dimensional scrutiny of the turbo-detector. By deriving the analytical terminology of the ML distributions under the Gaussian approximation, we confirm that the bit error rate (BER) performance of the turbo-detector converges to the BER performance of the coded additive white Gaussian noise (AWGN) channel at high signal to noise ratio (SNR), for any frequency selective channel.

***Keywords*—**MAP, ML, SNR, Decoder, BER, Coded transmission.

I. Introduction

IN the field of communication equalization techniques, such as feed forward and feedback adaptive filters, have been established to be efficient in mitigating polarization mode dispersion (PMD) for optical communications systems [1], [2]. These equalizers are typically implemented by tapping delay lines, and their coefficients are modernized such that the mean square error (MSE) or a further error statistic is minimized. Maximum-likelihood detection based techniques, such as maximum likelihood sequence estimation (MLSE) or maximum a posteriori (MAP) detection, are recently anticipated for PMD mitigation [3]-[7]. Maximum likelihood sequence (MLS) estimator bases its judgment on the inspection of a sequence of receiving signals, and find for the best path through a trellis that maximizes the joint probability of receiving signals. MAP detector, on the other hand, makes decisions on a symbol-by-symbol basis and is best in the sense that it minimizes the probability of bit errors. Both MAP detector and MLS estimator are better-quality to equalizers that rely on error metrics such as the MSE, as they openly minimize the errors in a symbol or sequence [5], [9]. However, they require knowledge of channel characteristics as well as of noise statistics. Previous implementations of maximum-likelihood based equalization techniques for optical communication systems have important limitations that they rely on generating lookup tables through histograms for noise statistics, a very challenging task at the very low bit error rates (BER) of 10^-8 or less at which optical communications systems operate.

An important source of degradation in high data rate communication systems is the presence of Inter symbol interference (ISI) between consecutive data symbols which is due to the frequency selectivity of mobile radio channels. To combat the effects of ISI, an equalizer has to be used. The optimal equalizer, in the sense of minimum sequence error rate (SER) or bit error rate (BER) is based on maximum a posteriori (MAP) detection. We distinguish two criteria, MAP sequence detection and MAP symbol detection. When no a priori information on the transmitted data is available, MAP detection turns into maximum likelihood (ML) detection. Efficient algorithms exist for the MAP sequence detection, for example the SER optimizing Viterbi algorithm [9], and MAP symbol detection, for example the BER optimizing Bahl–Cocke–Jelinek–Raviv (BCJR) algorithm [12]. These algorithms are interesting since their complexity grows linearly rather than exponentially with the sequence size. The performance of both algorithms is almost In order to improve the quality of the transmission; an error correction code is generally used, together with an equalizer. At the receiver, a solution achieving a good complexity/performance tradeoff is to use an iterative receiver consisting of a soft-input soft-output (SISO) equalizer and a SISO decoder [11], following the idea of turbo-codes [10]. The basic idea behind iterative processing is to exchange extrinsic information among the equalizer and the decoder in order to achieve successively refined performance.

In this paper, we consider a turbo-detector composed of an equalizer and a decoder using the symbol MAP criterion [8]. A natural question concerns the achievable performance of such a turbo-detector. It is believed (due to simulations) that the scheme converges to the performance without ISI.

The aim of this work is to show that the BER performance of the turbo-detector converges to the BER performance of the coded AWGN channel at high SNR. This work looks at the issue of BER performance to compare MAP turbo coding with ML (maximum likelihood) detection and results also have shown Computation of PSD for Raised Cosine and Rectangular Pulse.

II. Power Spectrum Density (PSD)

Power spectral density (PSD) introduces a function in which the strength of the variations (energy) as a function of frequency. Means in other words, Power spectral density (PSD) indicates the strength of the variations is strong and weak at which frequency [14]. The PSD has unit of energy per frequency (width) and Come into possession of energy within a

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specific frequency range by integrating PSD within that frequency range. The method of FFT or computing autocorrelation function is done directly for the computation of Power spectral density (PSD) and then transforming it [15].

In statistical signal processing, the goal of spectral density estimation (SDE) is to estimate the spectral density (also known as the power spectral density) of a random signal from a sequence of time samples of the signal. Intuitively speaking, the spectral density characterizes the frequency content of the signal. The purpose of estimating the spectral density is to detect any periodicities in the data, by observing peaks at the frequencies corresponding to these periodicities [17].

SDE should be distinguished from the field of frequency estimation, which assumes that a signal is composed of a limited (usually small) number of generating frequencies plus noise and seeks to find the location and intensity of the generated frequencies. SDE makes no assumption on the number of components and seeks to estimate the whole generating spectrum.

Here we suppose a stationary stochastic process x(t). Our target to determine with on its property in frequency, the function of a real variable will have a Fourier transform [13].

$$\chi(f) = F\{\chi(t)\} = \int_{-\infty}^{\infty} \chi(t) e^{-2\lambda ft} dt \tag{1}$$

$$\chi(f) = \sum_{n=-\infty}^{\infty} \chi_{n^{e^{-2\lambda ifn}}}, -1/2 \leq f \geq 1/2 \tag{2}$$

$$\chi_n = F^{-1}\{\chi(f)\} = \int_{-1/2}^{1/2} \chi(t) e^{-2\lambda ifn} df \tag{3}$$

Since a problem there with the definitions that we had a just showed.

A. Maximum Likelihood and MAP

The estimation technique works by updating the previous probability of some parameters following the achievement of new information [19]-[22], [18]. The result is known as Posterior Probability distribution. In the current situation, the new information is a sample X of size n. The posterior probability density of the parameters, note $P(\theta \mid X)$, is known by applying Bayes' theorem:

$$P(\theta \mid X) = l(\theta \mid X) * \frac{P(\theta)}{P(X)} \tag{4}$$

in which $l(\theta \mid X)$ is the likelihood of the information under the supposed parameter set θ, and $P(\theta)$ models the prior information existing on the parameter θ (that is, $\{\gamma, \beta, \alpha\}$). The priors can be any type as long as they are a distribution model.

The probability of the observed data termed as $P(X)$, ensuring that the posterior distribution has an area of 1 of normalization. It is given by $\int_\theta l(\theta \mid X) P(\theta) d\theta$. The evaluating of this constant is occasionally clumsy because it frequently includes solving multiple integrals that can only be estimated numerically. The complication of its computing might describe why the Bayesian method is seldom used in Psychology and may describe the preference of social scientists for the simpler MLE method.

By dividing the P(X) and multiplying the two top surfaces, the new third result is achieved. This is the posterior distribution. The important is that the distribution in the left column has two long tails, one is in the increasing direction γ and other one in the increasing direction β, by using a right column, the tails are nearly non-existent. We will return to this when we inspect the effect of priors in a later section

The Posterior distribution can be summarized by calculating central tendency statistics. Frequently the mean is calculated. Though, the median and the mode are also potentially valuable in statistics and we will discuss that the mode is the most consistent for a weibull distribution. To locate the mode of the posterior distribution, to look for the maximum of the function over the three parameters θ can be executed:

$$\max_{\theta \in \Theta} P(\theta \mid X) = \max_{\theta \in \Theta} \frac{l(\theta \mid X) * P(\theta)}{P(X)} \tag{5}$$

Because $P(X)$ is a constant independent of the parameters, it can be dropped, so that the mode is equally well localized by (3).

$$\max_{\theta \in \Theta} l(\theta \mid X) * P(\theta) \tag{6}$$

where Θ is the domain of the parameters (i.e. $R^+ * R^+ * R$) for the parameters γ, β and α correspondingly). Equation (3) is known as the MAP estimator, in this situation where there is no prior, i.e. when all the parameters are equally likely, $P(\theta)$ becomes a constant and therefore can be dropped from the equation and see in (4).

$$\max_{\theta \in \Theta} l(\theta \mid X) \tag{7}$$

This above equation is accurately the MLE solution. It is an exceptional case of estimation if the mode is extracted from the posterior and if there is no prior. Equation (3) is more interesting, the MAP estimator, which is a look for the mode of the likelihood weighted by the priors. If we deliberate a search for the logarithm of $l(\theta \mid X) * P(\theta)$, we achieved

$$\max_{\theta \in \Theta} (\log l(\theta \mid \mathrm{X}) + \log \mathrm{P}(\theta)) \tag{8}$$

$$= \max_{\theta \in \Theta} (\log l(\theta \mid \mathrm{X}) + \log \mathrm{P}(\theta)) \tag{9}$$

B. Raised Cosine Spectrum

A family of spectra that satisfy the Nyquist Theorem is the raised cosine family whose spectra are [16]

$$Z(f)=\begin{cases} T_s \\ \frac{T_s}{2}\left\{1+\cos\left[\frac{\pi T_s}{\beta}\left(|f|-\frac{1-\beta}{2T_s}\right)\right]\right\} \\ 0 \end{cases} \tag{10}$$

$$0\le|f|\le\frac{1-\beta}{2T_s} \qquad ,\frac{1-\beta}{2T_s}\le|f|\le\frac{1+\beta}{2T_s}, \qquad |f|\le\frac{1-\beta}{2T_s}$$

where the parameter roll-off factor β is a real number in the interval $0\le\beta\le$ the bandwidth of the spectrum, Since the spectrum is zero for $|f|\ge\frac{1+\beta}{2T_s}$, The baseband pulse is $\frac{1+\beta}{2T_s}$. For bandpass QAM modulation, the bandwidth is twice

$$BW=\frac{1+\beta}{T_s}=(1+\beta)R_s \tag{11}$$

Whereas R_s is the transmitted symbol rate. The ideal Low-pass rectangular spectrum is the special case where $\beta=0$ which has a pass-band Bandwidth equal to the symbol rate.

The corresponding time domain signal is

$$z(t)=\frac{\cos\left(\lambda\beta\frac{t}{T_s}\right)}{1-\left(2\beta\frac{t}{T_s}\right)^2}\times\frac{\sin\lambda\frac{t}{T_s}}{\lambda\frac{t}{T_s}} \tag{12}$$

Hence examine that $z(t)$ has Zero-crossings at $t=\pm T_s,\pm 2T_s$....... The time series corresponding to special case $\beta=0$ (the ideal low-pass rectangular spectrum is $\sin\left(\frac{\lambda t}{T_s}\right)\Big/\left(\frac{\lambda t}{T_s}\right)$ just as expected.

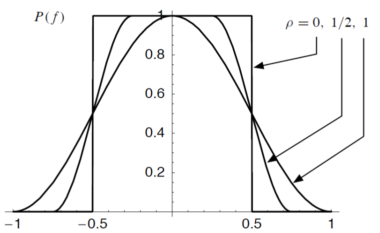

Fig. 1 Normalized Frequency f/w, RC spectrum, $p(f)$ for various values

III. Results and Discussion

The report shows a Bit Error Performance comparison of Maximum Likelihood detection scheme with Maximum A Posteriori detection scheme. The QAM symbols were mapped using two pulses, raised cosine and rectangular pulse. 10000 symbols were taken from the simulation. The transmitted data were passed through a channel gain and received at the receiver. The received signal was mixed with noise so we process the received signal through matched filter and then detected the signal using two detection schemes that are ML and MAP detection. The simulation results are as follows:

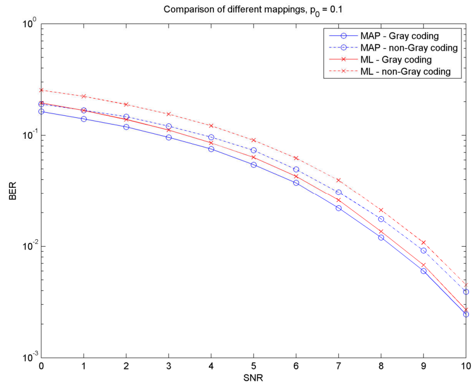

Fig. 2 Comparison of ML and MAP detection

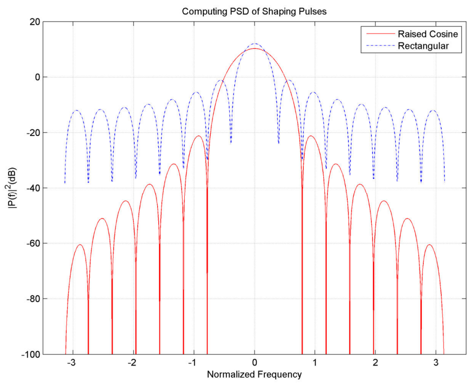

Fig. 3 Computation of PSD for Raised Cosine and Rectangular Pulse

IV. Conclusion

The result concludes performance of Gray coding for both detections is superior to non-Gray which is coded under the same conditions. The MAP detection works effectively as compared to the ML detection for different values of the SNR and enables us to achieve better performance with less Bit Error Rate. The raised cosine pulse achieves more attenuation with varying frequency and has a wider main lobe as compared to the rectangular pulse which varies quickly in its Power Spectral Density for the given frequency range section.

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