International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
Vol:8, No:10, 2014

Control Configuration Selection and Controller Design for Multivariable Processes Using Normalized Gain

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***Abstract*—**Several of the practical industrial control processes are multivariable processes. Due to the relation amid the variables (interaction), delay in the loops, it is very intricate to design a controller directly for these processes. So first, the interaction of the variables is analyzed using Relative Normalized Gain Array (RNGA), which considers the time constant, static gain and delay time of the processes. Based on the effect of RNGA, relative gain array (RGA) and NI, the pair (control configuration) of variables to be controlled by decentralized control is selected. The equivalent transfer function (ETF) of the process model is estimated as first order process with delay using the corresponding elements in the Relative gain array and Relative average residence time array (RARTA) of the processes. Secondly, a decentralized Proportional-Integral (PI) controller is designed for each ETF simply using frequency response specifications. Finally, the performance and robustness of the algorithm is comparing with existing related approaches to validate the effectiveness of the projected algorithm.

***Keywords*—**Decentralized control, interaction, Multivariable processes, relative normalized gain array, relative average residence time array, steady state gain.

I. Introduction

CONTROLLING of multivariable processes is significantly intricate than those process which have single input and single output. During the past several years, despite of the lot of theoretical developments in analysis and design of multivariable process control systems, a decentralized control technique is still extensively using in many of industrial control processes [1], [2], because of simple implementation, proficient maintenance, simple tuning approaches and robust performance even under model mismatches and uncertainties. It is soundly identified that the decentralized controllers are intrinsically further robust, even under disturbances, interactions and controller loop failures.

The fundamental step, which is predominantly used in design of a decentralized control, is the identification of control configuration. The pioneering effort of Bristol [3], introduced the RGA and it is the first method proposed for control configuration. Moreover, RGA pairing criterion considers only the static gain of the process, which may result in incorrect measures of interaction and consequently the loop pairing decisions.

To overcome the boundaries of RGA, dynamic RGA (DRGA) was proposed, which employs the transfer function model in place of static gain matrix to estimate RGA [4], [5]. However, DRGA is a lot controller dependent which makes more complex to understand by practice engineers. Xiong et al. [6] was projected an Effective Relative Gain Array (ERGA). However, the computation of ERGA depends on critical frequency of individual transfer function elements [6], [7], different assortment criteria for critical frequency point's effect in different REGAs, subsequently; it causes suspicions in selection of control configurations.

Recently, He et al [8] was projected a Relative Normalized Gain Array (RNGA), which considers both static and transient behavior of process.

In this paper we used, the RNGA based pairing rules in combination with the RGA and Niederlinski's index (NI). According to the values of RNGA, NI the control configuration is selected. Hence, Equivalent Transfer Functions (ETF) of model is approximated by the corresponding RGA, RNGA and RARTA [8], [9]. Controllers for each selected loop are designed individually based on constraints of gain margin and phase margin [10], [11]. The structure of decentralized control for interactive multivariable processes with closed loop is shown in Fig. 1.

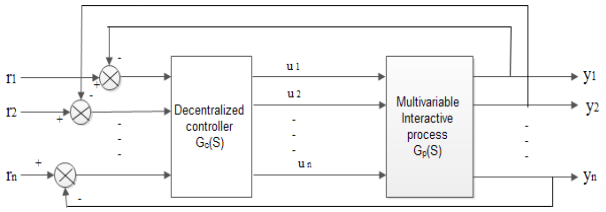

Fig. 1 Decentralized control of interactive multivariable processes

II. Selection of Control Configuration

The efficiency of decentralized control system is exclusively depends on pairing of controlled and manipulated variable. Here it is done by the integration of relative normalized gain array, relative gain array and Niederlinski's Index (RNGA-RGA-NI). This is explained in the following sub sections.

A. Relative Gain Array

The Relative Gain Array (RGA) is defined and computed as follows: Let $G_p(s)$ is the $n \times n$ multivariable process,

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$$G_p(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \dots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \dots & g_{1n}(s) \\ \dots & \dots & \dots & \dots \\ g_{n1}(s) & g_{n2}(s) & \dots & g_{nn}(s) \end{bmatrix} \quad (1)$$

Assume that, each element of the $G_p(S)$ be represented by first order process with delay time (FOPDT) model as,

$$g_{ij}(s) = \frac{k_{ij}}{\tau_{ij}s+1} e^{-\theta_{ij}s} \quad ; \ i, j = 1, 2, ...n$$

Therefore, RGA (Λ) for $n \times n$ systems is,

$$\Lambda = G_p(0) \otimes G_p^{-T}(0) = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \dots & \lambda_{2n} \\ \dots & \dots & \dots & \dots \\ \lambda_{n1} & \lambda_{n2} & \dots & \lambda_{nn} \end{bmatrix} \quad (2)$$

If the value of RGA (Λ) is greater than 0.5 and approaches in the direction of unity, the interaction also leads between corresponding pairs [4]. The pairing which directs the instability is avoided by means of Niederlinski's theorem. The Niederlinski's index (NI) for the control configuration above is denoted by N(G) and expressed as,

$$N(G) = \frac{|G_p(0)|}{\pi g_{ij}} \qquad i, j = 1, 2, 3, ...n \quad (3)$$

where $|G_p(0)|$ represents determinant of matrix $G_p(0)$ and πg_{ij} represents product of diagonal elements of $G(0)$ for a centralized control system. For a stability of composite nonlinear system, NI should be greater than zero.

B. Relative Normalized Gain Array (RNGA)

The process normalized gain $(K_{N,ij})$ for each particular transfer function element $(g_{ij}(s))$ is defined as [9],

$$K_{N,ij} = \frac{k_{ij}}{\sigma_{ij}} = \frac{k_{ij}}{\tau_{ij} + \theta_{ij}} \quad (4)$$

Here σ_{ij} is the average residence time (ART) and it represents the fastness of controlled variable to manipulated variable. The RNGA (ϕ) is expressed using process normalized gain, and hence, it can be determined as,

$$\phi = K_N \otimes K_N^{-T} \quad (5)$$

where '$\otimes$' is element by element product. Thus the RGA-NI-RNGA rules for selection of control configuration are developed as [8]: i) All paired Relative Gain Array elements should be positive, ii) Corresponding Niederlinski's index (NI) is positive, iii) The paired Relative Normalized Gain Array elements must closest to 1.0 and iv) The huge RNGA value elements must be avoided.

III. Equivalent Transfer Function Model (ETF)

To obtain ETF between when all other loops are open and closed, we first define the Relative average residence time (γ_{ij}) as the relation of the loop y_i-u_i average residence time, when all other loops are closed and open,

$$\gamma_{ij} = \frac{\hat{\sigma}_{ij}}{\sigma_{ij}} = \frac{\phi_{ij}}{\lambda_{ij}} \quad (6)$$

The relative average residence time (γ_{ij}) in array form, known as relative average residence time array (RARTA) and it is expressed as,

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \quad (7)$$

By (4) and (6), it can be rewritten as,

$$\hat{\sigma}_{ij} = \gamma_{ij}\sigma_{ij} = \gamma_{ij}\tau_{ij} + \gamma_{ij}\theta_{ij} \quad (8)$$

By using RNGA (ϕ), RARTA (Γ) and RGA (Λ), ETF when all other loops open is written as,

$$\hat{g}_{ij}(s) = \hat{k}_{ij} \frac{1}{\hat{\tau}_{ij}s+1} e^{-\hat{\theta}_{ij}s} \quad (9)$$

where $\hat{\theta}_{ij}$ is the delay time of the equivalent transfer function (ETF). From this, ETF when all other loops closed becomes,

$$\hat{g}_{ij}(s) = \frac{k_{ij}}{\Lambda_{ij}} \frac{1}{\gamma_{ij}\tau_{ij}s+1} e^{-\gamma_{ij}\theta_{ij}s} \quad (10)$$

IV. Decentralized Control Design

The controller (PI) of each loop is considered as in the standard form:

$$g_{c,i}(s) = k_{p,i} + \frac{k_{i,i}}{s} \quad (11)$$

Similarly, ETF of the main loop (i.e. diagonal element) of $\hat{G}(s)$ is represented as,

$$\hat{g}_{ii}(s) = \frac{\hat{g}_{ii}(0)}{\hat{\tau}_{ii}s+1} e^{-\hat{\theta}_{ii}s} \quad ; \text{Where} \quad \hat{g}_{ii}(0) = \frac{k_{ij}}{\lambda_{ij}} \quad (12)$$

Then, the open loop system transfer function is,

$$g_{c,i}(s)\hat{g}_{ii}(s) = k\frac{\hat{g}_{ii}(0)}{s} e^{-\hat{\theta}_{ii}s} \quad (13)$$

Abbreviating the gain margin (GM) and phase margin (PM) specifications as $A_{m,i}$ and $\psi_{m,i}$, their crossover frequencies as $\omega_{g,i}$ and $\omega_{p,i}$, respectively, we get,

$$\arg[g_{c,i}(j\omega_{g,i})\hat{g}_{ii}(j\omega_{g,i})] = -\pi \quad (14)$$

$$A_{m,i}\left|g_{c,i}(j\omega_{g,i})\hat{g}_{ii}(j\omega_{g,i})\right| = 1 \quad (15)$$

$$\left|g_{c,i}(j\omega_{p,i})\hat{g}_{ii}(j\omega_{p,i})\right| = 1 \quad (16)$$

$$\psi_{m,i} = \pi + \arg[g_{c,i}(j\omega_{p,i})\hat{g}_{ii}(j\omega_{p,i})] \quad (17)$$

By substitution and oversimplification, we obtain,

$$\omega_{gii}\hat{\theta}_{ii} = \frac{\pi}{2} \qquad A_{mi} = \frac{\omega_{g,i}}{k\hat{g}_{ii}(0)}$$

$$\omega_{p,i} = k\hat{g}_{ii}(0); \qquad \psi_{mi} = \frac{\pi}{2} - \omega_{p,i}\hat{\theta}_{ii}$$

which results,

$$\psi_{mi} = \frac{\pi}{2}(1 - \frac{1}{A_{mi}}) \quad \text{and} \quad k = \frac{\pi}{2A_{mi}\hat{\theta}_{ii}\hat{g}_{ii}(0)}$$

By these relations, some of possible gain margin (GM) and phase margin (PM) values are listed in Table I.

TABLE I
TYPICAL VALUES OF GAIN AND PHASE MARGINS

ψ_{mi}	$\pi/4$	$\pi/3$	$3\pi/8$	$2\pi/5$
A_{mi}	2	3	4	5

The controller parameters are computed as [8],

$$\left\{ k_{p,i} = \frac{\pi\hat{\tau}_{ii}}{2A_{m,i}\hat{\theta}_{ii}\hat{g}_{ii}(0)},\ k_{i,i} = \frac{\pi}{2A_{m,i}\hat{\theta}_{ii}\hat{g}_{ii}(0)} \right\} \quad (18)$$

V. Performance and Robustness

In design of control system, large values of GM and PM leads to sluggish closed loop response, while small values result in less sluggish and more oscillatory [12]. The choices for GM and PM should also replicate model exactness and expected process changeability. The entire response, from t=0 sec until steady state has been reached, can be used for the formulation of a dynamic performance criterion. These are integral of absolute error (IAE) and integral of square error (ISE):

$$\left\{ \begin{array}{l} IAE = \int_0^\infty |e(t)|dt \\ ISE = \int_0^\infty e^2(t)dt \\ where\ \ e(t) = r(t) - y(t) \end{array} \right\} \quad (19)$$

whereas IAE penalizes little errors, ISE penalizes huge errors in the processes. The GM and PM values used, quantify the amount of uncertainty that can be tolerated. Here $GM \geq 3$ and $PM \geq 60^0$ are selected as robustness boundaries [13], [14].

VI. Simulation Results

Example.1: Consider the process ISPR (Industrial scale polymerization reactor) proposed by Chien et al. [15] is,

$$G_p(s) = \begin{bmatrix} \frac{22.89}{4.572s+1}e^{-0.2s} & \frac{-11.64}{1.807s+1}e^{-0.4s} \\ \frac{4.689}{2.174s+1}e^{-0.2s} & \frac{5.80}{1.801s+1}e^{-0.4s} \end{bmatrix}$$

The RGA (Λ), RNGA (ϕ), NI values for main diagonal and off diagonal pairing of Example.1 is given in Table II.

TABLE II
THE RGA, RNGA, NI VALUES OF DIAGONAL PAIRINGS.

S.No	Control configuration	RGA(Λ)	RNGA(ϕ)	NI
1	(1,1)-(2,2)	0.7087	0.5482	1.4111
2	(1,2)-(2,1)	0.2913	0.4518	-3.4323

The results of RGA-NI-RNGA, suggests the pair of u_1-y_1, u_2-y_2 and corresponding equivalent transfer function as per (10), can be written as,

$$\hat{g}_{11}(s) = \frac{32.3003}{3.5368s+1}e^{-0.1547s} \qquad \hat{g}_{22}(s) = \frac{8.1844}{1.3932s+1}e^{-0.3094s}$$

The phase and gain margin chosen for the controller design are, 72^0 and 5.0 respectively. The proposed Decentralized controllers (RNGA-DCC) together with other related methods are listed in Table III.

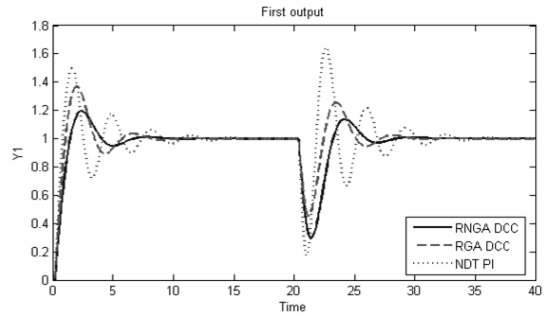
TABLE III
DECENTRALIZED CONTROLLER FOR EXAMPLE.1

Control loop	RNGA-DCC		RGA-DCC		NDT-PI [13]	
	$k_{p,ii}$	$\tau_{i,ii}$	$k_{p,ii}$	$\tau_{i,ii}$	$k_{p,ii}$	$\tau_{i,ii}$
Control Loop 1	0.2222	3.5368	0.263	1.42	0.3561	1.3920
Control Loop 2	0.1727	1.3932	0.163	1.77	0.2838	1.4374

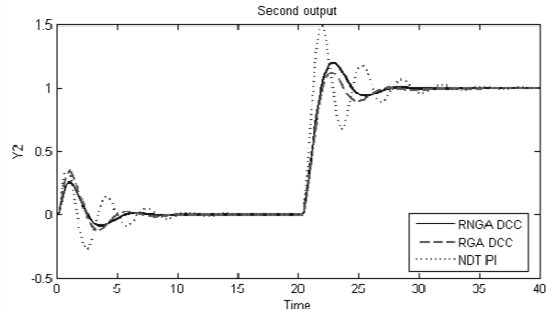
Fig. 2 and Table IV show the performance for different tuning methods with sequential set point changes in loops.

TABLE IV
PERFORMANCE OF EXAMPLE.1 (M_p% IS OVERSHOT AND T_R IS RISE TIME)

Tuning Method	Input(u)-output(y)	IAE	ISE	M_p (%)	T_r
RNGA-DCC	u_1-y_1	2.638	1.248	19.5	1.225
	u_2-y_2	2.157	1.052	19.4	1.760
RGA-DCC	u_1-y_1	2.592	1.003	37.0	0.945
	u_2-y_2	2.296	1.114	11.5	1.900
NDT-PI	u_1-y_1	4.134	1.702	49.0	0.7424
	u_2-y_2	3.18	1.223	48.9	1.1700



(a)



(b)

Fig. 2 Response of example.1 with set point changes at t=0, and t=20 (a) first output (b) second output

Example.2: Consider a higher dimensional process given by [16],

$$G(s) = \begin{bmatrix} \frac{11.9e^{-5s}}{21.7s+1} & \frac{15.3e^{-5s}}{337s+1} & \frac{-2.1e^{-5s}}{10s+1} \\ \frac{37.0e^{-5s}}{500s+1} & \frac{76.7e^{-5s}}{28s+1} & \frac{-5.0e^{-5s}}{10s+1} \\ \frac{93.0e^{-5s}}{500s+1} & \frac{-66.7e^{-5s}}{166s+1} & \frac{-103.3e^{-5s}}{23s+1} \end{bmatrix}$$

The RGA (Λ), K_N, RNGA (ϕ) and RARTA (Γ) of process is calculated as,

$$\Lambda = \begin{bmatrix} 2.896 & -1.1446 & -0.0449 \\ -1.3147 & 2.0677 & 0.2470 \\ 0.1252 & 0.0769 & 0.7979 \end{bmatrix},$$

$$K_N = \begin{bmatrix} 4.4569 & 0.4473 & -0.14 \\ 0.0733 & 2.3242 & -0.3333 \\ 0.1841 & -0.3900 & 3.6785 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 1.0022 & -0.0040 & 0.0017 \\ -0.0031 & 1.0193 & -0.0162 \\ 0.0009 & -0.0154 & 1.0145 \end{bmatrix},$$

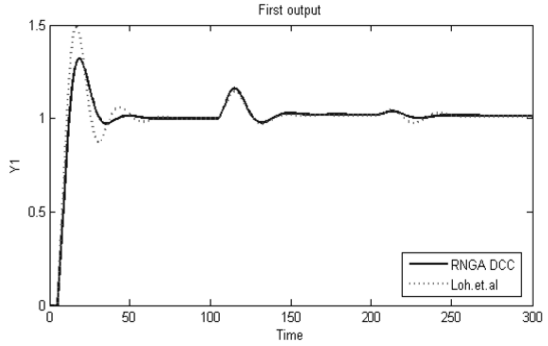
$$\Gamma = \begin{bmatrix} 0.6932 & 0.3847 & -0.0779 \\ 0.4686 & 0.7609 & -0.2295 \\ -0.1618 & -0.1456 & 1.3074 \end{bmatrix}$$

The suggested pairing according to the RGA, RNGA and NI (0.4759>0) is u_1-y_1/u_2-y_2/u_3-y_3 and corresponding RNGA-DCC parameters together with the related methods are listed in Table V.

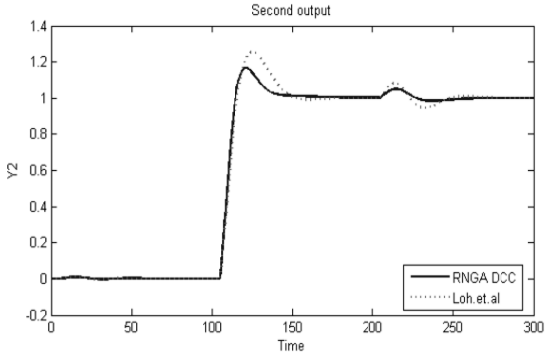
TABLE V
DECENTRALIZED CONTROL FOR EXAMPLE.2

Control loop	Proposed (RNGA-DCC)		Loh and Vasnani-PI	
	$k_{p,ii}$	$\tau_{i,ii}$	$k_{p,ii}$	$\tau_{i,ii}$
Loop 1	0.0418	15.0418	0.0181	14.4936
Loop 2	0.0406	21.3047	0.0335	14.4936
Loop-3	-0.0151	15.0710	-0.0260	14.4936

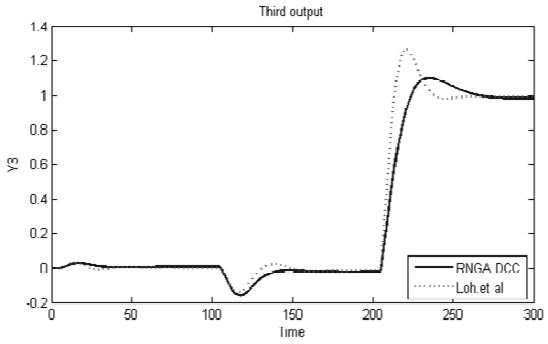
Fig. 3 and Table VI show the performance for different tuning methods with set point changes.



(a)



(b)



(c)

Fig. 3 Response of example.2 with set point changes at t=0, t=100 and t=200 (a) first output (b) second output (c) third output

TABLE VI
PERFORMANCE OF EXAMPLE.2 (M_p% IS OVERSHOT AND T_R IS RISE TIME)

Tuning Method	Input(u)-output(y)	IAE	ISE	M_p(%)	T_r
RNGA-DCC	u_1-y_1	17.48	8.533	32.00	11.39
	u_2-y_2	13.62	8.25	16.70	13.40
	u_3-y_3	21.02	10.89	10.10	20.20
Loh.et.al	u_1-y_1	18.79	8.992	48.60	10.14
	u_2-y_2	17.22	9.392	25.40	14.40
	u_3-y_3	17.04	8.905	26.90	12.60

VII. CONCLUSIONS

In this paper, RNGA DCC is intended for multivariable processes. The control relationship is selected based on RNGA. The coupling is done by the integrity of RGA-NI-RNGA rules. The ETF of the process was determined with the assist of RGA, RNGA and RARTA. Consequently, the decentralized controllers designed simply by means of single loop frequency response approach. Here, two different dimensional industrial processes are considered for study of the effectiveness of proposed method. Simulation results of these processes shows that the projected RNGA DCC provides the overall better performance comparing to the other related approaches in terms of IAE, ISE, rise time and overshoot. The pro of this method is simple to use by field engineers and more significant for high dimensional processes with modest interaction and even under model mismatches.

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