Numerical Investigation of the Effect of Number of Waves on Heat Transfer in a Wavy Wall Enclosure

Ali Reza Tahavvor, Saeed Hosseini, Afshin Karimzadeh Fard

Abstract—In this paper the effect of wall waviness of side walls in a two-dimensional wavy enclosure is numerically investigated. Two vertical wavy walls and straight top wall are kept isothermal and the bottom wall temperature is higher and spatially varying with cosinusoidal temperature distribution. A computational code based on Finite-volume approach is used to solve governing equations and SIMPLE method is used for pressure velocity coupling. Test is performed for several different numbers of undulations. The Prandtl number was kept constant and the Ra number denotes that the flow is laminar. Temperature and velocity fields are determined. Therefore, according to the obtained results a correlation is proposed for average Nusselt number as a function of number of side wall waves. The results indicate that the Nusselt number is highly affected by number of waves and increasing it decreases the wavy walls Nusselt number; although the Nusselt number is not highly affected by surface waviness when the number of undulations is below one.

Keywords—Cavity, natural convection, Nusselt number, wavy wall.

I. INTRODUCTION

NATURAL convection heat transfer phenomena is widely used in engineering application and scientific issues due to its reliability, low cost, low noise and geometrical simplicity. Wavy walled structures have a wide range of appliance in engineering fields such as heat exchangers, nuclear reactor systems, geology and also biology. Many researchers studied the issue in recent years.

For example the influence of wave amplitude, and number of the waves on the natural convection inside a solar collector with water based nanofluid was analyzed numerically by Nasrin et al. [1]. They found convection form has higher effect on heat transfer compared to radiation form. Also the structure of the fluid streamlines and isotherms within the solar collector is found to significantly depend upon amplitude and number of the waves. The effect of the undulated hot wall on the heat transfer by natural convection in an inclined square cavity has been investigated numerically [2]. The results showed that the undulation of the hot wall affects the fluid flow and heat transfer rate. Dalal and Das [3] numerically studied laminar natural convection inside a two-dimensional enclosure with three flat and one wavy wall. Top wall

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temperature profile was assumed sinusoidal. The problem was solved by SIMPLE algorithm and deferred QUICK scheme. The obtained results shows that the heat transfer mode remains conductive up to Ra = 1000. As the Ra number increases, the mode of heat transfer changes from conduction to convection. A numerical prediction of heat transfer and fluid flow characteristics inside an enclosure bounded by two isothermal wavy walls has been carried out [4]. The finite-volume method is used for governing equation discretization. For a certain value of the Grashof number higher heat transfer is observed at lower aspect ratios defined by amplitude/average width.

Laminar steady natural convection and fluid flow in a wavy enclosure are investigated numerically by Abdelkader et al. [5]. They reported two circulation cells are obtained in the enclosure cavity of wave in different directions especially at the small aspect ratio. Mahmud and Sadrul Islam [6] numerically investigated fluid flow and heat transfer inside a wavy enclosure. They presented volume average entropy generation rate and also contour of Bejan number as results. Unsteady heat transfer and fluid flow characteristics in an enclosure are investigated by Rostami [7]. The enclosure contained two vertical wavy walls, the results show that the Nusselt number increases by reduction in amplitude, therefore maximum value of the Nusselt number obtains in rectangular form. Another obtained result is that at initial times heat transfer is mainly by conduction, but as the time goes on, convection heat transfer is more dominant.

Das and Mahmud [8] investigated thermal and hydrodynamic behaviors of fluid within an enclosure with top and bottom wavy isothermal walls. The streamlines and isothermal lines for different amplitude-wavelength ratios were presented as results. They found that the amplitude-wavelength ratio has no significant effect on average heat transfer rate. Numerical results of natural convection heat transfer inside an enclosure subjected to steady sinusoidal temperature boundary condition on one wall and constant temperature on other three walls are presented by Dalal and Das [9]. One of the results shows for small Ra, it is possible to increase Nusselt number on the wavy wall by increase of amplitude. Heat transfer by natural convection within a wavy enclosure with uniform internal heat generation is studied numerically [10]. The obtained results showed that the flow field and heat transfer characteristics were affected by the amplitude of the wavy wall and the values of the internal to external Rayleigh number. By increasing the amplitude, higher Nusselt number were predicted.

Natural convection within a cavity made of two horizontal

straight walls and two vertical wavy walls filled with a fluid-saturated porous medium has been numerically examined by Misirlioglu et al. [11]; an interesting outcome was that the local Nusselt number of vertical walls could even be negative by appropriate surface waviness, aspect ratio, and Ra number. That means vertical walls may even become negative; negative Nusselt number means the heat generated in the porous medium cannot be transferred through the porous medium from the hot wall to the cold wall. Natural convection in an enclosure filled with fluid-saturated porous media containing two wavy vertical walls is studied numerically, Sompong and Witayangkurn [12]. From the results it was observed that the increase of wave amplitude affects the flow intensity inside the cavity.

Rahimi et al. [13] numerically studied Natural convection of mixture of nanoparticles and water near in a rectangular enclosure. They showed that the average Nusselt number increases by increasing nanoparticle volume fraction. The effect of Grashof number and nanoparticles volume fraction on fluid flow and natural convection heat transfer within a wavy enclosure is numerically investigated [14]. They reveal that the nanoparticles can be used for decreasing the entropy generation. Morsli and Sabeur-Bendehina [15] investigated the natural convection and entropy generation inside a wavy wall square cavity. The total entropy generation is found to be increasing with increasing undulation number. The natural convection hat transfer and fluid flow in an inclined wavy porous cavity has been numerically studied [16] it was found that the rate of heat transfer increases as angle of inclination increases. Obayedullah et al. [17] have been studied the natural convection in a rectangular enclosure with sinusoidal temperature distribution at the bottom wall. Cho et al. [18] and Mansour and Bakier [19] have been studied natural convection within a wavy enclosed cavity filled with nanofluid numerically. Both articles are the same. The obtained results shows that by tuning the wavy-surface geometry the heat transfer performance could be optimized.

Cho et al. [20] performed a numerical investigation on natural convection heat transfer performance and entropy generation within an enclosure containing nanofluids. An important result was the total energy generation can be minimized and the Nusselt number can be maximized via an appropriate tuning of the wavy surface geometry parameters. The effect of geometry on transition to unsteadiness has been studied by Ramgadia and Saha [21] they reported by increasing the Reynolds number; the steady flow shows a decrease in thermal performance factor while an increase in thermal performance factor is noticed when the flow is unsteady. Das et al. [22] numerically studied the effect of aspect ratio and surface waviness in an enclosure with top and bottom wavy walls. The results reported that natural convection heat transfer is changed considerably when surface waviness changes and also it depends on the aspect ratio of the domain.

Due to authors review of literature the effect of undulation on heat transfer of the fluid within an enclosure containing two vertical walls is not studied yet and the subject is very

functional for industries. In the present work the bottom wall temperature distribution is assumed cosinusoidal and other three walls are isothermal.

II. METHODOLOGY

A. Problem Specification

Consider a cavity with vertical isothermal wavy walls; the upper wall is kept at constant temperature. Both top and side walls have a temperature of 273 K. The bottom wall temperature is spatially varying with cosinusoidal temperature distribution; its value in the middle of the wall is maximum (293 K) and reduces to its minimum value (273 K) at side walls. The bottom wall distribution is illustrated in Fig. 1.
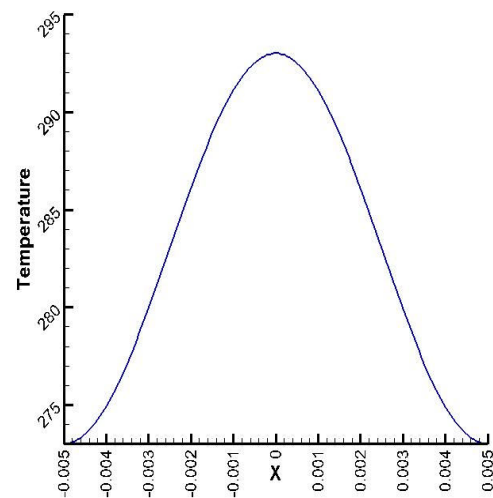

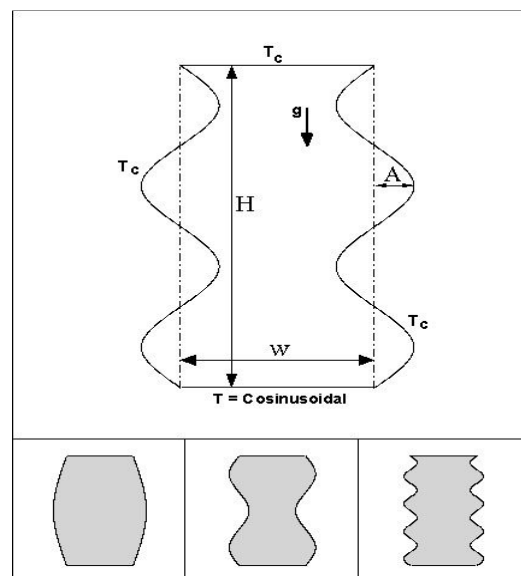Fig. 1 Temperature distribution of the bottom wall

Fig. 2 Problem geometry

Several cases have been studied. Number of undulations is varied between 0 and 4. Fluid in the cavity is air and the

Prandtl number was 0.71 where the Ra number varied from 3600 to 7400 due to cavity geometry change by different amplitudes. According to range of the Ra number the problem is laminar. The geometry is symmetric, therefor half of cavity has been considered as the case. Fig. 2 provides the problem geometry where T_c is the colder wall and T_h is the hot bottom wall. A is amplitude and N is number of the waves.

B. Governing Equations

Natural convection in a wavy walled enclosure can be expressed by mass, momentum and energy conversations equations in differential form. The problem is considered steady-state and two-dimensional w. The governing equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_C) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where the wavy walls are kept at constant temperature and no-slip, no-jump and no-permeability condition are considered for all walls. The bottom wall temperature varies as follows:

$$TH = 10\,(1 - \cos(2 \cdot \pi \cdot 100(y+0.005)))+273 \tag{5}$$

by assuming:

$$X = x/L$$
$$Y = y/L$$
$$U = u\,L/\alpha$$
$$V = v\,L/\alpha$$
$$P = pL2/\rho\alpha2$$
$$Pr = \nu/\alpha$$
$$\theta = (T\text{-}TC)/(TH\text{-}TC)$$
$$Ra = g\beta(TH\text{-}TC)L3 \cdot Pr / \nu2$$

The dimensionless form of (1)-(4) is presented as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{6}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{7}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + RaPr\theta \tag{8}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{9}$$

C. Computation Methods

Domain is discretized according to Delauny triangulation method [23]. Fig. 3 shows the grid type and distribution. Also dimensionless governing equations (6)-(9) have been solved using finite volume approach. SIMPLE algorithm is used for velocity and pressure coupling and discretization scheme of momentum and energy equations is the second order upwind [24].
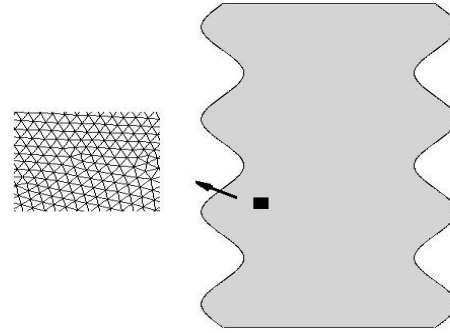

Fig. 3 Grid type and distribution

III. RESULTS AND DISCUSSION

A numerical study has been carried out to determine the influence of number of the waves on average Nusselt number of the wavy walls. The Prandtl number is kept constant at 0.71, Ra number varies from 3600 to 7400. Numbers of undulations are varied from 0 to 4 by step of 0.5.

A. Grid Study

The grid independency test has been performed for several cases with several grid sizes. Fig. 4 shows grid studies due to average Nusselt number for three cases. As showed in the figure for grid sizes above 100000 nodes, Nusselt number values have no significant variations; therefore the problem is grid independent.
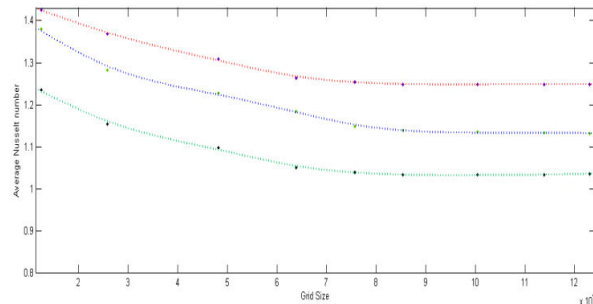

Fig. 4 Grid independency test Top: N=1.5, Middle: N=1.5, Bottom: N=3

B. Effect of Number of Waves

Wavy surface can affect the total heat transfer rate and average Nusselt number. Effect of number of side walls waves on Nusselt number is investigated numerically for 8 different cases. Fig. 5 shows the temperature field of the enclosure for different number of undulation. Amplitude of the waves is 0.1. As the waviness increases from 0 to 4 the hot region increases and covers more space of the enclosure. It means that the side walls Nusselt number in higher number of undulation are lower. All walls except the bottom one is kept at cooler constant temperature. When the heat from the bottom wall goes higher it means the wavy wall Nusselt number is reduces. The effect of surface waviness on side walls Nusselt number is a result of locked out air near the wavy wall. It reduces the contact area and causes the heat to go higher.
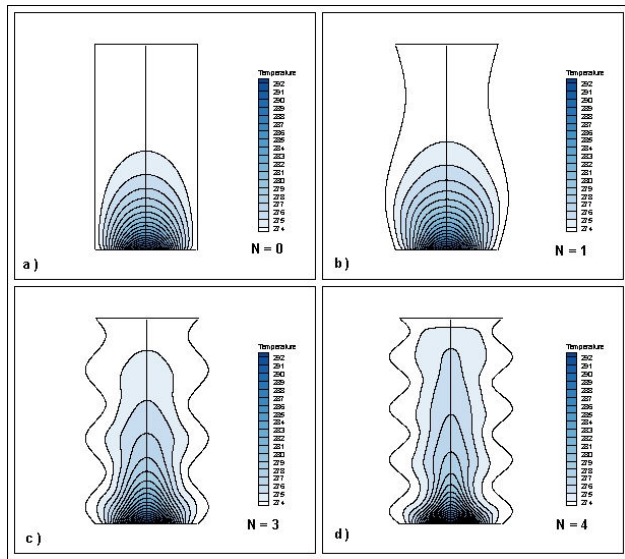
Fig. 5 Temperature field for A=0.1

Fig. 6 demonstrates the average Nusselt number vs. number of waves on the side walls .it reveals that the Nusselt number decreases by increasing the number of waves. When the amplitude is low the surface waviness doesn't affect the Nusselt number significantly because it acts like the straight wall and the locked out air is minimum. Amplitude of the side walls waves also affects the Nusselt number and total heat transfer rate of the enclosure. As the amplitude of waves increases the volume of locked out air increases and it causes a reduction in the wavy wall average Nusselt number as a result of reduction in contact area. When the number of wall waves is below 1 the effect on the Nusselt number is less observable. This occurs because of less skewness of wavy walls respect to the vertical plane. Small angles hold lower amount of air and its reduction effect to the Nusselt number diminishes.

Fig. 6 shows the effect of number of waves on the Nusselt number. The vertical axis shows the Nusselt number where the number of waves varies from 0 to 4.
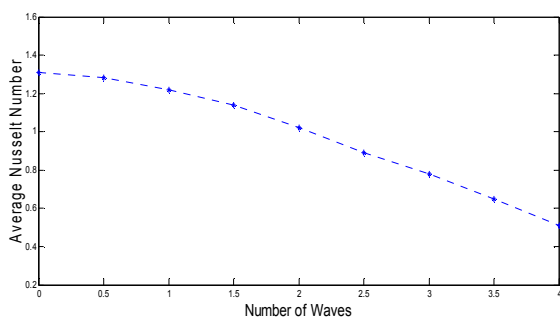

Fig. 6 Number of waves vs. average Nusselt number

C. Average Nusselt Number Correlation

Following correlation is proposed for wavy walls average Nusselt number respect to number of waves.

$$\ln Nu = a + b\,N^2 \qquad (9)$$

The coefficients, a and b are 0.2610553 and -0.0579134. For this equation the statistical parameters R^2 and Sum Square Error are 0.9978 and 0.00986 respectively.

IV. CONCLUSION

In this study natural convection in a two-dimensional wavy enclosure has been investigated numerically. Side wavy walls and top straight wall were kept isothermal where the temperature distribution of the bottom straight wall had a cosinusoidal form. The Prandtl number was 0.71 and the Ra number varies from 3400 to 76000 due to geometry change. Several different number of side walls waves from 0 to 4 has been investigated. Finite volume approach is used to solve the dimensionless governing equations. Some important points from obtained results are:

- As the waviness increases, the hot region increases and covers more space of the enclosure.
- Waviness on side walls locks out the air near the wavy wall. It reduces the contact area and causes the heat to go higher.
- When the amplitude is low the surface waviness doesn't affect the Nusselt number significantly because it acts like the straight wall and the locked out air is minimum.
- A correlation is proposed for average Nusselt number in terms of number of waves and amplitude as (9)

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