

# An Optimization Model for Natural Gas Supply Chain through a Cost Approach under Uncertainty

A. Azadeh, Z. Raoofi

**Abstract**—Natural gas, as one of the most important sources of energy for many of the industrial and domestic users all over the world, has a complex, huge supply chain which is in need of heavy investments in all the phases of exploration, extraction, production, transportation, storage and distribution. The main purpose of supply chain is to meet customers' need efficiently and with minimum cost. In this study, with the aim of minimizing economic costs, different levels of natural gas supply chain in the form of a multi-echelon, multi-period fuzzy linear programming have been modeled. In this model, different constraints including constraints on demand satisfaction, capacity, input/output balance and presence/absence of a path have been defined. The obtained results suggest efficiency of the recommended model in optimal allocation and reduction of supply chain costs.

**Keywords**—Cost Approach, Fuzzy Theory, Linear Programming, Natural Gas Supply Chain.

## I. INTRODUCTION

SUPPLY chain management has been one of the most important and challenging issues over the recent years. By definition, a supply chain is a network of suppliers, producers, product storage warehouses and distribution centers which would be organized in a way that the process of procurement and receiving parts and raw materials, converting these raw materials into finished products, and distribution of products among customers would be facilitated [1]. Gas supply chain is a huge network of equipment, infrastructures and complex processes which includes a range of operations from extraction of gas to its delivery to the customers. In this chain, suppliers and customers are connected to each other over a long distance and natural gas should flow through this network with a suitable pressure. From the perspective of natural gas supply chain, few papers have considered all the levels of natural gas supply chain. Reference [2] considered a multi-period mix integer linear model for gas supply chain at the three levels of producers, transmission companies and local distribution companies. Reference [3] proposed a single-objective, multi-period mix integer nonlinear mathematical model for multi-echelon gas supply chain and solved their intended distribution programming using a hierarchical algorithm. Reference [4] focused on modeling gas pipelines flow and obtained maximum network flow using linear mathematical programming and through Brazil's database. As mentioned, most of the studies regarding mathematical modeling of natural gas merely focus on one part of supply

chain. In addition; they did not consider uncertainty in their models.

## II. NATURAL GAS SUPPLY CHAIN

Gas industry has particular characteristics which distinguish it from other industries. The strategic importance of the product, fluctuating prices and political pressures affect the gas supply chain. Gas supply chain comprises of three main parts of extraction and refining, transportation network and distribution network, which in turn complicates the issue, because each of these three parts has its own challenges. Exploration and extraction is the first level in the gas supply chain. Then, the produced sour gas is processed and refined. Through nationwide high pressure pipelines and numerous compressor stations, the processed sweet gas is transmitted to consumption centers. Along this path, the demand of main gas customers including power plants and major industries is met and eventually, after going through city gate stations, natural gas enters cities and is delivered to residential customers and commercial-small industries customers. [5]

## III. PROBLEM DEFINITION

In this study, modeling natural gas supply chain has been conducted at five levels. At the first level, two types of suppliers (gas well and importation), at the second level refinery, at the third level compressor station, at the fourth level city gate station and finally at the fifth level, six groups of customers (including oil well injection, exportation, industrial customers, power plants, residential customers and commercial-small industries customers) have been considered. Storage tank, as one of the components of the supply chain, has also been modeled. This network has been formulated with the aim of minimizing supply chain economic costs and in the form of a multi-period fuzzy linear mathematical programming model in which the one-year horizon and periods have been considered discreetly and in the form of six-month periods.

In this chain, gas is transmitted to the refinery after being extracted from a gas well; however, a portion is allocated to sour gas injection to oil wells. After refining the gas, the refinery sends it to the compressor station where a portion is allocated to sweet gas injection to the oil wells. Importation directly enters the network and, along with the gas produced in the refinery, enters the compressor station. Therefore, compressor stations receive the gas from the refinery, origin of importation or another station and deliver it to the storage tank or another station or meet the exportation-industrial-power plant demand or deliver it to the city gate station. This means

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that in order to supply gas to the major gas customers (exportation-industrial-power plant) there is no need for city gate stations. The storage tank, as one of the components of the chain, stores gas in hot seasons and in cold seasons is used for peak shaving. Finally, through city gate stations, gas is supplied to residential customers and commercial-small industries customers. Fig. 1 shows the general scheme of this chain.

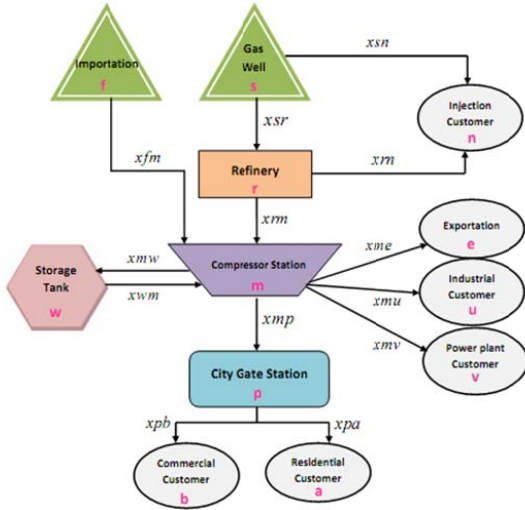


Fig. 1 A schematic of the proposed supply chain problem

#### IV. MATHEMATICAL MODELING

TABLE I  
SETS AND INDICES

Symbol	Quantity
$t$	Time Period $\tau = \{1, 2, \dots,  \tau \}$ $t \in \tau$
$s, f$	Suppliers Set ( $s$ : Gas Well, $f$ : Importation)
$r$	Refineries Set
$m$	Compressor Stations Set
$p$	City Gate Stations Set
$n$	Injection Customer
$e$	Exportation
$u$	Industrial Customer
$v$	Power plant Customer
$a$	Residential Customer
$b$	Commercial-small industrial Customer
$w$	Storage Tank Set
$i$	Starting Nodes $i \in \{s \cup f \cup r \cup m \cup p\}$
$j$	Finishing Nodes $j \in \{r \cup m \cup p \cup n \cup e \cup u \cup v \cup a \cup b\}$

Decision variables of the problem are defined as  $x_{ij}^{jt}$  which indicates the amount of the gas transmitted from the “i”th to the “j”th level in the  $t$  period for different “i”s and “j”s.

TABLE II  
COST PARAMETERS

Symbol	Quantity
$\widetilde{cs}_{st}$	Cost of supply by gas well per unit in period $t$
$\widetilde{cf}_{ft}$	Cost of supply by importation per unit in period $t$
$\widetilde{cr}_{rt}$	Cost of Production by refinery per unit in period $t$
$\widetilde{cm}_{mt}$	Cost of operation by compressor station per unit in period $t$
$\widetilde{cp}_{pt}$	Cost of operation by city gate station per unit in period $t$
$\widetilde{cw}_{wt}$	Cost of operation in storage tank per unit in period $t$
$\widetilde{c}_0$	Transportation cost per product unit per distant unit

TABLE III  
CAPACITY & DEMAND PARAMETERS

Symbol	Quantity
$\widetilde{sc}_{st}$	Capacity of gas well in period $t$
$\widetilde{fc}_{ft}$	Capacity of importation in period $t$
$\widetilde{rc}_{rt}$	Capacity of refinery in period $t$
$\widetilde{mc}_{mt}$	Capacity of compressor station in period $t$
$\widetilde{pc}_{pt}$	Capacity of city gate station in period $t$
$\widetilde{wc}_w$	Capacity of storage tank
$\widetilde{nd}_{nt}, \widetilde{ed}_{et}, \widetilde{ud}_{ut}, \widetilde{vd}_{vt}, \widetilde{ad}_{at}, \widetilde{bd}_{bt}$	Demand of each kind of customers in period $t$

TABLE IV  
ROUTE PARAMETERS

Symbol	Quantity
$lij$	length of the route between node $i$ and node $j$
$oij$	Hardness coefficient of the route between node $i$ and node $j$
$\lambda_{ij}$	1, when there is a route between node $i$ and node $j$ , 0, otherwise
$Q_{ij}^{\min}$	Minimum flow rate between node $i$ and node $j$
$Q_{ij}^{\max}$	Maximum flow rate between node $i$ and node $j$
$\xi_r, \xi_m, \xi_p$	Efficiency coefficient of refinery, compressor station & city gate station

TABLE V  
OBJECTIVE FUNCTION

Formulation	N
$\begin{aligned} \text{Min} Z_1 = & \sum_s \sum_r \sum_t x_{sr}^{st} (\widetilde{cs}_{st} + l_{sr}^{SR} o_{sr}^{SR} \widetilde{c}_0) + \sum_s \sum_n \sum_t x_{sn}^{st} (\widetilde{cs}_{st} + l_{sn}^{SN} o_{sn}^{SN} \widetilde{c}_0) \\ & + \sum_r \sum_m \sum_t x_{rm}^{rt} (\widetilde{cr}_{rt} + l_{rm}^{RM} o_{rm}^{RM} \widetilde{c}_0) + \sum_f \sum_m \sum_t x_{fm}^{ft} (\widetilde{cf}_{ft} + l_{fm}^{FM} o_{fm}^{FM} \widetilde{c}_0) \\ & + \sum_r \sum_n \sum_t x_{rn}^{rt} (\widetilde{cr}_{rt} + l_{rn}^{RN} o_{rn}^{RN} \widetilde{c}_0) + \sum_m \sum_{m'} \sum_t x_{mm'}^{mt} (\widetilde{cm}_{mt} + l_{mm'}^{MM'} o_{mm'}^{MM'} \widetilde{c}_0) \\ & + \sum_m \sum_p \sum_t x_{mp}^{mt} (\widetilde{cm}_{mt} + l_{mp}^{MP} o_{mp}^{MP} \widetilde{c}_0) + \sum_e \sum_t x_{me}^{mt} (\widetilde{cm}_{mt} + l_{me}^{ME} o_{me}^{ME} \widetilde{c}_0) \\ & + \sum_m \sum_u \sum_t x_{mu}^{mt} (\widetilde{cm}_{mt} + l_{mu}^{MU} o_{mu}^{MU} \widetilde{c}_0) + \sum_m \sum_v \sum_t x_{mv}^{mt} (\widetilde{cm}_{mt} + l_{mv}^{MV} o_{mv}^{MV} \widetilde{c}_0) \\ & + \sum_w \sum_t x_{mw}^{wt} (\widetilde{cm}_{mt} + l_{mw}^{MW} o_{mw}^{MW} \widetilde{c}_0) + \sum_w \sum_t x_{wm}^{wt} (\widetilde{cw}_{wt} + l_{wm}^{WM} o_{wm}^{WM} \widetilde{c}_0) \\ & + \sum_p \sum_a \sum_t x_{pa}^{pt} (\widetilde{cp}_{pt} + l_{pa}^{PA} o_{pa}^{PA} \widetilde{c}_0) + \sum_p \sum_b \sum_t x_{pb}^{pt} (\widetilde{cp}_{pt} + l_{pb}^{PB} o_{pb}^{PB} \widetilde{c}_0) \end{aligned}$	(1)

TABLE VI  
CONSTRAINTS

Formulation	N
$\sum_s xsn_{st} + \sum_r xrm_{rt} \geq \widetilde{nd}_{nt} \quad \forall n, t$	(2)
$\sum_m xme_{met} \geq \widetilde{ed}_{et} \quad \forall e, t$	(3)
$\sum_m xmu_{mut} \geq \widetilde{ud}_{ut} \quad \forall u, t$	(4)
$\sum_m xmv_{mvt} \geq \widetilde{vd}_{vt} \quad \forall v, t$	(5)
$\sum_p xpa_{pat} \geq \widetilde{ad}_{at} \quad \forall a, t$	(6)
$\sum_p xpb_{pbt} \geq \widetilde{bd}_{bt} \quad \forall b, t$	(7)
$\sum_r xsr_{rst} + \sum_n xsn_{nt} \leq \widetilde{sc}_{st} \quad \forall s, t$	(8)
$\sum_m xfm_{fmt} \leq \widetilde{fc}_{ft} \quad \forall f, t$	(9)
$\sum_m xrm_{mt} + \sum_n xrm_{nt} \leq \widetilde{rc}_{rt} \quad \forall r, t$	(10)
$\sum_p xmp_{mpt} + \sum_w xmw_{mwt} + \sum_e xme_{met}$ $+ \sum_u xmu_{mut} + \sum_v xmv_{mvt} + \sum_{m'} xmm'_{mm't} \leq \widetilde{mc}_{mt} \quad \forall m, t$	(11)
$\sum_a xpa_{pat} + \sum_b xpb_{pbt} \leq \widetilde{pc}_{pt} \quad \forall p, t$	(12)
$\sum_m \sum_{t'=1}^t xmw_{mwt'} - \sum_m \sum_{t'=1}^t xwm_{wmt'} \geq 0 \quad \forall w, t$	(13)
$\sum_m \sum_{t'=1}^t xmw_{mwt'} - \sum_m \sum_{t'=1}^t xwm_{wmt'} \leq \widetilde{wc}_w \quad \forall w, t$	
$\xi_r \sum_s xsr_{rst} = \sum_m xrm_{mt} + \sum_n xrm_{nt} \quad \forall r, t$	(14)
$\xi_m (\sum_r xrm_{mt} + \sum_f xfm_{fmt} + \sum_w xwm_{wmt} + \sum_{m'} xmm'_{mm't})$ $= \sum_p xmp_{mpt} + \sum_w xmw_{mwt} + \sum_e xme_{met}$ $+ \sum_u xmu_{mut} + \sum_v xmv_{mvt} + \sum_{m'} xmm'_{mm't} \quad \forall m, t$	(15)
$\xi_p \sum_m xmp_{mpt} = \sum_a xpa_{pat} + \sum_b xpb_{pbt} \quad \forall p, t$	(16)
$xsr_{st} \leq M \lambda_{sr}^{SR}, xsn_{st} \leq M \lambda_{sn}^{SN}, xrm_{mt} \leq M \lambda_{rm}^{RM}$ $xfm_{fmt} \leq M \lambda_{fm}^{FM}, xrm_{mt} \leq M \lambda_{rm}^{RN}, xmm'_{mm't} \leq M \lambda_{mm'}^{MM'}$ $xmp_{mpt} \leq M \lambda_{mp}^{MP}, xme_{met} \leq M \lambda_{me}^{ME}, xmu_{mut} \leq M \lambda_{mu}^{MU}$ $xmv_{mvt} \leq M \lambda_{mv}^{MV}, xmw_{mwt} \leq M \lambda_{mw}^{MW}, xwm_{wmt} \leq M \lambda_{wm}^{WM}$ $xpa_{pat} \leq M \lambda_{pa}^{PA}, xpb_{pbt} \leq M \lambda_{pb}^{PB}$	(17)

Equation (1) shows the objective function, the objective function of this problem has been considered as the cost of supplying at each level and the cost of transmission to the next level. Equations (2) through (7) represent the constraints on fulfilling various customers' demands. Constraints on capacity have also been modeled in (8) through (13). Also, (14)

through (16) consider the constraints on input/output balance. Finally, (17) expresses the constraints on presence/absence of a path in the model. Parameter  $\lambda$  represents the presence or absence of a certain path.

## V. CONSIDERING UNCERTAINTY IN THE PROBLEM AREA

Given the dynamic and complex nature of the factors affecting decision-making area in the supply chain management, such decisions face a high level of uncertainty. Failure to adopt an appropriate approach for considering this problem and dealing with it can severely affect the supply chain performance. In this research problem, due to lack of access to objective and historical data, insufficiency of these data and also impossibility of assigning exact numerical values to these parameters, most of the parameters affecting the problem have a fuzzy and non-deterministic nature. Thus, in the proposed model, all the parameters related to demand, cost parameters and parameters related to capacity have been considered as a triangular possibility distribution function. In fact, model parameters have been expressed in the form of fuzzy numbers with triangular distributions such as  $\tilde{A} = (A^p, A^m, A^o)$  which represent the most pessimistic, most likely and most optimistic values of these parameters respectively [6], [7]. It is noteworthy that in the proposed model, to represent parameters which face uncertainty in the decision-making area, “~” sign has been used.

In the literature, several methods have been presented for conversion of possibilistic models with imprecise coefficients in the objective function as well as constraints into the deterministic model [8]-[10]. In the presented solution in this research, in order to convert the proposed fuzzy linear model into its equivalent deterministic model, the approach proposed by [8] has been used due to its high efficiency. Due to prolongation of Contents, just two constraints of defuzzy model was presented here. Other ones are the same.

$$\sum_s xsn_{st} + \sum_r xrm_{rt} \geq \beta \left( \frac{nd_{nt}^2 + nd_{nt}^3}{2} \right) + (1-\beta) \left( \frac{nd_{nt}^1 + nd_{nt}^2}{2} \right) \quad \forall n, t$$

$$\sum_r xsr_{rst} + \sum_n xsn_{nt} \leq \beta \left( \frac{sc_{st}^1 + sc_{st}^2}{2} \right) + (1-\beta) \left( \frac{sc_{st}^2 + sc_{st}^3}{2} \right) \quad \forall s, t$$

## VI. NUMERICAL EXAMPLE

To verify the proposed model, a small-sized problem with random data and within reasonable intervals has been solved using GAMS 23.2 software. The considered network has been considered as 2 gas wells, 1 origin of importation, 1 refinery, 3 compressor stations, 2 city gate stations, 1 storage tank, 2 injection customers, 1 exportation customer, 1 industrial customer, 1 power plant customer, 3 residential customers and 3 commercial-small industries customers. The general scheme of the abovementioned network is as shown in the below figure.

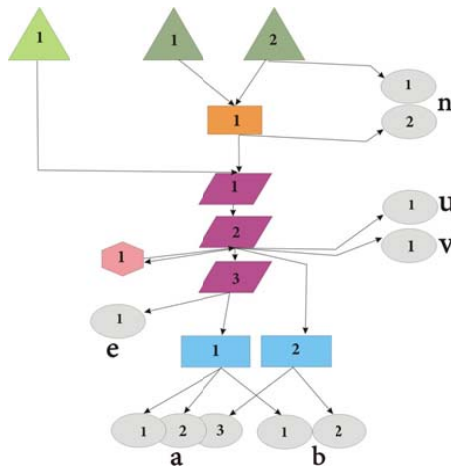


Fig. 2 The general scheme of the example problem

In order to sensitivity analysis of the model, the costs of the gas supply chain for each unit at different levels has been reduced up to 10 percent and as a result of this reduction, 116.2171 million Dollars were saved on economic costs.

One of the important parameters in studying the performance of the possibilistic programming model is ( $\beta$ ) parameter which represents the minimum acceptable feasibility degree defined by the decision-maker for satisfying the constraints. Sensitivity analysis of various ( $\beta$ ) parameter values and the amount of the objective function have been shown in the below table.

TABLE VII  
SENSITIVITY ANALYSIS OF THE POSSIBILISTIC PROGRAMMING MODEL FOR VARIOUS  $\beta$  VALUES

$\beta$ -value	$\beta=0.5$	$\beta=0.6$	$\beta=0.7$	$\beta=0.8$	$\beta=0.95$	$\beta=1$
objective function (million Dollars)	820.23 6	880.77 6	941.48 7	984.83 6	1045.8 55	1035.88 8

As it is seen, as the degree of satisfying constraints increases; costs of fulfilling the customers' demand also increase. In fact, the possibility of finding a better answer for the objective function (a lower  $z$  value) requires considering a lower  $\beta$  value. Therefore, the decision-maker faces two contradictory objectives; improving the amount of the objective function which is possible through a lower  $\beta$  value and improving the degree of satisfying constraints which is possible through a higher  $\beta$  value.

## VII. CONCLUSION

In this research, the general structure of natural gas supply chain has been identified and a suitable multi-echelon, multi-period fuzzy linear mathematical model has been developed with the aim of minimizing economic costs at all the levels of the chain. Also in the proposed model, possibility theory and fuzzy numbers have been used to express the uncertainty of the problem. The outputs obtained from the model regarding the optimal amount of transmission between different levels

indicate rationality and validity of the proposed model. Also, using the results of the model, besides providing a clear picture of the total chain costs, makes it possible to calculate the cost of production, refining and transmission for each customer and this really helps to have a proper and numerical understanding of the issue of costs in the natural gas supply chain.

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