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Eccentric Connectivity Index, First and Second Zagreb Indices of Corona Graph

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Abstract**—**The eccentric connectivity index based on degree and eccentricity of the vertices of a graph is a widely used graph invariant in mathematics.

In this paper, we present the explicit eccentric connectivity index, first and second Zagreb indices for a Corona graph and sub division-related corona graphs.

Keywords**—**Corona graph, Degree, Eccentricity, Eccentric Connectivity Index, First Zagreb index, Second Zagreb index and Subdivision graphs.

I. Introduction

CRITICAL step in pharmaceutical drug design continues to be the identification and optimization of compounds in a rapid and cost effective way. An important tool in this work is the prediction of physic-chemical, pharmacological and toxicological properties of a compound directly and its molecular structure [2], [6]. This analysis is known as the study of the Quantitative Structure Activity Relationship (QSAR). In chemistry, a molecular graph represents the topology of a molecule, by considering how the atoms are connected. This can be modeled by a graph, where the points represent the atoms, and the edges symbolize the covalent bonds. Relevant properties of these graph models are studied, giving rise to numerical graph invariants. The parameters derived from this graph-theoretic model of a chemical structure are being used not only in QSAR studies pertaining to molecular design and pharmaceutical drug design, but also in the environmental hazard assessment of chemicals.

All the graphs considered in this paper are finite, undirected and simple. We refer [7] for terminology and notations. A *graph* $G=(V,E)$ is a set of finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set of G is denoted by $V(G)$, and the edge set is denoted by $E(G)$. The edge $e=\{u,v\}$ is said to join the vertices u and v. If $e=\{u,v\}$ is an edge of a graph G, then u and v are adjacent vertices, whereas u and e are incident, as are v and e [3], [7].

The *degree* of a vertex v in a graph G is the number of edges of G incident with v, which is denoted by $\deg_G(v)$ or simply by $\deg v$. A vertex of degree 0 in G is called an *isolated vertex* and a vertex of degree 1 is an *end vertex* of G. The *distance* $d_G(u,v)$ from a vertex u to a vertex v in a connected graph G, or simply $d(u,v)$, is the minimum of the lengths of the $u-v$ paths in G. The *eccentricity,* $ec(u)$ of a vertex $u \in V(G)$ is the maximum distance between u and any other vertex in G [1].

The eccentric connectivity index of any graph G is calculated using:

$$\xi^{(C)}(G)=\sum \deg(v)\cdot ec(v);\text{ where } v\in V(G)$$

Suppose that G is ``a connected graph, we define the subdivision related graphs namely subdivision graph $S(G)$ and the subdivision-related graph $R(G)$ [8] as follows:

Subdivision Graph: $S(G)$ is the graph obtained by inserting an additional vertex in each edge of G. Equivalently, each edge of G is replaced by a path of length 2.

Subdivision-related Graph: $R(G)$ is obtained from G by adding a new vertex corresponding to each edge of G, then joining each new vertex to the end vertices of the corresponding edge. Another way to describe $R(G)$ is to replace each edge of G by a triangle.

Corona Graph: Let $G_1(n_1,m_1)$ and $G_2(n_2,m_2)$ be two graphs. The Corona graph $G_1 * G_2$ is the graph obtained by taking n_1 copies of G_2 and for each i, making all vertices in the i^{th} copy of G_2 adjacent with neighbors of $v_i (i=1,2,...,n)$ of G_1 [4].

II. Eccentric Connectivity Index of Corona Graph

In this section, we derived an expression for the eccentric connectivity index of corona graph $C_n * K_m$ and the subdivision related Corona graph.

Theorem 1. Let C_n be a cycle with n vertices and K_1 be a complete graph with one vertex, then the eccentric connectivity index of the corona graph $C_n * K_1$ is

$$\xi^{(C)}(C_n * K_1)=\begin{cases}2n^2+3n, \textit{when } n\geq 3 \textit{ and } n \textit{ is odd}\\ 2n^2+5n, \textit{when } n>3 \textit{ and } n \textit{ is even}\end{cases}$$

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Proof: Here, C_n be a cycle with n vertices and K_1 be a complete graph with one vertex. The corona graph $C_n * K_1$ is the graph obtained by taking n copies of K_1 and then joining the i^{th} vertex of C_n to every vertex in the i^{th} copy of K_1.

The corona graph $C_n * K_1$ contains n vertices in the cycle C_n and n pendant vertices.

For all values of $n \geq 3$ and n is odd,

The eccentricity of n vertices is computed as $\frac{n+1}{2}$ and the eccentricity of the pendant vertices is computed as $\left(\frac{n+1}{2}+1\right)$.

For all values of $n > 3$ and n is even,

The eccentricity of n vertices is computed as $\frac{n}{2}+1$ and the eccentricity of the pendant vertices are computed as $\left(\frac{n}{2}+2\right)$.

Therefore, in general,

$$\xi^{(C)}(C_n * K_1) = \begin{cases} 2n^2+3n, & when\ n \geq 3\ and\ n\ is\ odd \\ 2n^2+5n, & when\ n > 3\ and\ n\ is\ even \end{cases}$$

Theorem 2. Let C_n be a n-cycle and K_m be a complete graph with m vertices, then the eccentric connectivity index of $C_n * K_m$ is given by

$$\xi^{(C)}(C_n * K_m) = \begin{cases} \frac{1}{2}\left[n(m+2)(n+1)+m(2n)(n+3)\right] \\ when\ n \geq 3\ and\ n\ is\ odd \\ \begin{cases} \frac{1}{2}\left[(m+2)n(n+2)+m(2n)(n+4)\right] \\ when\ n > 3\ and\ n\ is\ even \end{cases} \end{cases}$$

Proof: Let C_n be a n-cycle and K_m be a complete graph with m vertices. The graph $C_n * K_m$ is obtained by taking n copies of K_m and joining i^{th} vertex of C_n to every vertex in the i^{th} copy of K_m.

For all values of $n > 2, m > 0$, all vertices of C_n in $C_n * K_m$ have degree $(m+2)$. Each vertex of n copies of K_m has degree m.

For $m = 1$, by Theorem 1, the index is calculated as follows:

$$\xi^{(C)}(C_n * K_1) = \begin{cases} 2n^2+3n, when\ n \geq 3\ and\ n\ is\ odd \\ 2n^2+5n, when\ n > 3\ and\ n\ is\ even \end{cases}$$

For $m = 2$, the eccentricity of n vertices in C_n is computed as $\frac{n+1}{2}$ when $n > 2$ and n is odd. It is $\frac{n}{2}+1$ when $n > 3$ and n is even.

The eccentricity of $2n$ vertices of n copies of K_2 is $\left(\frac{n+1}{2}+1\right)$

It is $\frac{n}{2}+2$ when $n > 3$ and n is even.

Thus the eccentric connectivity index for $C_n * K_2$ is given by,

$$\xi^{(C)}(C_n * K_2) = \begin{cases} 4n^2+8n, when\ n > 2\ and\ n\ is\ odd \\ 4n^2+12n, when\ n > 3\ and\ n\ is\ even \end{cases}$$

Proceeding like this, for general $n\ and\ m$, the index for $C_n * K_m$ is obtained as follows:

For $n > 2$ and n is odd, $m > 0$, the degree of n vertices in C_n is $(m+2)$ and the eccentricity of n vertices in C_n is computed as $\frac{n+1}{2}$.The remaining vertices of n copies of K_m have degree m and eccentricity $\frac{n+1}{2}+1$

For $n > 3$ and n is even, $m > 0$, the degree of n vertices in C_n is $(m+2)$ and the eccentricity of n vertices in C_n is computed as $\frac{n}{2}+1$.The remaining vertices of n copies of K_m have degree m and eccentricity $\frac{n}{2}+2$

Therefore, in general, for any $n\ and\ m$, the index of $C_n K_m$ is computed as

$$= \begin{cases} \frac{1}{2}\left[n(m+2)(n+1)+m(2n)(n+3)\right] \\ when\ n \geq 3\ and\ n\ is\ odd \\ \begin{cases} \frac{1}{2}\left[(m+2)n(n+2)+m(2n)(n+4)\right] \\ when\ n > 3\ and\ n\ is\ even \end{cases} \end{cases}$$

III. First Zagreb Index of Corona Graph

The first Zagreb index [5] of the corona graph, the explicit expression for the values of first Zagreb index of subdivision graph of corona graph and for the subdivision-related corona graph are derived in this section.

Theorem 3. Let C_n and K_1 be two graphs, then the first Zagreb index of corona graph is $C_n * K_1$ is

$$M_1(C_n * K_1) = 10n, \text{ for all } n \geq 3$$

Proof: C_n and K_1 are two graphs with n vertices and single vertex respectively. The corona graph operation $C_n * K_1$ is the graph obtained by taking n copies of K_1 and then joining i^{th} vertex of C_n to every vertex in the i^{th} copy of K_1.

The corona graph $C_n * K_1$ contains n vertices in the cycle C_n and n pendant vertices. Let the vertices of C_n be labeled as $u_1, u_2, ..., u_n$ and the vertices of n copies of K_1 are labeled as $v_1, v_2, ..., v_n$ respectively. Therefore, the corona graph $C_n * K_1$ contains totally $2n$ vertices.

In general, for all the values of $n \geq 3$

Among $2n$ vertices, the n vertices of cycle C_n are of degree 3 and n vertices of K_1 are of degree 1.

Hence, the first Zagreb index of the corona graph $C_n * K_1$ is calculated as

$M_1(C_n * K_1) =$ degree of vertices of C_n + degree of vertices of n copies of K_1

$$= \sum_{i=1}^{n} (\deg u)^2 + \sum_{i=1}^{n} (\deg v)^2 = n(3^2) + n(1^2) = 9n + n = 10n$$

Theorem 4 Let C_n and K_1 be two graphs, then the first Zagreb index of subdivision graph of corona graph is

$$M_1(S(C_n * K_1)) = 18n \text{ for all } n \geq 3.$$

Proof: The subdivision graph of corona graph $S(C_n * K_1)$ contains n vertices in the cycle C_n and n pendant vertices. Let the vertices of C_n be labeled as $u_1, u_2, ..., u_n$, the vertices of n copies of K_1 are labeled as $v_1, v_2, ...v_n$ and the subdivision vertices are labeled as $e_1, e_2, ...e_n, e_{n+1}, ...e_{2n}$ respectively. Therefore, the corona graph $S(C_n * K_1)$ contains totally $4n$ vertices.

In general, for all the values of $n \geq 3$

Among $2n$ vertices, the n vertices of cycle C_n are of degree 3, n vertices of K_1 are of degree 1 and the subdivision vertices are of degree 2. Hence, the first Zagreb index of the corona graph $S(C_n * K_1)$ is calculated as

$M_1(S(C_n * K_1)) =$ degree of vertices of C_n + degree of vertices of n copies of K_1 + degree of subdivision vertices of

$$S(C_n * K_1) = \sum_{i=1}^{n} (\deg u)^2 + \sum_{i=1}^{n} (\deg v)^2 + \sum_{i=1}^{2n} (\deg e)^2$$

$$= n(3^2) + n(1^2) + 2n(2^2) = 9n + n + 8n = 18n$$

Theorem 5. Let C_n and K_1 be two graphs, then the first Zagreb index of subdivision – related graph of corona graph $C_n * K_1$ is

$$M_1(R(C_n * K_1)) = 48n \text{ for all } n \geq 3.$$

Proof: The subdivision – related graph of corona graph $R(C_n * K_1)$ contains n vertices in the cycle C_n and n pendant vertices. Let the vertices of C_n be labeled as $u_1, u_2, ..., u_n$, the vertices of n copies of K_1 are labeled as $v_1, v_2, ..., v_n$ and the subdivision vertices are labeled as $e_1, e_2, ...e_n, e_{n+1}, ..., e_{2n}$ respectively. Therefore, the corona graph $S(C_n * K_1)$ contains totally $4n$ vertices.

In general, for all the values of $n \geq 3$

Among $2n$ vertices, the n vertices of cycle C_n are of degree 6, n vertices of K_1 and the subdivision vertices are of degree 2. Hence, the first Zagreb index of the corona graph $R(C_n * K_1)$ is calculated as

$M_1(R(C_n * K_1)) =$ degree of vertices of C_n + degree of vertices of n copies of K_1 + degree of subdivision vertices of

$$R(C_n * K_1) = \sum_{i=1}^{n} (\deg u)^2 + \sum_{i=1}^{n} (\deg v)^2 + \sum_{i=1}^{2n} (\deg e)^2$$

$$= n(6^2) + n(2^2) + 2n(2^2) = 36n + 4n + 8n = 48n$$

IV. Second Zagreb Index Of Corona Graph

We determine the second Zagreb index of the corona graph $C_n * K_1$ in this section. We also derive the explicit expression for the second Zagreb index of subdivision graph of corona graph $C_n * K_1$ and also for the subdivision-related graph of corona graph $C_n * K_1$.

Theorem 6. Let C_n and K_1 be two graphs, then the second Zagreb index of corona graph $C_n * K_1$ is

$$M_2(C_n * K_1) = 12n \text{ for all } n \geq 3.$$

Proof: C_n and K_1 are two graphs with n vertices and one vertex respectively. The corona graph $C_n * K_1$ is the graph obtained by taking n copies of K_1 and then joining i^{th} vertex of C_n to every vertex in the i^{th} copy of K_1.

The corona graph $C_n * K_1$ contains n vertices in the cycle C_n and n pendant vertices. Let the vertices of C_n be labeled as $u_1, u_2, ..., u_n$ and the vertices of n copies of K_1 are labeled as $v_1, v_2, ..., v_n$ respectively. Therefore, the corona graph $C_n * K_1$ contains totally $2n$ vertices.

In general, for all the values of $n \geq 3$

Among $2n$ vertices, the n vertices of cycle C_n are of degree 3 and n vertices of K_1 are of degree 1. The corona graph $C_n * K_1$ contains n edges in the cycle C_n and n edges joining i^{th} vertex of C_n to every vertex in the i^{th} copy of K_1.

Hence, the second Zagreb index of the corona graph $C_n * K_1$ is calculated as

$$M_2(C_n * K_1) = \sum_{\substack{u_i, u_j \in E(G) \\ i \neq j}} (\deg u_i) \cdot (\deg(u_j) + \sum_{u_i, u_j \in E(G)} (\deg u_i) \cdot (\deg v_i)$$
$$= n(\deg u_i \cdot \deg u_j) + n(\deg u_i \cdot \deg v_j) = n(3 \cdot 3) + n(3 \cdot 1)$$
$$= 9n + 3n = 12n$$

Theorem 7 Let C_n and K_1 be two graphs, then the second Zagreb index of subdivision graph of corona graph is

$$M_2(S(C_n * K_1)) = 20n \text{ for all } n \geq 3.$$

Proof: C_n and K_1 are two graphs with n vertices and one vertex respectively. The subdivision graph of corona graph $S(C_n * K_1)$ contains n vertices in the cycle C_n and n pendant vertices. Let the vertices of C_n be labeled as $u_1, u_2, ..., u_n$, the vertices of n copies of K_1 are labeled as $v_1, v_2, ..., v_n$ and the subdivision vertices are labeled as $e_1, e_2, ..., e_n, e_{n+1}, ..., e_{2n}$ respectively. Therefore, the corona graph $C_n * K_1$ contains totally $4n$ vertices.

In general, for all the values of $n \geq 3$

Among $2n$ vertices, the n vertices of cycle C_n are of degree 3, n vertices of K_1 are of degree 1 and the subdivision vertices are of degree 2. Each edge in the cycle C_n and edges joining i^{th} vertex of C_n to every vertex in the i^{th} copy of K_1 are subdivided into 2 edges by introducing subdivision vertex e_i between the edges of the original graph. Hence there are totally 4n edges in $S(C_n * K_1)$.

Hence, the second Zagreb index of the corona graph $S(C_n * K_1)$ is calculated as

$$M_2(C_n * K_1) = \sum_{u,e \in E(G)} (\deg u) \cdot (\deg(e) + \sum_{v,e \in E(G)} (\deg v) \cdot (\deg e)$$
$$= 3n(3.2) + n(2.1) = 20n.$$

Theorem 8. Let C_n and K_1 be two graphs, then the second Zagreb index of subdivision – related graph of corona graph is

$$M_2(R(C_n * K_1)) = 88n \text{ for all } n \geq 3.$$

Proof: C_n and K_1 are two graphs with n vertices and single vertex respectively. The subdivision – related graph of corona graph $R(C_n * K_1)$ contains n vertices in the cycle C_n and n pendant vertices. Let the vertices of C_n be labeled as $u_1, u_2, ..., u_n$, the vertices of n copies of K_1 are labeled as $v_1, v_2, ..., v_n$ and the subdivision vertices are labeled as $e_1, e_2, ...e_n, e_{n+1}, ..., e_{2n}$ respectively. Therefore, the corona graph $S(C_n * K_1)$ contains totally $4n$ vertices.

In general, for all the values of $n \geq 3$

Among $2n$ vertices, the n vertices of cycle C_n are of degree 6. Each edge in the cycle C_n and edges joining i^{th} vertex of C_n to every vertex in the i^{th} copy of K_1 are subdivided into 2 edges by introducing subdivision vertex e_i between the edges of the original graph. Also the edges of original graph $C_n * K_1$ are also retained. Hence there are totally 6n edges in $R(C_n * K_1)$.

Hence, the second Zagreb index of subdivision related graph of the corona graph $R(C_n * K_1)$ is calculated as

$$M_2(R(C_n * K_1))$$
$$= \sum_{\substack{u_i, u_j \in E(G) \\ i \neq j}} (\deg u_i) \cdot (\deg(u_j) + \sum_{u_i, u_j \in E(G)} (\deg u_i) \cdot (\deg v_i)$$
$$+ \sum_{u,e \in E(G)} (\deg u) \cdot (\deg(e) + \sum_{v,e \in E(G)} (\deg v) \cdot (\deg e)$$
$$= n(6.6) + n(6.2) + 3n(6.2) + n(2.2)$$
$$= 36n + 12n + 36n + 4n = 88n.$$

V. CONCLUSION

The eccentric connectivity index for the corona graph $C_n * K_m$, the first and second Zagreb indices for the corona graph $C_n * K_1$ and its subdivision graph and subdivision related graphs are derived in this paper. Further research work can be carried on to obtain the eccentric connectivity index of subdivision and subdivision related corona graph $C_n * K_m$.

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