

# Stability Analysis of Three-Lobe Journal Bearing Lubricated with a Micropolar Fluids

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**Abstract**—In this paper, the dynamic characteristics of a three-lobe journal bearing lubricated with micropolar fluids are determined by the linear stability theory. Lubricating oil containing additives and contaminants is modelled as micropolar fluid. The modified Reynolds equation is obtained using the micropolar lubrication theory. The finite difference technique has been used to determine the solution of the modified Reynolds equation. The dynamic characteristics in terms of stiffness, damping coefficients, the critical mass and whirl ratio are determined for various values of size of material characteristic length and the coupling number. The computed results show that the three-lobe bearing lubricated with micropolar fluid exhibits better stability compared with that lubricated with Newtonian fluid. According to the results obtained, the effect of the parameter micropolar fluid is remarkable on the dynamic characteristics and stability of the three-lobe bearing.

**Keywords**—Three-lobe bearings, Micropolar fluid, Dynamic characteristics, Stability analysis.

## I. INTRODUCTION

ADVANCES in technology and in many practical lubrication applications necessitate the development of improved lubricants where the Newtonian fluids constitutive approximation is not a satisfactory engineering approach to lubrication problems. The experimental results support the achievement of better lubricating effectiveness on blending small amount of long-chained additives with the Newtonian lubricants. Micropolar fluids obtained from the general microfluids by imposing the assumption of the skew symmetry of the gyration tensor and the microisotropic property are the simplest subclass of microfluids in which microstructure is still present [1]. A number of theories of the microcontinuum have been developed to explain the behavior of these fluids as polymeric fluids [2]. The study of the flow behaviours using the theory of micropolar lubrication was initiated with the problem of a two-dimensional slider bearing [3]. Shukla et al. [4] derived the generalized Reynolds equation for micropolar fluids with application to one-dimensional slider bearing. The infinitely long journal bearing lubricated with micropolar fluids was studied by PraKash et al. [5]. Singh and Sinha [6] made a detailed order of magnitude study and obtained the same form of Reynolds equation for the three-dimensional case. Hauang et al. [7], [8] presented the static and dynamic characteristics of finite-width journal bearings lubricated with micropolar fluids. The effects of micropolar lubricants and three-dimensional irregularities in hydrodynamic journal bearings were studied by Lin [9].

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Das et al. [10] studied the effect of the misalignment on hydrodynamic journal bearing lubricated with a micropolar fluids. The dynamic characteristics of journal bearings lubricated with micropolar fluids were presented by Das et al. [11]. The current trend in the design of high-speed rotating machinery is to have a good dynamic stability. In such applications, multi-lobe bearings are often used because these have better dynamic stability than systems with plain journal bearings. The three-lobe bearing is a widely used non-circular bearing. In practical, all the lobes are manufactured with equal arc length. The theoretical analysis of three-lobe bearings was first presented by Pinkus [12]. The stability characteristics and general transient motion of a vertical three-lobe bearing were presented by Falkenhagen et al. [13]. The stability criteria for a multi-lobe bearing were developed by Lund et al. [14] based on linearization of the Reynolds equation by small perturbation theory. Sinhasan et al. [15] compared some three-lobe bearing configurations on the basis of the theoretical static and dynamic characteristics. The three-lobe bearing design data, which include both static and dynamic characteristics, were reported by Malik et al. [16]. Static and dynamic characteristics of 6 types of multi-lobe journal bearings in turbulent flow regime have been studied by Abdul-Wahed et al. [17]. Prabhu [18] made an investigation to evaluate the experimental performance of misaligned cylindrical and three-lobe journal bearings. A numerical procedure incorporating cavitation modeling in the prediction of dynamic coefficients for four multi-lobe bearings is presented by Rao et al. [19]. Prabhakaran Nair et al. [20] presented an analysis of the deformation effect of the bearing liner on the static and dynamic characteristics of an elliptical journal bearing with a micropolar lubricant. Rahmatabadi et al. [21] studied the static characteristics of a noncircular journal bearing (two-lobe, three-lobe and four-lobe) lubricated with a micropolar fluids. In Recent investigation, Chetti [22] presented the effect of micropolar fluids on the dynamic characteristics of a four-lobe journal bearing. In the present work, dynamic characteristics in terms of stiffness, damping coefficients, the critical mass and whirl ratio are determined for a three-lobe journal bearing lubricated with a micropolar and Newton fluids. The effects of different values of micropolar parameter on the dynamic characteristics of the journal bearing are presented.

## II. MODIFIED REYNOLDS EQUATION

Under the usual assumptions made for the lubrication film, the assumptions of the absence of body forces, body couples and constancy of characteristic coefficients across the film of

the micropolar fluid, the modified Reynolds equation is written in the following form [8]:

$$\frac{\partial}{\partial \theta} \left[ \bar{G} \frac{\partial \bar{p}}{\partial \theta} \right] + \left( \frac{R}{B} \right)^2 \frac{\partial}{\partial z} \left[ \bar{G} \frac{\partial \bar{p}}{\partial z} \right] = 6 \frac{\partial \bar{h}}{\partial \theta} + 12 \frac{\partial \bar{h}}{\partial t} \quad (1)$$

where

$$\bar{G}(N, \bar{h}, L) = \bar{h}^{-3} + 12 \frac{\bar{h}}{L^2} - 6 \frac{N \bar{h}^{-2}}{L} \coth \left( \frac{NL \bar{h}}{2} \right) \quad (2)$$

and

$$\mu = \mu_v + \frac{1}{2} k_v, \quad \ell = \left( \frac{\gamma}{4\mu} \right)^{1/2}, \quad N = \left( \frac{k_v}{2\mu + k_v} \right)^{1/2} \quad (3)$$

$N$  and  $\ell$  are two parameters distinguishing a micropolar fluid from a Newtonian fluid.  $N$  is a dimensionless parameter called the coupling number which couples the linear and angular momentum equations arising due to the microrotational effect of the suspended particles in the fluid.

$\ell$  represents the interaction between the micropolar fluid and the film gap and is termed as the characteristics length of the micropolar fluid.

$O_{L,3}$  are the centres of lobes 1, 2 and 3 respectively of the bearing. The bearing is provided with three oil supply grooves which the size of each groove is 20 deg. The eccentricity ratios and attitude angles of the individual lobes of three-lobe bearing are given by the following relations:

The eccentricity ratios and the attitude angles of each lobe for the bearing are given by:

$$\varepsilon_1^2 = \varepsilon^2 + \delta^2 - 2\varepsilon\delta \cos \left( \frac{\pi}{3} - \phi \right) \quad (4)$$

$$\varepsilon_2^2 = \varepsilon^2 + \delta^2 - 2\varepsilon\delta \cos \phi$$

$$\varepsilon_3^2 = \varepsilon^2 + \delta^2 - 2\varepsilon\delta \cos \left( \frac{\pi}{3} + \phi \right)$$

$$\phi_1 = \frac{4\pi}{3} + \sin^{-1} \left[ \frac{\varepsilon}{\varepsilon_1} \sin \left( \frac{\pi}{3} - \phi \right) \right] \quad (5)$$

$$\phi_2 = \sin^{-1} \left( \frac{\varepsilon}{\varepsilon_2} \sin \phi \right)$$

$$\phi_3 = \frac{2\pi}{3} - \sin^{-1} \left[ \frac{\varepsilon}{\varepsilon_3} \sin \left( \frac{\pi}{3} + \phi \right) \right]$$

The non-dimensional fluid film thickness for each lobe is given by:

$$\bar{h}_i = 1 + \varepsilon_i \cos(\theta - \phi_i), \quad i=1,2,3 \quad (6)$$

where

$$\bar{h} = \frac{h}{c}, \quad \varepsilon = \varepsilon(1 - \delta)$$

### B. Boundary Conditions

The pressure boundary conditions in dimensionless form are:

$$\bar{P} = 0 \text{ at } \bar{z} = 0, \bar{z} = 1, \theta = \theta_{s1}, \theta = \theta_{s2} \text{ and } \theta = \theta_{s3} \quad (7a)$$

$$\frac{\partial \bar{P}}{\partial \theta} = 0 \text{ at } \theta = \theta_{t1}, \theta = \theta_{t2} \text{ and } \theta = \theta_{t3} \quad (7b)$$

$$\bar{P} = 0 \text{ for } \theta_{e1} \geq \theta \geq \theta_{t1}, \theta_{e2} \geq \theta \geq \theta_{t2} \text{ and } \theta_{e3} \geq \theta \geq \theta_{t3} \quad (7c)$$

Equation (7a) is result from the fact that the ends of the bearing are exposed to the ambient pressure, while (7b) and (7c) are the Reynolds (Swift-Stieber) conditions.

### C. Stability Analysis

The linearized equations of the disturbed motion of the journal centre are [17]:

$$M \ddot{x} + K_{xx} x + C_{xx} \dot{x} + K_{xy} y + C_{xy} \dot{y} = 0 \quad (8)$$

$$M \ddot{y} + K_{yx} x + C_{yx} \dot{x} + K_{yy} y + C_{yy} \dot{y} = 0$$

where the fluid film stiffness and damping coefficients are respectively given by:

$$\begin{pmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{pmatrix} = - \left\{ \frac{\partial}{\partial x} \right\} \begin{bmatrix} W_x & W_y \end{bmatrix} \quad (9)$$

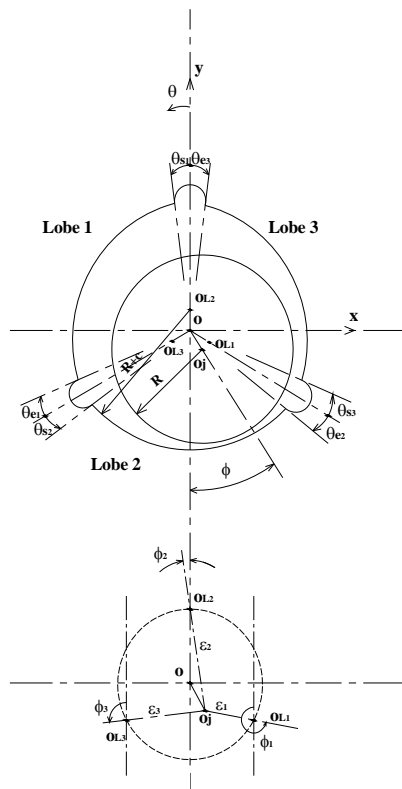


Fig. 1 Four-lobe journal bearing

### A. Bearing Geometry and Boundary Conditions

The three-lobe bearing configuration is shown in Fig. 1. This bearing comprises three lobes whose centres of curvature are not in the geometric centre of the bearing.  $O_{L,1}$ ,  $O_{L,2}$  and

$$\begin{pmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{pmatrix} = - \begin{Bmatrix} \partial/\partial x \\ \partial/\partial y \end{Bmatrix} \begin{bmatrix} w_x & w_y \end{bmatrix} \quad (10)$$

Equation (8) is used to study the stability of the bearing system and can be written as:

$$\begin{bmatrix} K_{xx} - Mv^2 + ivC_{xx} & K_{xy} + ivC_{xy} \\ K_{yx} + ivC_{yx} & K_{yy} - Mv^2 + ivC_{yy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad (11)$$

For a nontrivial solution the determinant must vanish and equating the real and imaginary parts to zero gives:

$$\overline{M}\gamma^2 = \frac{\overline{C}_{xx}\overline{K}_{yy} + \overline{C}_{yy}\overline{K}_{xx} - \overline{C}_{xy}\overline{K}_{yx} - \overline{C}_{yx}\overline{K}_{xy}}{\overline{C}_{xx} + \overline{C}_{yy}} \quad (12)$$

$$\gamma^2 = \frac{(\overline{K}_{xx} - \overline{M}\gamma^2)(\overline{K}_{yy} - \overline{M}\gamma^2) - \overline{K}_{yx}\overline{K}_{xy}}{\overline{C}_{xx}\overline{C}_{yy} - \overline{C}_{xy}\overline{C}_{yx}} \quad (13)$$

The critical mass  $\overline{M}_c$  and the whirl ratio  $\gamma$  are calculated from (12) and (13).  $\overline{M}_c$  is the critical mass parameter above which the bearing is unstable.

#### D. Solution Procedure

Applying the central finite-difference scheme to (1), the value of any pressure is given by:

$$A_0\overline{P}_{i,j} + A_1\overline{P}_{i+1,j} + A_2\overline{P}_{i-1,j} + A_3\overline{P}_{i,j+1} + A_4\overline{P}_{i,j-1} = B_{i,j} \quad (14)$$

where the values of the constants  $A_i$  can be defined by comparing this equation with (1).

Firstly, an initial value of the attitude angle  $\phi$  is assumed in order to calculate the film thickness.

The modified Reynolds equation (1) is solved using the finite difference technique with Gauss Seidel method under the boundary conditions (7). The negative pressures are set to zero to account for Swift-Stieber boundary conditions. The iterative procedure is stopped when at each point the error in pressure between two successive iterations does not exceed 0.001. The horizontal and vertical components of the load are calculated for the three lobes of the bearing. If the magnitude of the horizontal component of the load becomes nearly zero (within specified limits of convergence) the results are accepted, otherwise the procedure is repeated with new value of attitude angle  $\phi$ . The dynamic bearing characteristics are computed only after establishing the attitude angle  $\phi$  (the static equilibrium position). By changing  $x$  and  $y$  by a small amount corresponding to the static equilibrium position, the stiffness coefficients are found. Similarly, by giving small values for  $\dot{x}$  and  $\dot{y}$  around the static equilibrium position, the damping coefficients can be calculated.

### III. RESULTS AND DISCUSSION

In the present analysis, the dynamic characteristics for a three-lobe journal bearing are computed for an ellipticity ratio of 0.5 and an aspect ratio  $R/B = 1$ . The values of coupling number  $N$  considered are; 0.3, 0.6 and 0.9. The values of the non-dimensional characteristics length  $L$  are ranging from 0 to 60. As the parameters  $L \rightarrow \infty$  or  $N \rightarrow 0$  the micropolar effects become insignificant and the fluid converts to Newtonian fluid. To establish the validity of the solution algorithm and the computer program used in the present study, the stiffness and damping coefficients of three-lobe journal bearing lubricated with Newtonian fluid are compared with the published results in [13]. It is seen that the present results obtained are in good agreement with the published results. The variation of the direct stiffness coefficients ( $K_{xx}$ ,  $K_{yy}$ ) with non-dimensional characteristics length  $L$  for various values of coupling number  $N$  are shown in Figs. 2 and 3.

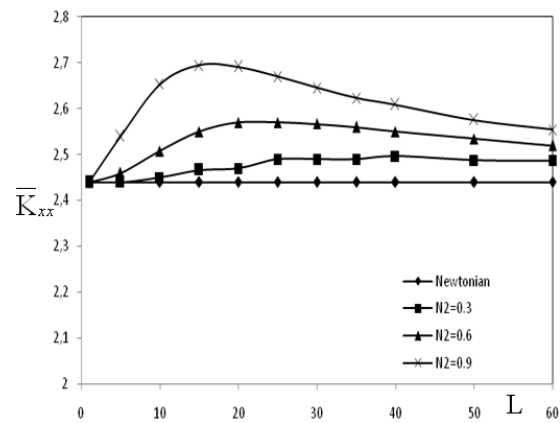


Fig. 2 Variation of  $\overline{K}_{xx}$  with  $L$  for different values of  $N^2$

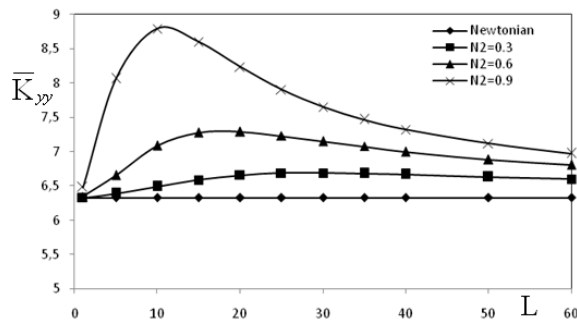


Fig. 3 Variation of  $\overline{K}_{yy}$  with  $L$  for different values of  $N^2$

It has been observed that the direct stiffness coefficients increase with increasing of  $N$  for given value of  $L$ . The direct stiffness coefficients for the micropolar fluid are higher than those for the Newtonian fluid. Figs. 4 and 5 present the effects of the non-dimensional length  $L$  and the coupling number  $N$  on the cross stiffness coefficients ( $K_{xx}$ ,  $K_{yy}$ ). It can be observed that the increasing of  $N$  produces an increase of  $K_{yx}$ , while this observation can be noted at higher values of  $L$  ( $L > 10$ ) for  $K_{xy}$ .

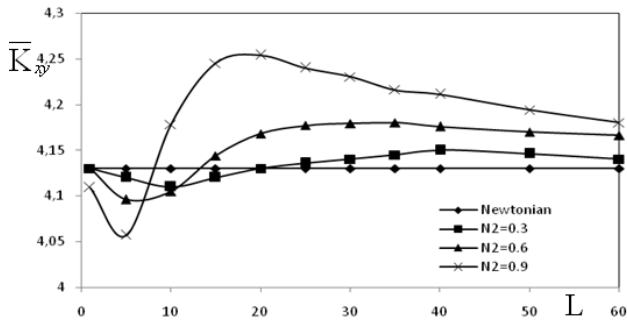


Fig. 4 Variation of  $\bar{K}_{xy}$  with  $L$  for different values of  $N^2$

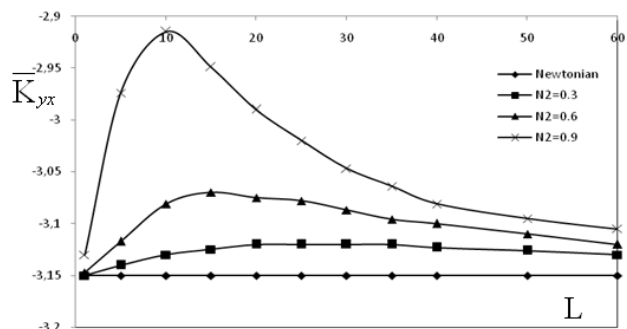


Fig. 5 Variation of  $\bar{K}_{yx}$  with  $L$  for different values of  $N^2$

The variation in direct damping coefficients ( $C_{xx}$ ,  $C_{yy}$ ) with non-dimensional characteristics length  $L$  for various values of coupling number  $N$  is illustrated in Figs. 6 and 7. It can be seen that the values of  $C_{xx}$  decreases while the values of  $C_{yy}$  increases with an increase of  $N$ .

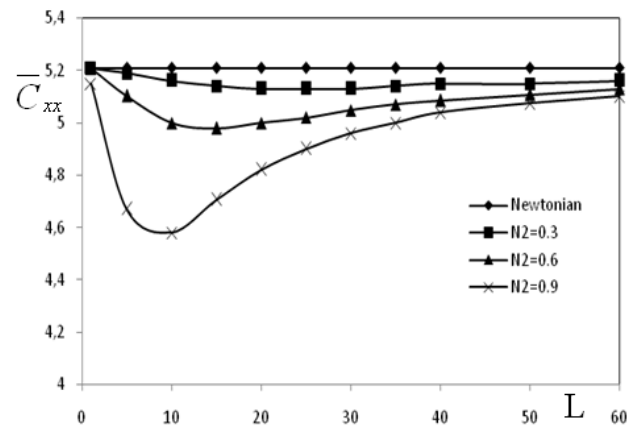


Fig. 6 Variation of  $\bar{C}_{xx}$  with  $L$  for different values of  $N^2$

The effect of non-dimensional characteristics length  $L$  and the coupling number  $N$  on the cross damping coefficients  $C_{xy}$  are shown in Fig. 8. From the figure, it can be seen that the cross damping increases with an increase of  $N$  for any values of  $L$ . Fig. 9 exhibits the variation of the critical mass with non-dimensional characteristics length  $L$  for various values of the coupling number  $N$ . The critical mass is observed to be

increased as  $N$  increases for all values of  $L$ . The critical mass for the micropolar fluid is higher than that for the Newtonian fluid.

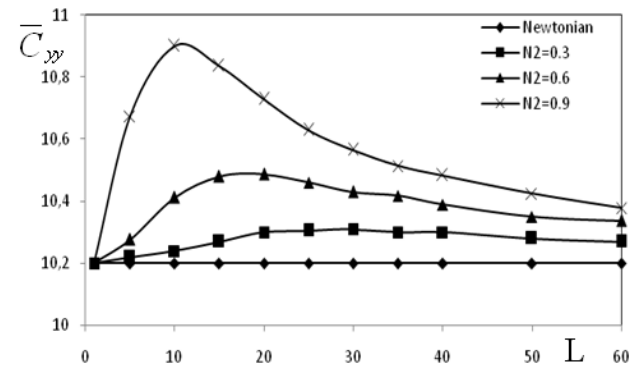


Fig. 7 Variation of  $\bar{C}_{yy}$  with  $L$  for different values of  $N^2$

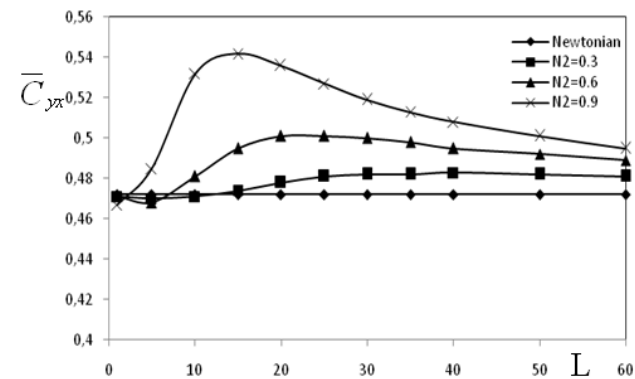


Fig. 8 Variation of  $\bar{C}_{xy} = \bar{C}_{yx}$  with  $L$  for different values of  $N^2$

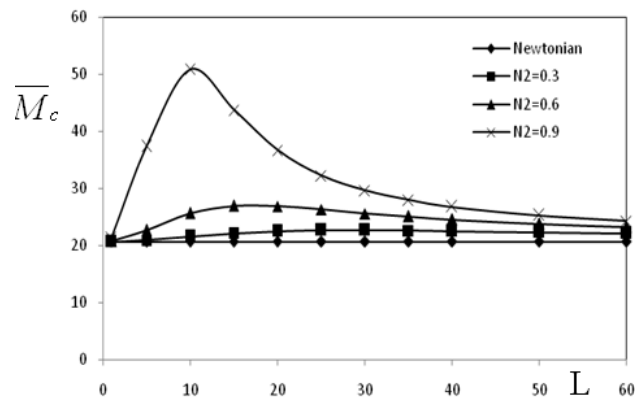


Fig. 9 Variation of  $\bar{M}_c$  with  $L$  for different values of  $N^2$

The effect of the parameters of micropolar fluid  $N$  and  $L$  on the whirl ratio is shown in Fig. 10. It is observed that the whirl ratio is found to be decreased with the increase of  $N$ . At high values of  $L$ , the critical mass and the whirl ratio for micropolar fluid approach to those of the Newtonian fluid.

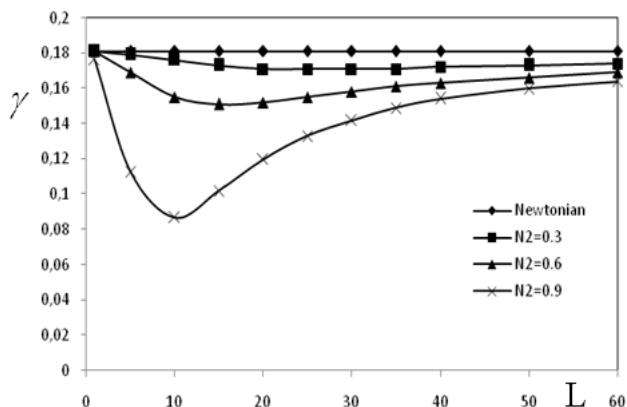


Fig. 10 Variation of  $\gamma$  with  $L$  for different values of  $N^2$

#### IV. CONCLUSIONS

Based on the results presented in this paper, the following conclusions can be made:

- 1) The critical mass increases while the whirl ratio decreases with an increase of the coupling number  $N$  for the three-lobe journal bearing.
- 2) In general, the stiffness and damping coefficients increase with increasing the coupling number  $N$ .
- 3) In general, the stiffness and damping coefficients for the three-lobe journal bearing lubricated with a micropolar fluid are higher than those for Newtonian fluid.
- 4) At high values of  $L$ , the stiffness coefficients, damping coefficients, the critical mass and the whirl ratio for micropolar fluid converge to those for Newtonian fluid.
- 5) The stability of the three-lobe journal bearing is improved by using a micropolar fluid compared to a Newtonian fluid.

#### NOMENCLATURE

$c$	major clearance
$\bar{C}_{ij}$	dimensionless damping coefficients
$e$	eccentricity
$e_p$	ellipticity, $c - c_m$
$h$	oil film thickness
$\bar{K}_{ij}$	dimensionless stiffness coefficients
$L$	bearing ratio
$N$	coupling number
$\ell$	material length
$M$	mass of journal
$P$	pressure
$R$	journal radius
$W$	bearing load
$\bar{W}$	dimensionless bearing load $W(c/R)^2 / \mu_t RL$
$x, y, z$	circumferential, radial, and axial co-ordinates respectively
$\delta$	ellipticity ratio, $e_p/c$
$\varepsilon$	eccentricity ratio based on major clearance, $e/c$
$\bar{\varepsilon}$	eccentricity ratio based on minor clearance

$\theta$	angular coordinate
$\theta_{e1}, \theta_{e2}, \theta_{e3}$	angular coordinates at the end of bearing pads
$\theta_{s1}, \theta_{s2}, \theta_{s3}$	angular coordinates at the start of bearing pads
$\theta_{t1}, \theta_{t2}, \theta_{t3}$	angular coordinates at the trailing edges
$\mu$	lubricant viscosity
$k_v$	spin viscosity
$\mu_v$	viscosity coefficient for Newtonian fluid
$\nu$	whirl frequency
$\gamma$	whirl ratio
$\phi$	attitude angle
$\omega$	angular velocity of the journal

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