

# Design and Development of Constant Stress Composite Cantilever Beam

Vinod B. Suryawanshi, Ajit D. Kelkar

**Abstract**—Composite materials, due to their unique properties such as high strength to weight ratio, corrosion resistance, and impact resistance have huge potential as structural materials in automotive, construction and transportation applications. However, these properties often come at higher cost owing to complex design methods, difficult manufacturing processes and raw material cost. Traditionally, tapered laminated composite structures are manufactured using autoclave manufacturing process by ply drop off technique. Autoclave manufacturing though very powerful suffers from high capital investment and higher energy consumption. As per the current trends in composite manufacturing, Out of Autoclave (OoA) processes are looked as emerging technologies for manufacturing the structural composite components for aerospace and defense applications. However, there is a need for improvement among these processes to make them reliable and consistent. In this paper, feasibility of using out of autoclave process to manufacture the variable thickness cantilever beam is discussed. The minimum weight design for the composite beam is obtained using constant stress beam concept by tailoring the thickness of the beam. Ply drop off techniques was used to fabricate the variable thickness beam from glass/epoxy prepregs. Experiments were conducted to measure bending stresses along the span of the cantilever beam at different intervals by applying the concentrated load at the free end. Experimental results showed that the stresses in the beam at different intervals were constant. This proves the ability of OoA process to manufacture the constant stress beam. Finite element model for the constant stress beam was developed using commercial finite element simulation software. It was observed that the simulation results agreed very well with the experimental results and thus validated design and manufacturing approach used.

**Keywords**—Beams, Composites, Constant Stress, Structures.

## I. INTRODUCTION

IN the recent past, demand for lightweight and high strength materials has resulted into an extensive use of advanced composite materials for variety of different structural applications including aerospace, energy, automotive, sports industries etc. [1]. Among the different classes of composite materials, fiber reinforced polymer (FRP) composites are to date the most popular and perhaps the most versatile ones. Different processing technologies such as the hand lay-up, filament winding, compression molding, pultrusion, autoclave, Resin Transfer Molding (RTM), Vacuum Assisted Resin Transfer Molding (VARTM), and Sheet Molding Compound (SMC) are regularly used for the manufacturing of FRP [2].

Vinod B. Suryawanshi is with the Nanoengineering Department, Joint School of Nanoscience and Nanoengineering Greensboro, NC 27401, USA (e-mail: vbsuryaw@aggies.ncat.edu).

Ajit D. Kelkar is with the Nanoengineering Department, Joint School of Nanoscience and Nanoengineering Greensboro, NC 27401, USA (e-mail: ajitkelkar@gmail.com).

Many industries are willing switch to Fiber Reinforced Plastics (FRPs) from metals and other materials because composite materials are markedly superior to other materials. However, manufacturing of composites is very different than that for metals and hence, many companies struggle with realizing the full benefits of composites structures. Improvements in manufacturing techniques for fiber reinforced polymer composite materials and structures are needed to meet the cost and performance targets to enable wider adoption across different type of industries [3]. Due to benefits such as less capital investment, low operating costs, and ability to produce large size components, Out of autoclave (OoA) processes with vacuum-bag only prepregs, are emerging as the leading composite manufacturing technology. Manufacture of tapered composite structures is challenging task. Tapered structures used for aerospace applications are usually manufactured to good level of accuracy by ply drop off technique using autoclave process. However, there is need to explore the possibility of using ply drop off method with OoA process to manufacture good quality tapered structures.

Composite structures are subjected different types of loading conditions such as tensile, flexural, torsion and fatigue etc. Cantilever beam structures with distributed or end loads are very common in construction and transportation applications. Usually, the composite cantilever structures are manufactured with constant cross section along the axis of the beam. Shape optimization of structures helps to find the shape which is optimal in the sense that it minimizes a certain cost functional while satisfying given constraints [4]. Both, analytical [5] and numerical methods [6] have been used for optimization of cantilever beam. Minimum weight design for cantilever beam can be obtained by varying the thickness along the span with maximum at fixed end and minimum at free end [7].

This paper presents minimum weight composite cantilever beam design using constant stress approach. Classical beam theory [8] was used to tailor the thickness of over the span cantilever beam keeping stress as constant. Out of autoclave process was then used to manufacture proposed variable thickness laminated composite cantilever beam. Finally, finite element modeling of the beam is presented to validate the design methodology and manufacturing process.

## II. DESIGN OF CONSTANT STRESS BEAM

A cantilever beam with uniform cross section subjected to point load at the free end has maximum and minimum bending stresses at fixed and free respectively. The weight of the beam can be reduced by tailoring the thickness of the beam keeping

the bending stress under permissible limit over the span of the beam. From beam theory, analytical solution for cantilever beam with variable thickness is given by (1):

$$t = \sqrt{\frac{6P(L-x)}{b\sigma}} \quad (1)$$

where, L is span of the beam, P is the end load, x is axial distance from the fixed end, b is the width of beam cross section and  $\sigma$  is the allowable bending stress. Using (1) a constant stress cantilever beam for a design load of 22.5 N and allowable bending stress of 35 N/mm<sup>2</sup> was designed. The length (L) and width (b) of the beam fixed at 254 mm and b=25.4 mm respectively and thickness (t) across the length was calculated.

### III. EXPERIMENTAL WORK

#### A. Materials

Unidirectional glass epoxy preregs, purchased from Mitsubishi Rayon, Carbon fibers and Composites, California, USA are used to manufacture the constant stress cantilever beam.

#### B. Fabrication of Composite Panel

Composite panel of dimensions 300 mm x 125 mm with thickness variable thickness along the longer dimension was fabricated from glass/epoxy preregs using out of autoclave process. Ply drop off technique [9] was used to achieve thickness variation as per the design calculations described in previous section. Each ply was debulked for 5 minutes, followed by vacuum bagging the layup as per the bagging sequence shown in Fig. 1. The vacuum bag is cured under pressure of 29 mm of hg as per the preregs manufacturer recommended cure cycle of 250 °C for 3 hours with ramp rate of 5°C/min. Minimum and maximum thickness of the panel measured as 1.524 mm and 6.528 mm was achieved using 4 and 39 number of plies respectively. Cantilever beam specimen shown in Fig. 3 with size 280 mm x 25.4 mm was cut using water jet cutting machine from the panel.

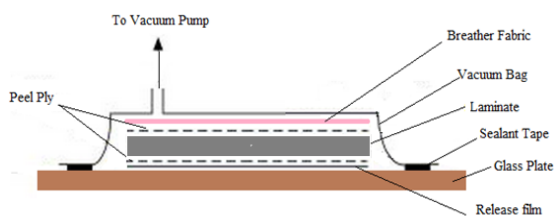


Fig. 1 Layup sequence for vacuum bagging

#### C. Testing of Composite Beam

The objective of the test is to measure the bending stresses in the cantilever beam, at the different interval along the span of the beam. Four strain gauges each as shown in schematic in Fig. 2 were mounted on top and bottom face of the beam respectively. Fig. 3 shows actual test specimen with strain gauges mounted on the composite beam. The data acquisition

system consists of A2 signal conditioning and amplifying system (Micro-Measurements, a Vishay Precision Group, USA) and the computer. The strain gauges are connected to the A2 signal conditioning unit, and the signal condition unit is interfaced with the computer using ether net cable. The voltage readings corresponding to each strain gauge can be recorded through the graphical user interface (GUI) on computer.

The cantilever beam was tested under four different end load conditions 9.81N, 19.62 N, 29.43 N and 39.24 N. Excitation voltage of 5 mV was applied. Table I shows the experimental data consisting of output voltage for eight strain gauges at the different applied loads. The experimental setup used for the purpose is shown in Fig. 4.



Fig. 2 Schematic showing numbering of strain gauges



Fig. 3 Test specimen for measuring bending stresses



Fig. 4 Test setup

#### D. Material Properties of Glass/Epoxy Composite Material

To calculate bending stresses in the beam using measured strains during the experiment and to build the finite element model for the cantilever beam under consideration, material properties are required. Unidirectional glass/epoxy composite specimens were prepared as per ASTM standard D3039 (standard test method for tensile properties of polymer matrix

composite materials) [10]. The material properties obtained during the testing of specimen, along with material properties from the literature [11] are shown Table I.

TABLE I  
MEASURED VOLTAGE FOR STRAIN GAUGES AT DIFFERENT APPLIED LOADS,  
MILLIVOLT

Strain Gauge	Applied Load, N			
	9.81	19.62	29.43	39.24
1	0.678	1.342	2.008	2.667
2	0.626	1.241	1.855	2.458
3	0.68	1.349	2.012	2.661
4	0.783	1.556	2.317	3.062
5	0.783	1.548	2.318	3.082
6	0.72	1.425	2.135	2.837
7	0.801	1.582	2.365	3.133
8	0.851	1.693	2.526	3.344

TABLE II  
MATERIAL PROPERTIES OF GLASS/EPOXY COMPOSITE

Property	Unit	Value
Modulus of Elasticity, $E_{xx}$	N/m <sup>2</sup>	$4.16 \times 10^{10}$
Modulus of Elasticity, $E_{yy}$	N/m <sup>2</sup>	$1.05 \times 10^{10}$
Modulus of Elasticity, $E_{zz}$	N/m <sup>2</sup>	$1.05 \times 10^{10}$
Modulus of Rigidity, $G_{xx}$	N/m <sup>2</sup>	$3.15 \times 10^9$
Modulus of Rigidity, $G_{xz}$	N/m <sup>2</sup>	$4.61 \times 10^9$
Modulus of Rigidity, $G_{yz}$	N/m <sup>2</sup>	$4.61 \times 10^9$
Poisson's ratio, $\nu_{12}$	--	0.31
Poisson's ratio, $\nu_{13}$	--	0.33
Poisson's ratio, $\nu_{23}$	--	0.33

#### IV. FINITE ELEMENT MODELING

The Finite Element model for the constant stress beam is developed using the commercial finite element software ANSYS. Orthotropic material properties used for the model are given in Table II. Two node beam element, BEAM 188 was used. The dimensions of the beam are obtained from the test specimen fabricated for measurement of bending stresses in the beam. The thickness of the beam was measured at 10 equal intervals along the length. The beam with 10 tapered sections is modeled. Single element per section was used to mesh the model. Fixed boundary conditions were applied at one end, while concentrated load was applied at the other end as shown in Fig. 5. Simulation results were obtained for different values of the applied loads similar to the one used during the experiments.

#### V. RESULTS AND DISCUSSION

##### A. Experimental Results

From the experimental data shown in Table I, the percentage strain at each strain gauge locations is computed. The stresses in the beam are obtained using the constitutive stress-strain relation using Modulus of Elasticity (Table II) along the longitudinal direction of the beam. The computed bending stresses on top face of the beam and obtained from strain gauges 1-4, are shown in Table III while, Table IV shows the bending stresses at the bottom face of the beam.

From the stress data shown in Tables III and IV it can be seen that there is very little variation in the stresses across the length of the beam. The maximum deviation of the stresses was found to be 9.6%. The deviation can be due to the staircase like structure produced due the nature of the manufactured process. This can be minimized by reducing the step size used for increase of the thickness during layup of the prepregs. During this study the step size was used as 25 mm, i.e. thickness variation was achieved in 10 steps.

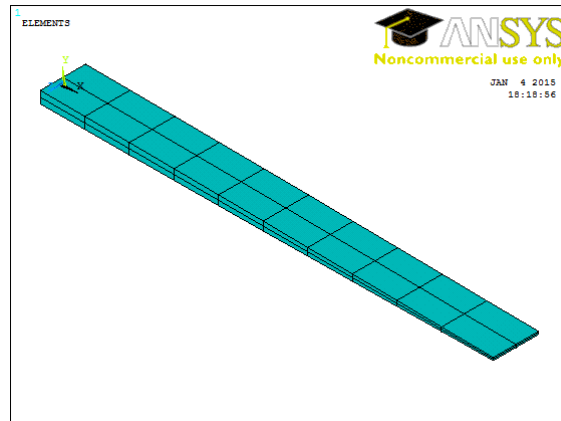


Fig. 5 Finite element model for constant stress beam

TABLE III  
BENDING STRESSES AT TOP FACE OF BEAM

Tensile stresses at strain gauge positions shown in Fig. 2, N/mm <sup>2</sup>					Mean
Load, N	1	2	3	4	
9.81	14.09	13.01	14.13	16.27	14.38
19.62	27.88	25.78	28.03	32.33	28.51
29.43	41.72	38.54	41.80	48.14	42.55
39.24	55.41	51.07	55.28	63.61	56.34

TABLE IV  
BENDING STRESSES AT BOTTOM FACE OF BEAM

Compressive stresses at strain gauge positions shown in Fig. 2, N/mm <sup>2</sup>					Mean
Load, N	5	6	7	8	
9.81	-16.27	-14.96	-16.64	-17.68	-16.39
19.62	-32.16	-29.60	-32.87	-35.17	-32.45
29.43	-48.16	-44.36	-49.13	-52.48	-48.53
39.24	-64.03	-58.94	-65.09	-69.47	-69.47

##### B. Simulation Results

The finite element simulation for the proposed cantilever was carried out for four load cases with at applied load of 9.81 N, 19.62N, 29.42 N and 39.24 N. Fig. 6 shows the results of simulations for the finite element model for the applied load of 19.62 N. It can be seen that the stresses across the length of the beam are constant except for slight variation in the last section near the free end. The variation is due to the fact that, the beam thickness at the free end was modified to positive value than zero during the design stage. This modification was required to support the end load at the free end. In all four simulations showed constant stress along the beam length, this

further validates the design as well as manufacturing procedure adopted for the constant stress composite cantilever beam discussed in this work. Finally, the comparison of finite element simulation results with that of experimental results is presented in Table V for all the load cases. It may be noted that stresses on top and bottom face of the beam using simulation are same, while the experimental values are different for the two faces. This is due to the reason that the beam is modeled as symmetric about the mid plane during the simulation, where as actual design of the beam has taper only along the top face. This approximation was done in order to facilitate use of beam element, as during the initial stages of simulations beam element provided much accurate solution to the problem than the solid element. Also, since the thickness of the beam is very less as compared to other two dimensions this approximation should not alter the problem formulation substantially. Overall, the experimental results agree very well with the finite element simulation results.

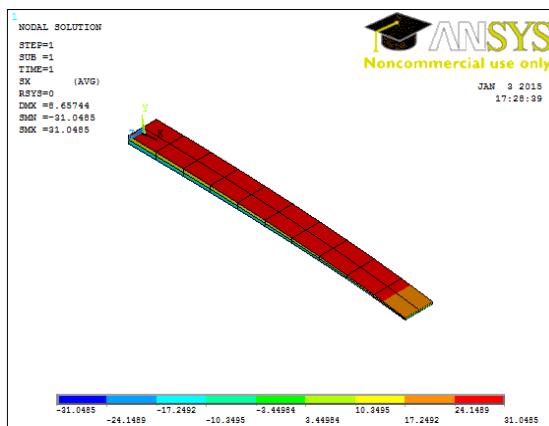


Fig. 6 Stress plot at Load P = 19.62 N

TABLE V  
COMPARISON OF EXPERIMENTAL AND SIMULATION RESULTS

Applied Load, N	Mean stress, N/mm <sup>2</sup>		Stress using simulation, N/mm <sup>2</sup>
	Top Face	Bottom Face	
9.81	14.38	-16.39	15.53
19.62	28.51	-32.45	31.05
29.43	42.55	-48.53	46.57
39.24	56.34	-69.47	62.1

## VI. CONCLUSION

Minimum weight design for composite cantilever beam using constant stress method is discussed in this paper. The constant stress beam variable thickness was successfully fabricated using OoA process. Minimal variation in measured bending stresses along the span of the beam validates the design and manufacturing of the proposed constant stress composite cantilever beam. The finite element simulations for the cantilever agreed very well with the experimental results, this further validates the design and manufacturing procedure.

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**Vinod B. Suryawanshi** is a doctoral candidate in Nanoengineering Department at North Carolina A&T State University Greensboro NC, USA. Previously, he has worked as an Assistant Professor in Mechanical Engineering Department at VJTI Mumbai, INDIA. He has published 1 international journal, 8 international conference and 7 national conference articles in the area of computational design and manufacturing. He is recipient of Graduate research assistantship award from North Carolina A&T State University from Fall 2012 till date.



**Dr. Ajit D. Kelkar** is a Professor and Chair of Nanoengineering department at Joint School of Nanoscience and Nanoengineering. He also serves as an Associate Director for the Center for Advanced Materials and Smart Structures. For the past twenty five years he has been working in the area of performance evaluation and modeling of polymeric composites and ceramic matrix composites. He has worked with several federal laboratories in the area of fatigue, impact and finite element modeling of woven composites including US Army, US Air force, NASA-Langley Research Center, National science Foundation, Office of Naval Research, and Oak Ridge National Laboratory. In addition he has collaborated with Rice University, Texas A&M University, Tuskegee University, Air Force Institute of Technology, University of Dayton, Florida State University, Prairie View A&M University, University of Delaware, Texas State University, University

of Minnesota, University of California, and San Diego. His expertise are in the area of low cost fabrication and processing of woven composites using VARTM process, fatigue and impact testing of composites, analytical modeling of woven composites. Presently he is involved in the development of nano engineered multifunctional materials using XD CNTs and electro spun fiber materials. He is also involved in reengineering of several H-46 and H-47 helicopter components for NAVAIR using out of autoclave processing. In the past he has worked on the one step processing of Composite Armored Vehicle using low cost VARTM method in consortium with University of Delaware-CCM and UC San Diego. In the modeling area he is working on blast simulations for the Humvee vehicles subjected to various TNT blasts loadings and atomistic modeling of polymers embedded with CNTs and alumina nanoparticles. He is also involved in high velocity impact modeling of ceramic matrix composites and polymeric matrix composites embedded with electrospun nanofibers. He has published over two hundred papers in these areas. In addition he has edited a book in the area of Nano Engineered materials. He is member of several professional societies including ASME, SAMPE, AIAA, ASM, and ASEE.