Optimal Placement of Phasor Measurement Units Using Gravitational Search Method

Satyendra Pratap Singh, S. P. Singh

Abstract—This paper presents a methodology using Gravitational Search Algorithm for optimal placement of Phasor Measurement Units (PMUs) in order to achieve complete observability of the power system. The objective of proposed algorithm is to minimize the total number of PMUs at the power system buses, which in turn minimize installation cost of the PMUs. In this algorithm, the searcher agents are collection of masses which interact with each other using Newton's laws of gravity and motion. This new Gravitational Search Algorithm based method has been applied to the IEEE 14-bus, IEEE 30-bus and IEEE 118-bus test systems. Case studies reveal optimal number of PMUs with better observability by proposed method.

Keywords—Gravitational Search Algorithm (GSA), Law of Motion, Law of Gravity, Observability, Phasor Measurement Unit.

I. INTRODUCTION

SEVERAL decisions are taken on the basis of available measurements in day to day operation and control of power systems. Introduction of phasor measurement units (PMUs) have facilitated producing time synchronized measurements of voltage and current signals facilitating real time synchronized measurements [1], [2]. The present and possible future applications of phasor measurement units have been well documented in [3]. As technology advances, the time frame of synchronized information has been steadily reduced from minutes to microseconds. However, determination of locations of PMUs in the system in order to get desired measurements with least number is a challenging job. For the dynamic view of power grid, synchrophasor measurement is the most important real time measuring device.

Several algorithms and approaches have been published in the literature for the optimal PMU placement (OPP) in power system. Initial work on PMU development and utilization has been reported by [2]-[4]. An algorithm which finds the minimal set of PMU placement needed for power system has been developed in [5], [6] where the graph theory and simulated annealing method have been used to achieve the goal. In [7] a strategic PMU placement algorithm is developed to improve the bad data processing capability of state estimation. Providing selected buses with PMUs can make the entire system observable. This will only be possible by proper placement of PMUs among the system buses. The authors in [8]-[13] developed an optimal placement algorithm for PMUs by using integer linear programming. In [13], author proposed binary integer programming for the OPP which provide the suitable constrained for the power systems with two adjacent injection measurements. In [14]-[16], a genetic algorithm is used to find out the optimal locations of PMUs. Reference [16] is a combination of Immunity algorithm and genetic algorithm. In [17], a tabu search method is proposed for the OPP problem in which augment incidence matrix is used for the observability analysis of PMU. A recursive tabu search method is suggested in [18] which is more superior than multiple tabu search. In [19], [20], a binary search method is proposed to find the minimum placement of PMUs.

In [21], authors presented a binary particle swarm optimization (PSO) methodology for optimal placement of PMUs when using a mixed measurement set. A modified binary PSO algorithm is suggested in [22] for optimal placement of PMUs for state estimation. In [23], author used an iterated local search method to minimize the size of the PMU configuration needed to observe the power system network. Authors in [24] proposed the multi-objective problem of PMU placement and used non-dominated sorting differential evolution algorithm based on pareto non-dominated sorting. An unconstrained nonlinear weighted least square algorithm is developed in [25] for OPP problem. In [26], OPP technique is proposed on the basis of the cost/benefit analysis.

A new heuristic Gravitational Search Algorithm (GSA) has been proposed in this investigation to minimize the number of PMUs in a power system. The Gravitational Search Algorithm is based on the Newton's Law of Gravity and mass interactions [27]. GSA has been found high quality performance in solving different optimization problems in the literature [28]-[30]. In GSA, agents are considered as objects and their performance is measured by their masses. The heavy masses correspond to good solutions and move more slowly and conversely light masses correspond to poor solutions and move toward heavy masses much faster [27]. The objective of proposed algorithm is to minimize the total number of PMUs at the power system buses, which minimize installation cost of the PMUs.

The paper is organized as follows: Section II gives the details of PMU technology in Power Systems and Section III states the problem of minimum PMU placement in the power system. Section IV explains the brief discussion of the GSA and presented the optimal PMU placement (OPP) by using GSA. Finally test results are given in Section V, and Section VI concludes the paper.

Satyendra Pratap Singh is with the Department of Electrical Engineering, Indian Institute of Technology (BHU) Varanasi, UP, India (e-mail: spsingh.rs.eee@iitbhu.ac.in).

S. P. Singh is with the Department of Electrical Engineering, Indian Institute of Technology (BHU) Varanasi, UP, India (e-mail: spsingh.eee@iitbhu.ac.in).

II. PMU TECHNOLOGY IN POWER SYSTEM

Phasor measurement technology was developed near the end of 1980s and the first product appeared in the market in the early 1990s. Phasor Measurement Units (PMUs) are used for Wide-Area Measurement System (WAMS) applications by power engineers and system operators as a time-synchronized tool. The PMUs measure time-synchronized voltage and current phasors that are time-stamped with high precision. They are equipped with Global Positioning Systems (GPS) receivers that allow the synchronization of the several readings taken at distant points. The PMUs at different buses send the data, through communication channels, to Phasor Data concentrators (PDC) and then PDC send that data to the control center for advance applications for power system control and protection, and for other applications, as shown in Fig. 1.

PMU-based measurements support real-time measurements of voltage and incident current phasors at observable system buses. The voltage phasors contain enough information to detect voltage-stability margin directly from their measurements.

III. OPTIMAL PMU PLACEMENT (OPP) PROBLEM

The OPP formulation based topological observability method finds a minimal set of PMUs such that a bus must be reached at least once by the PMUs. The optimal placement of PMUs for an N bus system is formulated as follows [9]:

$$Min \sum_{i=1}^{N} w_i x_i \qquad (1)$$

$$\text{Subject to, } AX \geq b \qquad (2)$$
$$X = [x_1 x_2 \ldots \ldots \ldots x_N]^T$$

where, N is total number of system buses, w_i is weight factor accounting to the cost of installed PMU at bus i, A is binary connectivity matrix of the system, X is a binary variable vector having elements x_i define possibility of PMUs on a bus i whose entries are defined as (3) and AX is a vector such that its entries are non-zero if the corresponding bus voltage is observable using the given measurement set and according to observability rules mentioned above. It ensures full network observability while minimizing the total installation cost of the PMUs, otherwise its entries are zero.

$$x_i = \begin{cases} 1 & \text{if a PMU is needed at bus } i \\ 0 & \text{otherwise;} \end{cases} \qquad (3)$$

The entries in A are defined as follows:

$$a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

and b is a vector whose entries are all ones as shown in (5).

$$b = [1\ 1 \ldots 1]^T \qquad (5)$$

After getting the optimal number of PMUs, expression for total observability (O_{total}) is taken from [11].
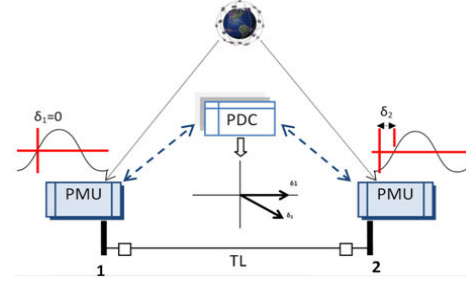

Fig. 1 PMU utilization in a power system

IV. GRAVITATIONAL SEARCH ALGORITHM (GSA)

Gravitational search algorithm is a stochastic algorithm that uses the concept of gravity and laws of motion to provide a suitable solution for an optimization problem. It is a well known fact that any two particles in the universe attract each other by a gravitational force directly proportional to the product of their masses and inversely proportional to the distance between them [27]. In GSA this concept is utilized along with the laws of motion, where agents are considered as objects and their performance is measured by their masses with gravitational force acting as a mode of communication between them.

A review of Newton's gravitational laws can gives a better understanding of GSA. Newton's first law of gravity can be stated mathematically as:

$$F = G\frac{M_1 M_2}{R^2} \qquad (6)$$

where, G is gravitational constant, F is the magnitude of the gravitational force, M_1 and M_2 are the masses of the 1st and 2nd particle respectively, and R is the distance between the two particles. Newton's second law says that a force applied to a particle is equal to the product of its mass and the particle acceleration (Ac). Mathematically:

$$Ac = \frac{F}{M} \qquad (7)$$

Both of the above mentioned laws can be rewritten as

$$F_{ij} = G\frac{M_{aj} \times M_{pi}}{R^2} \qquad (8)$$

$$Ac_i = \frac{F_{ij}}{M_{ii}} \qquad (9)$$

where M_{aj} represents the active gravitational mass (It is a measure of the strength of the gravitational field due to a particular object) of particle j and M_{pi} represents the passive gravitational mass (It is a measure of the strength of an object's interaction with the gravitational field) of particle i and F_{ij}, gravitational force that acts on mass i by mass j, is proportional to the product of the active gravitational mass of j and passive gravitational mass of i, and inversely proportional to the square of distance R between them. Also Ac_i (acceleration) is proportional to F_{ij} and inversely proportional to M_{ii}, inertial mass (It is a measure of an object resistance to change in its state of motion when a force is applied) of particle i.

Now, consider a system with Q agents (masses). We define the position of the i_{th} agent by:

$$X_i = \left(x_i^1, \ldots, x_i^d, \ldots, x_i^n\right) \text{ for i=1,2,3..,Q} \qquad (10)$$

where, x_i^d represents the position of i_{th} agent in the d_{th} dimension. At a specific time 't', the force acting on mass 'i' from mass 'j' can be defined as following:

$$F_{ij}^d = G(t)\frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon}\left(x_j^d(t) + x_i^d(t)\right) \qquad (11)$$

where M_{aj} is the active gravitational mass of agent j, M_{pi} the passive gravitational mass of agent i, $G(t)$ gravitational constant at time t, ε a small constant, and $R_{ij}(t)$ is the Euclidean distance between two agents i and j.

The total force acting on the i_{th} agent ($F_i^d(t)$) is calculated as follows:

$$F_i^d(t) = \sum_{j \in Kbest, j \neq i} rand_j F_{ij}^d(t) \qquad (12)$$

where $rand_j$ is a random number in the interval [0, 1] and k_{best} is the set of first K agents with the best fitness value and biggest mass. k_{best} is a function of time, with the initial value of K_0 at the beginning and decreasing with time. In such a way, at the beginning, all agents apply the force, and as time passes, k_{best} is decreased linearly and at the end there will be just one agent applying force to the others.

By the law of motion, the acceleration of agent i at time t, in direction d_{th} is given by:

$$Ac_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \qquad (13)$$

where M_{ii} is the inertial mass of i_{th} agent.

Furthermore, the next velocity of an agent is a function of its current velocity added to its current acceleration. Therefore, the next position and the next velocity of an agent can be calculated as follows:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + Ac_i^d(t) \qquad (14)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \qquad (15)$$

where $rand_i$ is a uniform random variable in the interval [0, 1]. The gravitational constant G, is initialized at the beginning of problem and will be decreased with time to control the search accuracy [27].

$$G(t) = G_o e^{-\alpha\frac{t}{T}} \qquad (16)$$

where α is a user specified constant, t is the current iteration and T is the total iterations. Gravitational and inertial masses are simply calculated by the fitness evaluation. Both of the masses namely gravitational and inertial are assumed to be equal and their values are calculated using the map of fitness. The gravitational and inertial masses are updated by using following equations:

$$M_{ai} = M_{pi} = M_{ii} = M_i \qquad (17)$$

where, i=1, 2, 3....,Q

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \qquad (18)$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{Q} m_j(t)} \qquad (19)$$

where $fit_i(t)$ represents the fitness value of the agent i at time t, and the *best(t)* and *worst(t)* in the population of agents respectively indicate the strongest and the weakest agent according to their fitness and can be defined as follows:

For a minimization problem:

$$best(t) = \min_{j \in \{1,..,Q\}} fit_j(t) \qquad (20)$$

$$worst(t) = \max_{j \in \{1,...,Q\}} fit_j(t) \qquad (21)$$

The detail procedure to apply the GSA based on Newton's Law of Gravity and Mass interactions for solving the OPP problem is as follow:

Step 1.   Read bus data and line data of the test system.
Step 2.   Obtain the connectivity matrix (A).
Step 3.   Initialize GSA parameters T, Q, G_o and α.
Step 4.   Identify the search space.
Step 5.   Generate initial population between minimum and maximum values of the control variables.
Step 6.   The fitness values of each agent in the population are calculated for the OPP problem.
Step 7.   Based on fitness value, update $G(t)$, best (t), worst (t) and M_i(t) for i=1, 2, ..., Q
Step 8.   Calculation of total force in different directions using (12).
Step 9.   Acceleration of each agent is modified using (13).
Step 10.   The velocity and position of each agent is updated using (14) and (15) respectively.
Step 11.   Repeat steps 6-10 until the stop criterion is reached.
Step 12.   Stop

V. CASE STUDY

The proposed formulation has been tested on IEEE 14-bus, IEEE 30-bus and IEEE 118-bus systems. In this paper, a simple case of power system has been applied to check the performance of proposed algorithm. The single line diagrams of IEEE 14-bus and IEEE 30-bus test systems obtained from DIgSILENT software are shown in Figs. 2 and 3 respectively. The simulations are executed on a computer having the following configuration: Intel core i7 CPU @ 3.40 GHz, 4 GB RAM.

Table I shows the chosen values of the parameters for the GSA used for the OPP problem solution. These values have been arrived at by various trials of OPP solutions on all the system tested. The values listed in Table I produced best performance in terms of finding the optimal solution and computational time.
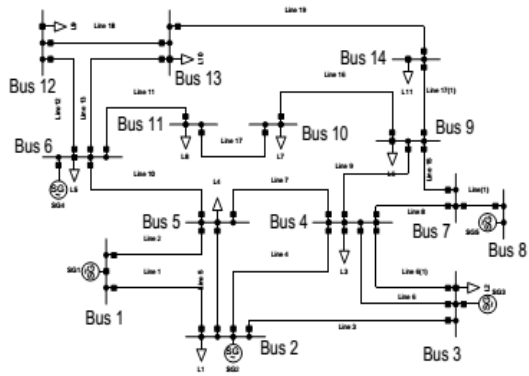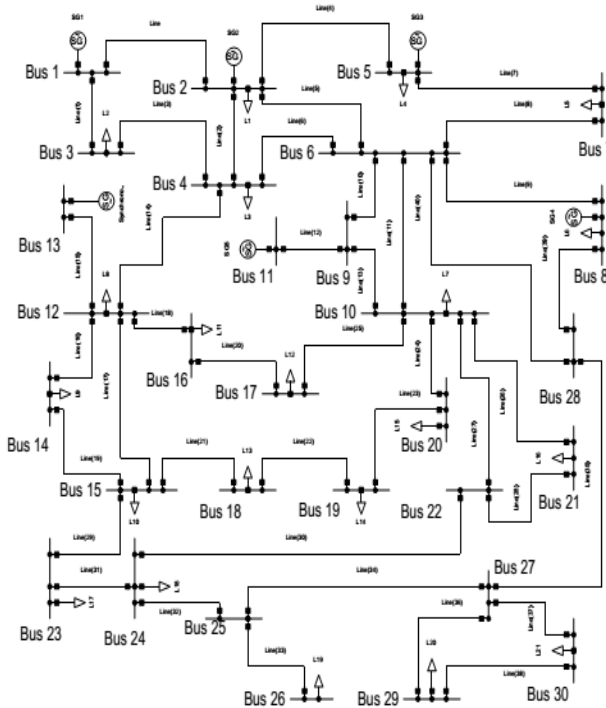
Fig. 2 IEEE 14-bus test system

Fig. 3 IEEE 30-bus test system

Table II shows the optimal number of PMUs and their locations which provide the full observability of power network. The convergence of GSA algorithm for IEEE 14-bus, IEEE 30-bus and IEEE 118-bus test systems by proposed method have been shown in Fig. 4. A steep decline in objective function value is observed in Fig. 4. It is observed from Fig. 4 that the GSA converged in ten iterations for IEEE 14-bus system and suggested four PMUs for observability of the system. Similarly IEEE 30-bus and IEEE 118-bus system converged in forty iterations and forty seven iterations and suggests ten PMUs and thirty two PMUs respectively.

TABLE I
GSA PARAMETER

G_0	α	$Iter_{max}$	Q
1	20	150	50

TABLE II
OPTIMAL LOCATION OF PMUS FOR THREE TEST SYSTEMS

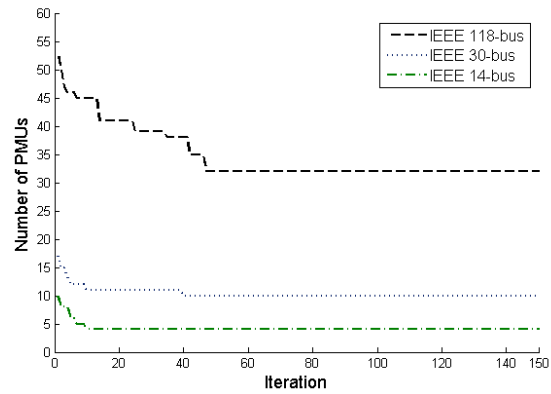| System configuration	Minimum no. of PMUs	Optimal PMU Locations
IEEE 14-bus	4	2, 6, 7, 9
IEEE 30-bus	10	2, 4, 6, 9, 10, 12, 15, 19, 25, 27
IEEE 118-bus	32	3, 5, 9, 11, 12, 17, 21, 25, 28, 34, 37, 41, 45, 49, 52, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114

Fig. 4 Convergence of GSA for all the test systems

The computation times of proposed OPP method for IEEE 14-bus, IEEE 30-bus and IEEE 118-bus systems are 0.61 sec, 0.76 sec, and 2.81 sec respectively. Fig. 4 also reveals that the number of iterations increases as the size of the system increases but it is not proportional to size of the system. The results of proposed algorithm have been compared with those obtained by the previously reported methods in the literature and shown in Table III. For all the test systems, minimum numbers of PMUs are same as the previously reported methods but the locations of PMUs by proposed method are different which provide the better observability of each bus as shown in Table IV. The total observability for IEEE 30-bus system produced by proposed method is 52 which is more than other methods.

TABLE III
COMPARISON OF OBTAINED RESULTS BY SEVERAL METHODS

Test System	IEEE 14-Bus	IEEE 30-Bus	IEEE 118-Bus
Proposed method	4	10	32
Generalized ILP [10]	4	10	--
Binary Search Algorithm [19]	4	10	--
WLS [25]	4	10	32

TABLE IV
COMPARISON OF OBTAINED RESULTS ON THE BASIS OF EACH BUS
OBSERVABILITY FOR IEEE 30-BUS

	Observability of each bus	Total observability
Proposed method	1 3 1 4 1 5 1 1 3 3 1 3 1 2 2 1 1 2 1 2 1 1 1 1 2 1 2 2 1 1	52
[10]	--------	--------
[19]	2 3 1 3 1 4 1 1 3 3 1 2 1 2 2 1 1 2 1 2 1 1 1 1 2 1 2 2 1 1	50
[25]	1 1 1 1 1 3 1 1 2 1 1 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	35

VI. CONCLUSION

This paper has proposed a Gravitational Search method for solving OPP problem, which minimizes the total number of PMUs for the complete observability of the power system. The method was applied on IEEE 14-bus, IEEE 30-bus and IEEE 118-bus test systems and its results are compared with other methods reported in the literature. The simulation results and fast convergence time indicate that the proposed algorithm satisfactorily provides observable system measurements with minimum number of PMUs. The observability of proposed method was found to be better than other methods.

REFERENCES

[1] V. Terzija, G. Valverde, D. Cai, P. Regulski, V. Madani, J. Fitch, S. Skok, M. M. Begovic, and A. Phadke, "Wide-Area Monitoring, Protection, and Control of Future Electric Power Networks*" Proc. of the IEEE*, Vol. 99, No. 1, Jan. 2011.

[2] A. G. Phadke, J. S. Thorp, and K. J. Karimi, "State Estimation with Phasor Measurements", *IEEE Transactions on Power Systems*, Vol. 1, No. 1, pp. 233- 241, February 1986.

[3] EPRI Final Rep., 1997 "Assessment of Applications and Benefits of Phasor Measurement Technology in Power Systems," *GE Power Syst. Eng.*,

[4] A. G. Phadke, "Synchronized phasor measurements in power systems", *IEEE Computer Applications in Power*, Vol. 6, Issue 2, pp. 10-15, April 1993.

[5] L. Mili, T. Baldwin and R. Adapa, "Phasor Measurement Placement for Voltage Stability Analysis of Power Systems." Proceedings of the 29th Conference on Decision and Control, Honolulu, Hawaii, Dec. 1990.

[6] T. L. Baldwin, L. Mili, M. B. Boisen, and R. Adapa, "Power System Observability With Minimal Phasor Measurement Placement", *IEEE Transactions on Power Systems,* Vol. 8, No. 2, pp. 707-715, May 1993.

[7] J. Chen and A. Abur, "Placement of PMUs to Enable Bad Data Detection in State Estimation." *IEEE Trans. on Power Systems*, Vol. 21, No. 4, pp. 1608-1615, Nov. 2006.

[8] B. Xu, A. Abur, "Observability analysis and measurement placement for systems with PMUs." *Proceedings of 2004 IEEE PES Conference and Exposition*, vol.2, pp: 943-946, 10-13 Oct. 2004.

[9] B. Gou, "Optimal placement of PMUs by integer linear programming," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1525–1526, Aug. 2008.

[10] B. Gou, "Generalized integer linear programming formulation for optimal PMU placement" IEEE Trans. Power Syst., 23: 1099-1104, Aug. 2008.

[11] S. P. Singh, S. P. Singh, "Optimal PMU Placement in Power System Considering the Measurement Redundancy*", Int. J. of Advances in Electronic and Electric Engineering*, vol. 4, no. 6, pp. 593-598, Jan. 2014.

[12] Sadegh Azizi, Ahmad Salehi Dobakhshari, S. ArashNezam Sarmadi, and Ali Mohammad Ranjbar, "Optimal PMU placement by an equivalent linear formulation for exhaustive search",*IEEE Trans. On Smart Grid*, Vol. 3, No. 1, pp. 174-182, March 2012.

[13] S. Mehdi Mahaei, M. Tarafdar Hagh, "Minimizing the number of PMUs and their optimal placement in power systems", Elect. Power Syst. Research, Vol. 83, pp. 66-72, 2012.

[14] B. Milosevic and M. Begovic, "Nondominated Sorting Genetic Algorithm for Optimal Phasor Measurement Placement." *IEEE Trans. On Power Systems*, vol. 18, No. 1, pp. 69-75, Feb. 2003.

[15] F. J. Marın, F. Garcıa-Lagos, G. Joya, and F. Sandoval, "Genetic algorithms for optimal placement of phasor measurement units in electric networks," Electron. Lett., vol. 39, no. 19, pp. 1403–1405, Sep. 2003.

[16] F. Aminifar, C. Lucas, A. Khodaei, and M. Fotuhi, "Optimal placement of phasor measurement units using immunity genetic algorithm," *IEEE Trans. Power Del.*, vol. 24, no. 3, pp. 1014–1020, Jul. 2009.

[17] J. Peng, Y. Sun, and H. F. Wang, "Optimal PMU placement for full network observability using Tabu search algorithm," *Elect. Power Syst.Res.*, vol. 28, pp. 223–231, 2006.

[18] N. C. Koutsoukis, N. M. Manousakis, P. S. Georgilakis, G. N. Korres, "Numerical observability method for optimal phasor measurement units placement using recursive Tabu search method", *IET Gener. Transm. Distrib.*, Vol. 7, Iss. 4, pp. 347–356, 2013.

[19] S. Chakrabarti, and E. Kyriakides, "Optimal Placement of Phasor Measurement Units for Power System Observability" *IEEE Trans. Power Syst.,* vol. 23, no. 3, pp. 1433-1440, Aug. 2008.

[20] R. J. Albuquerque and V. L. Paucar, "Evaluation of the PMUs Measurement ChannelsAvailability for Observability Analysis*", IEEE Trans. On Power Syst.*, Vol. 28, No. 3, Aug. 2013.

[21] A. Ahmadi, Y. A. Beromi, M. Moradi, "Optimal PMU placement for power system observability using binary particle swarm optimization and considering measurement redundancy", *Expert Systems with Applications*,Vol. 38, pp. 7263–7269, 2011.

[22] M. Hajiana, A. M. Ranjbar, T. Amraee, B. Mozafari, "Optimal placement of PMUs to maintain network observability using a modified BPSO algorithm", *Elec. Power & Energy Sys.*Vol. 33,pp. 28–34, 2011.

[23] M. Hurtgen and J.-C. Maun, "Optimal PMU placement using iterated local search", *Elec. Power & Energy Sys.*Vol. 32,pp. 857–860, 2010.

[24] C. Peng, H. Sun and J. Guo, "Multi-objective optimal PMU placement using a non-dominated sortingdifferential evolution algorithm", *Elec. Power & Energy Sys.*Vol. 32, pp. 886–892, 2010.

[25] N. M. Manousakis and G. N. Korres, "A Weighted Least Squares Algorithm for Optimal PMU Placement", *IEEE Trans. On Power Syst.*, Vol. 28, No. 3, Aug. 2013.

[26] F. Aminifar, M. Fotuhi-Firuzabad, and A. Safdarian, "Optimal PMU Placement Based on Probabilistic Cost/Benefit Analysis" *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 566-567, Feb. 2013.

[27] E. Rashedi, H. Nezamabadi-pour, S. Saryazdi, "A gravitational search algorithm", *Information Sciences*, vol. 179, pp. 2232-2248, 2009.

[28] S. Duman, U. Guvenc, N. Yorukeren, "Gravitational search algorithm for economic dispatch with valve-point effects" *IREE. Vol. 5, N. 6*, pp. 2890-2995, Dec. 2010.

[29] H. R. Hassanzadeh, M. Rouhani, "A multi-objective gravitational search algorithm" *Second ICCI, communication systems and networks conference*, pp. 7–12, 2010.

[30] C. Li, J. Zhou, "Parameters identification of hydraulic turbine governing system using improved gravitational search algorithm" *Energy Convers Manage*, vol. 52, Issue 1, pp. 374–381, Jan. 2011.