Effect of Out-of-Plane Deformation on Relaxation Method of Stress Concentration in a Plate with a Circular Hole

Shingo Murakami, Shinichi Enoki

Abstract— In structures, stress concentration is a factor of fatigue fracture. Basically, the stress concentration is a phenomenon that should be avoided. However, it is difficult to avoid the stress concentration. Therefore, relaxation of the stress concentration is important. The stress concentration arises from notches and circular holes. There is a relaxation method that a composite patch covers a notch and a circular hole. This relaxation method is used to repair aerial wings, but it is not systematized. Composites are more expensive than single materials. Accordingly, we propose the relaxation method that a single material patch covers a notch and a circular hole, and aim to systematize this relaxation method.

We performed FEA (Finite Element Analysis) about an object by using a three-dimensional FEA model. The object was that a patch adheres to a plate with a circular hole. And, a uniaxial tensile load acts on the patched plate with a circular hole. In the three-dimensional FEA model, it is not easy to model the adhesion layer. Basically, the yield stress of the adhesive is smaller than that of adherents. Accordingly, the adhesion layer gets to plastic deformation earlier than the adherents under the yield load of adherents. Therefore, we propose the three-dimensional FEA model which is applied a nonlinear elastic region to the adhesion layer. The nonlinear elastic region was calculated by a bilinear approximation. We compared the analysis results with the tensile test results to confirm whether the analysis model has usefulness. As a result, the analysis results agreed with the tensile test results. And, we confirmed that the analysis model has usefulness.

As a result that the three-dimensional FEA model was used to the analysis, it was confirmed that an out-of-plane deformation occurred to the patched plate with a circular hole. The out-of-plane deformation causes stress increase of the patched plate with a circular hole. Therefore, we investigated that the out-of-plane deformation affects relaxation of the stress concentration in the plate with a circular hole on this relaxation method. As a result, it was confirmed that the out-of-plane deformation inhibits relaxation of the stress concentration on the plate with a circular hole.

Keywords—Stress concentration, patch, out-of-plane deformation, Finite Element Analysis.

I. INTRODUCTION

THE most structural fatigue fractures are caused by the stress concentration that occurs due to notches and circular holes [1]. The stress concentration areas are fragile, because high stress occurs in the stress concentration areas. Furthermore, the state of the stress concentration varies according to the shape of a notch and a circular hole [2]. So, it is

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difficult to avoid the stress concentration. Therefore, relaxation of the stress concentration is important. There is a relaxation method that a composite patch covers a notch and a circular hole. This relaxation method is used to repair aerial wings [3], but it is not systematized. Composites are more expensive than single materials. Accordingly, the authors propose the relaxation method that a single material patch covers a notch and a circular hole, and aim to systematize this relaxation method. In the relaxation method that used a composite patch, the adhesion layer had been already studied. The study shows the following thing: In relaxation of the stress concentration, it is more effective to change the patch shape than changing adhesion state [4]. On the basis of the knowledge, there is a study [5] that suggested a suitable shape of the patch by a three-dimensional FEA model. This knowledge related to the relaxation method that used a composite patch. However, their knowledge can be applied to the relaxation method that used a single material patch. In the relaxation method, an adhesive is used for adhering single material patch. Basically, the yield stress of the adhesive is smaller than that of adherents. Accordingly, the adhesion layer gets to plastic deformation earlier than the adherents under the yield load of adherents. Therefore, we propose a three-dimensional FEA model which is applied a nonlinear elastic region to the adhesion layer. The nonlinear elastic region is calculated by a bilinear approximation. And then, we prove that the three-dimensional FEA model has usefulness.

In the relaxation method, a bending moment occurs to the patched plate with a circular hole due to the disturbance of the load transmission pass. This bending moment brings an out-of-plane deformation to the patched plate with a circular hole. The out-of-plane deformation causes stress increase of the patched plate with a circular hole. Therefore, we investigate that the out-of-plane deformation affects relaxation of the stress concentration in the plate with a circular hole on this relaxation method.

II. VERIFICATION OF THE ANALYSIS MODEL

A. Objects on Analyses and Tensile Tests

Objects on analyses and tensile tests are shown in Fig. 1. Type A is a plate with a circular hole. Type B is that a patch adhered to Type A. Material of the plate with a circular hole was A5052 (JIS-H4000, Aluminum alloy, Thickness 1.0 (mm)). Material of the patch was A1050 (JIS-H4000, Aluminum, Thickness 0.5 (mm)). Material properties of A5052 and A1050

are shown in Table I. Incidentally, the young's modules that are shown in Table I were measured by a tensile test.

(a) Type A   (b) Type B

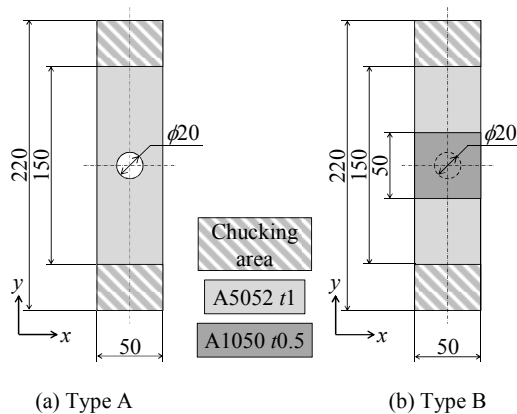Fig. 1 Schematic drawings of objects on analyses and tensile tests

TABLE I
MATERIAL PROPERTIES OF A5052 AND A1050

Material	Young's modulus (MPa)	Shearing modulus (MPa)	Poisson's ratio
A5052	52000	19549	0.33
A1050	52000	19549	0.33

B. Analysis Models

The analysis models are shown in Fig. 2. The analysis models were 1/4 model of the analysis objects. The analysis models were meshed with 20-nodes hexahedral element. Material properties of an acrylic plastic applied to the adhesion layer of the analysis models, because the acrylic adhesive is used to the adhesive in the tensile tests. In the tensile tests, there is possibility that the stress of plastic region occurs to the adhesion layer. Therefore, a nonlinear elastic region applied to the adhesive layer of the analysis model of Type B-1. The nonlinear elastic region was calculated by a bilinear approximation. Type B-2 was an analysis model that a liner elastic region applied to the adhesive layer. Analysis conditions of Type B-2 were same as that of Type B-1 except for the elastic region of the adhesive layer. Material properties of an acrylic plastic are shown in Table II. In the analysis models, the thickness of the adhesive layer was 0.1 (mm), and y axial tensile load acted on lower end of the plate with a circular hole. FEA software ANSYS 14.5 (ANSYS.inc) was used for the analysis.

TABLE II
MATERIAL PROPERTIES OF AN ACRYLIC PLASTIC

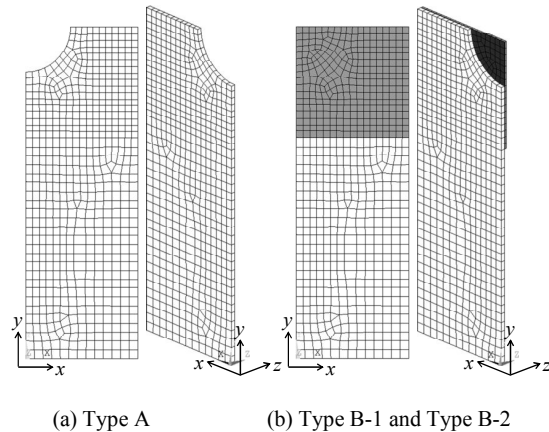| Material	Young's modulus (MPa)	Poisson's ratio	Strain (%)	Tangent modulus
Acrylic plastic	3140	0.35	0.07	499

(a) Type A   (b) Type B-1 and Type B-2

Fig. 2 Analysis models of Type A and Type B

C. Experimental Methods

Specimens of the tensile tests were made based on Type A and Type B that was shown in Fig. 1. The adhesive to glue the patch to the plate with a circular hole was an acrylic adhesive (HARDLOC M-372-20A/M-372US-20B, Denka Corporation, Tensile Shear Strength 19.0 (MPa)). The universal testing (UH-F500kNI, Shimadzu Corporation) was used for the tensile tests. Tension speed was 1.0 (mm/min). In the tensile tests, the strain of the near circular hole was measured by strain gauges (KFG-1-120-C1-16L5M2R, Kyowa Electronic Instruments Corporation).

D. Comparison of Test Results and Analysis Results

Specimens after the tensile tests are shown in Fig. 3. As shown in Fig. 3, the patch was transformed by the tensile load that acted on the patched plate with a circular hole. In other words, the tensile load that acted on the patched plate with a circular hole transferred through the adhesive layer to the patch.
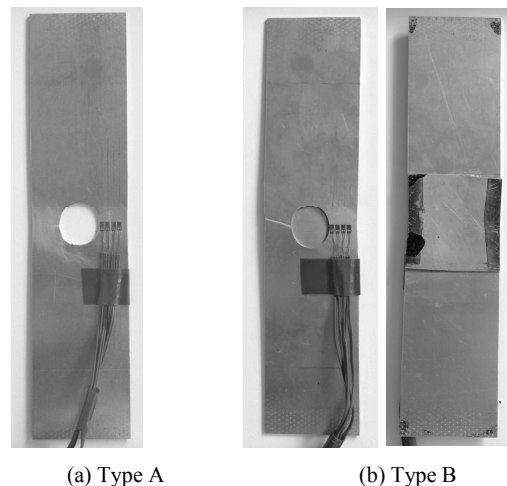

(a) Type A   (b) Type B

Fig. 3 Specimens after the tensile tests

In the tensile tests and FEA, the y axial stress distribution of Type A around the circular hole is shown in Fig. 4. And, the y axial stress distribution of Type B around the circular hole is

shown in Fig. 5. The tendencies of the analysis result agreed with that of the test result. In the case of Type A, the analytic values were larger than the test values. It shows good conformance, because it is a safe side on the viewpoint of design. In the case of Type B, the analytic values of Type B-1 were near the test values of Type B. The analytic values of Type B-2 were larger than the test values of Type B. Therefore, the analysis model that the nonlinear elastic region applied to the adhesive layer has usefulness.

In the test results, the stress concentration around the circular hole was relaxed by bonding the patch as shown in Figs. 4 and 5. Similarly, in the analysis results, the stress concentration around the circular hole was relaxed by bonding the patch. Therefore, it is confirmed that this relaxation method is effective in relaxation of the stress concentration.
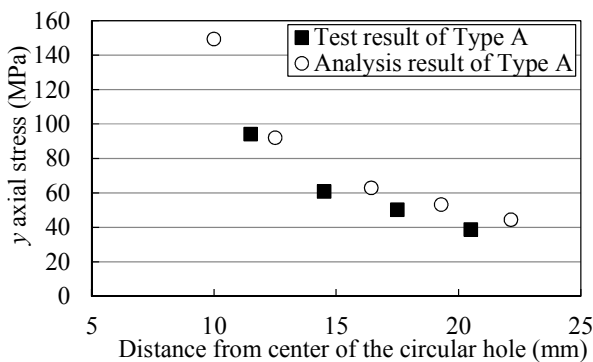

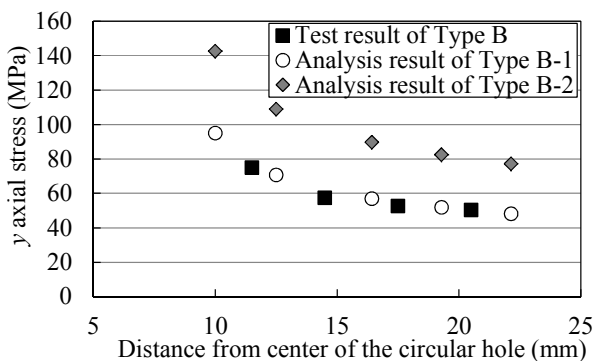Fig. 4 y axial stress distributions around the circular hole on Type A

Fig. 5 y axial stress distributions around the circular hole on Type B

III. EFFECT OF THE OUT-OF-PLANE DEFORMATION ON RELAXATION OF THE STRESS CONCENTRATION

A. Method to Reduce the Out-Of-Plane Deformation

The tensile load that acted on the patched plate with a circular hole transferred through the adhesive layer to the patch. Then, a bending moment occurs to the patched plate with a circular hole due to the disturbance of the load transmission pass. This bending moment brings the out-of-plane deformation and a bending stress to the patched plate with a circular hole. This bending stress causes stress increase of the patched plate with a circular hole. When the out-of-plane deformation becomes small, the bending stress reduces.

Therefore, we investigated effect of the out-of-plane deformation on relaxation of the stress concentration. The schematic drawing of Type C is shown in Fig. 6. The analysis model of Type C is shown in Fig. 7. Type C is the analysis model that a thickness of the patch is increased by bonding a circular patch to the patch of Type B. The deformation of Type C is different from that of Type B due to changing the thickness of the patch. In this analysis, the analysis object was Type C that a circular patch (A1050, thickness 0.5 (mm)) adhered to the patch of Type B. The analysis model was 1/4 model of the analysis object. The analysis model was meshed with 20-nodes hexahedral element. Material properties and the model of the adhesive layer were same as the analysis model of Type B-1.
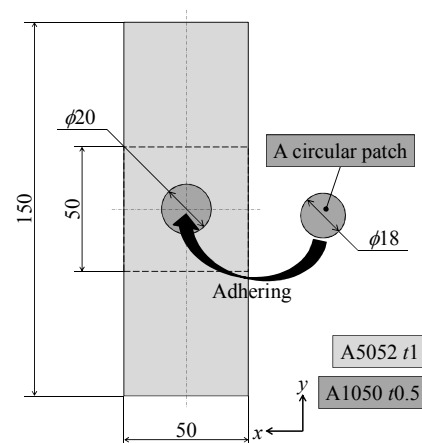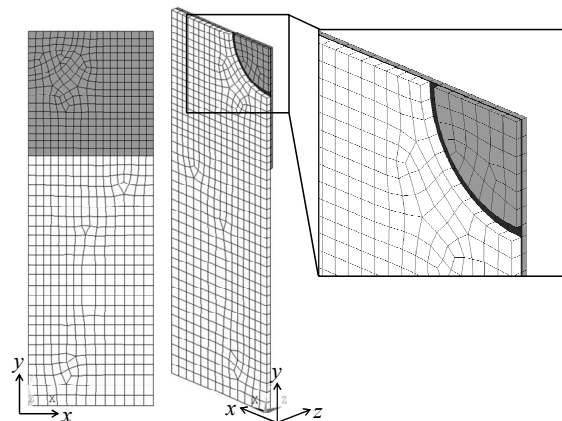

Fig. 6 Schematic drawing of Type C

Fig. 7 Analysis model of Type C

B. Relation between the Out-Of-Plane Deformation and Relaxation Rate of the Stress Concentration

Fig. 8 shows Mises stress distributions of the patched plate with a circular hole on Type B-1 and Type C. Fig. 9 shows the distributions of z axial displacement on Type B-1 and Type C. The α points in Fig. 9 are the vicinity that maximum Mises stress of the patched plate with a circular hole occurred. As shown in Fig. 9, the z axial displacement of Type C was smaller than that of Type B-1 on the α point. Moreover, the stress

concentration of Type C relaxed better than that of Type B-1. An elastic curve equation is the following equation:

$$\frac{d^2z}{dy^2} = -\frac{M}{EI} \qquad (1)$$

where M is the bending moment, E is young's modulus, I is the second moment of area. As shown in the elastic curve equation, z axial displacement depends on the bending moment and the second moment of area. Fig. 10 shows schematic diagrams of load share in Type B-1 and Type C. In Fig. 10, the bending moment that occurs to the A-A' section is in proportion to the moment distance L_{B-1} and L_C. The moment distance L_C was smaller than the moment distance L_{B-1}. Therefore, in Type C, the bending moment that occurs to the A-A' section was smaller than that of Type B-1. On the other hand, According to the elastic curve equation, z axial displacement becomes small, when the second moment of area becomes large. The second moment of area becomes large with increase of a thickness. In the case of Type C, the thickness of the A-A' section was increased by bonding the circular patch to the patch, and the z axial displacement was smaller than that of Type B-1. When the z axial displacement becomes small, the bending moment that occurs to the A-A' section becomes small. Accordingly, relaxation rate of the stress concentration was large in Type C. Therefore, it was found that the out-of-plane deformation inhibits relaxation of the stress concentration in the plate with a circular hole.

Fig. 11 shows Mises stress distributions around the circular hole on Type A, Type B-1 and Type C. As shown in Fig. 11, relaxation rate of the stress concentration improved on Type C that the thickness of the patch is larger than others. Therefore, it was found that increase of the patch thickness connects with improving relaxation rate of the stress concentration in the plate with a circular hole.
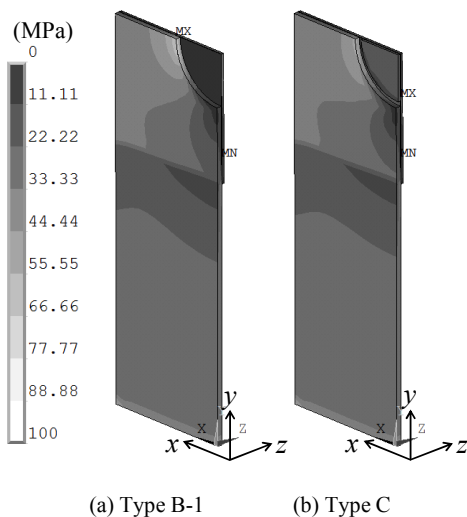

(a) Type B-1   (b) Type C

Fig. 8 Mises stress distributions of the patched plate with a circular hole on Type B-1 and Type C
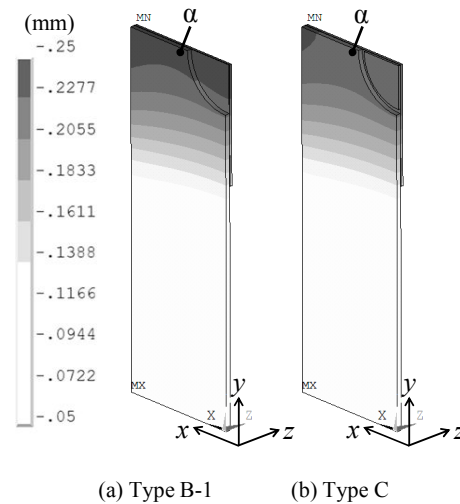
(a) Type B-1   (b) Type C

Fig. 9 Distributions of z axial displacement on Type B-1 and Type C
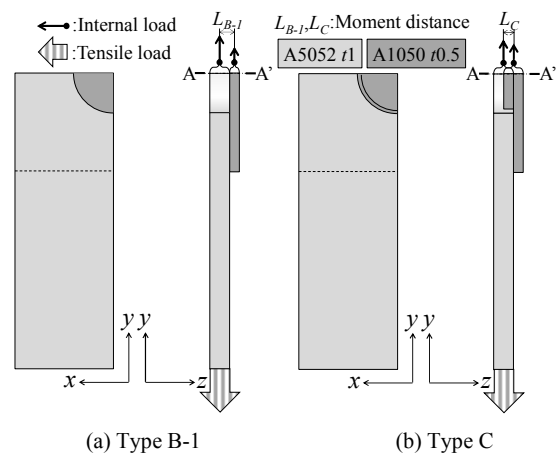

(a) Type B-1   (b) Type C

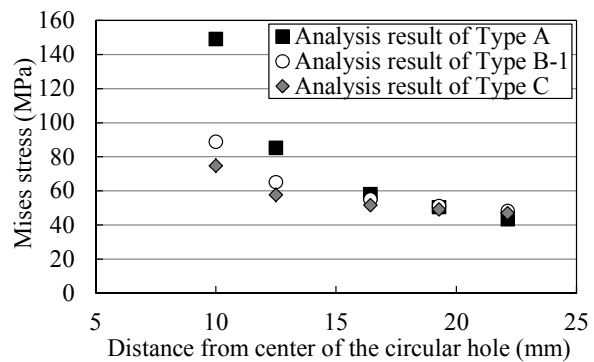Fig. 10 Schematic diagrams of load share on Type B-1 and Type C

Fig. 11 Mises stress distributions around the circular hole on Type A, Type B-1 and Type C

IV. CONCLUSION

The aim of this paper was to investigate that the out-of-plane deformation affects relaxation of the stress concentration in the relaxation method.

The following conclusions can be drawn:

1) The analysis model that the nonlinear elastic region applied to the adhesive layer has usefulness.
2) The out-of-plane deformation inhibits relaxation of the stress concentration in the plate with a circular hole.
3) Increase of the patch thickness is effective to reduce the out-of-plane deformation.
4) Increase of the patch thickness connects with improving relaxation rate of the stress concentration in the plate with a circular hole.

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