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Structural Reliability of Existing Structures: A Case Study

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***Abstract*—**A reliability-based methodology for the assessment and evaluation of reinforced concrete (R/C) structural elements of concrete structures is presented herein. The results of the reliability analysis and assessment for R/C structural elements were verified by the results obtained through deterministic methods. The outcomes of the reliability-based analysis were compared against currently adopted safety limits that are incorporated in the reliability indices β's, according to international standards and codes. The methodology is based on probabilistic analysis using reliability concepts and statistics of the main random variables that are relevant to the subject matter, and for which they are to be used in the performance-function equation(s) associated with the structural elements under study. These methodology techniques can result in reliability index β, which is commonly known as the reliability index or reliability measure value that can be utilized to assess and evaluate the safety, human risk, and functionality of the structural component. Also, these methods can result in revised partial safety factor values for certain target reliability indices that can be used for the purpose of redesigning the R/C elements of the building and in which they could assist in considering some other remedial actions to improve the safety and functionality of the member.

***Keywords*—**Concrete Structures, FORM, Monte Carlo Simulation, Structural Reliability.

I. Introduction

COMBINING the traditional structural engineering assessment approaches with reliability-based methods to evaluate the integrity and safety of existing R/C and steel buildings has gained momentum over the last few years, and the use of the concept has increased significantly. This concept is even more vital when used and conducted in cases where remedial actions and monitoring for defected R/C structural components are required. The rationale behind this combined approach for engineering solution to assess the structural integrity and safety can be an extremely difficult task, especially when it is not restricted to a specific area of the structure. This approach requires, but is not limited to, reviewing original documents, conducting visual inspection, performing destructive and nondestructive tests, using deterministic structural analysis, and applying a reliability-based analysis method.

The reliability-based design of any structure requires the consideration of the following three components: loads, structural strength, and methods of reliability analysis. These three components are essential for the development of reliability-based load and resistance factor design (LRFD). There are two primary approaches for reliability-based design and analysis [1]: (a) direct reliability-based design and (b) load and resistance factor design. The LRFD approach is called a Level 1 reliability method. Level 1 reliability methods utilize partial safety factors (PSFs) that are reliability based, but the methods do not require explicit use of the probabilistic description of the variables.

The direct reliability-based design and analysis methods use all available information on the basic random variables for strength and load effects, and do not simplify the limit state function(s) in any manner ([1] and [2]). These methods require performing spectral analysis and extreme analysis of the loads. In addition, linear or nonlinear structural analysis can be used to develop a stress frequency distribution. Then, stochastic load combinations can be performed. Linear or nonlinear structural analysis can then be used to obtain deformation and stress values. The appropriate loads, strength variables, and failure definitions need to be identified for each failure mode. Using reliability assessment methods, such as the first-order reliability method (FORM), reliability indices βs for all failure modes at all levels need to be computed and compared with target reliability indices β_Ts. The relationship between the reliability index β and the probability of failure is given by [2]-[4] as

$$p_f = 1 - \Phi(\beta) \tag{1}$$

where $\Phi(.)$ = cumulative probability distribution function of the standard normal distribution, and β = reliability index. Equation (1) assumes all the random variables in the linear limit state equation to have a normal probability distribution. For all practical purposes, (1) can be used to estimate the failure probability p_f with sufficient accuracy [2], [4].

The most commonly and widely used design format is given by [5] as

$$\phi R_n = \sum \gamma_i L_{ni} \tag{2}$$

where ϕ = the resistance reduction factor, R_n = strength, γ_i = partial load amplification factor for each type of load i, and L_{ni} = nominal load effect for each type of load effect i. Equation (2) implies the use of load amplification factors and resistance reduction factor (or called partial safety factors). The American Institute of Steel Construction (AISC), the American Concrete Institute (ACI), the U.S. Navy, and many other societies and industries have recently adopted and incorporated the use of this design format. Also, a

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recommendation for its use was provided by the National Institute of Standards and Technology, NIST [5].

In this study, two buildings were examined and evaluated. One building is a 7-y old recreational building, whereas the other is a 42-ys old office building. Deterministic structural analysis and design verification demonstrated that some beams in the recreational building and some columns in the office building did not satisfy the ACI318-1 [6] strength requirements.

II. Materials Factors

In this study, a total of 128 concrete cores were extracted and tested from both buildings; 91 cores from the recreational building and 37 cores from the office building. The concrete compressive strength f_c' was quantified based on statistical analysis and Monte Carlo simulation for the 128 concrete core tests that were obtained from randomly selected structural elements. It was concluded from the simulation and the probabilistic best-fit that the compressive strength of concrete f_c' is to follow a Lognormal distribution. Fig. 1 shows the probabilistic best-fit for the compressive strength of the 91 cores that were extracted from the recreational building.

Table I shows the probabilistic parameters for the concrete compressive strength in both buildings. For the reinforcing steel with Grade 420 MPa, a bias factor λ=1.13 and a coefficient of variation COV = 4% has been suggested [7].

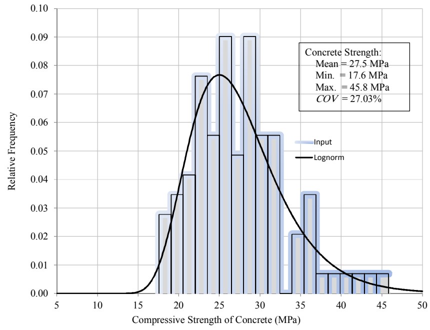

Fig. 1 Probabilistic best-fit of concrete cylinder compressive strength in the recreational building

TABLE I
Statistical Parameters of Concrete Compressive Strength f_c'.

Building	No. of cores	Mean (MPa)	Min. (MPa)	Max. (MPa)	*COV*	Distribution
Recreational	91	27.5	17.6	45.8	27.0%	Lognormal
Office	37	11.0	4.84	23.5	28.0%	Lognormal

III. Applied Loads

Structural loads (dead, live, and wind) were estimated based on the ASCE 7-10 [8], whereas the statistical parameters were estimated based on [5], [7], [9], as listed in Table II. Support settlements were measured and hence, loading due to support settlement were assumed to be deterministic.

TABLE II
Statistical Parameters of Structural Loads

Loading	Bias Factor, λ	COV	Distribution
Dead	1.05	0.29	Normal
Live	1.25	0.10	Normal
Wind	0.78	0.37	Type I

Tables III and IV give the nominal loads acting on selected weakened beams in the recreational building and selected weakened columns in the office building, respectively. The dead, live, and wind loads specified in these tables are the maximum values over a referenced return period of 50 y.

TABLE III
Nominal Moments Acting on Selected Beams in the Recreational Building

Beam	Dead Load Moment (kN.m)	Live Load Moment (kN.m)	Moment due to Support Settlement (kN.m)
B1	3092	1008	-
B2	3400	1145	-
B3	3693	1244	-
B4	2590	884.2	993.4
B5	2448	877.9	-

TABLE IV
Nominal Loads Acting on Selected Columns in the Office Building

Column	Load	Dead Load	Live Load	Wind Load
C1	Axial (kN)	4,150	641	84.7
	Moment (kN.m)	95	43	12.8
C2	Axial (kN)	450	54.8	-
	Moment (kN.m)	109	42.6	

IV. Fabrication and Professional Factors

Information that is also vital to reliability analysis is the statistics of fabrication and professional factors. A fabrication factor represents the variation in dimensions and geometry, whereas a professional factor represents the variation in the ratio of the actual resistance and what can be analytically predicted using accurate material strength and dimension values. Table V shows statistical data on fabrication parameters. Whereas Table VI provides information on professional factors.

TABLE V
Statistical Information on Fabrication Random Variables

	Bias Factor, λ	*COV*	Distribution Type
Width of Cross Section, *b*	1.01	0.04	Normal
Height of Cross Section, *h*	0.99	0.04	Normal

TABLE VI
Statistical Information on Professional Factor

	Bias Factor, λ	*COV*	Distribution Type
Beams in flexure	1.02	0.06	Normal
Tied columns	1.00	0.08	Normal

V. Ultimate Resistance of Flexural Members

The ultimate bending capacity (flexural resistance) of R/C members in flexure is determined using the following formula [10]:

$$M_n = A_s f_y \left(d - \frac{A_s f_y}{1.7 f_c' b} \right) \quad (3)$$

where M_n = ultimate moment capacity, A_s = cross sectional area of reinforcement steel, b = width of rectangular section of the beam, d = distance from the center of reinforcement to the upper edge of the rectangular section of the beam, f_y = yield strength of steel, and f_c' = compression strength of concrete. Using an electromagnetic device, the concrete cover in 30 beams from the recreational building was measured. The average concrete cover was found to be 33.6 mm with a standard deviation of 5.46 mm and had a normal distribution.

In reliability analysis, (3) constitutes the flexural strength of the limit state function. TABLE VIITable VII shows the geometrical properties (b and h) and reinforcement (A_s) for five beams in the recreational building. The deterministic analysis results for these beams showed that their flexural capacities were not adequate.

Based on the general form of (3) that represents strength, and also on the moments M_D, M_L, and M_{ss} due to dead, live, and support settlement load effects, respectively, the flexural performance or limit-state function for an R/C beam section can be given as shown in the following equation:

$$g = A_s f_y \left(d - \frac{A_s f_y}{1.7 f_c' b} \right) - M_D - M_L - M_{ss} = 0 \quad (4)$$

VI. Ultimate Resistance of R/C Columns

The resistance of a loaded column depends on the applied moment(s) and axial force(s). The limit state can be defined in the form of an interaction diagram [11]. A typical interaction diagram is shown in Fig. 2. In terms of the position of the load on the cross section of a column, there are two main types of R/C columns, namely, concentrically loaded column that carries no moment, as shown in Fig. 3 (a), and eccentrically loaded column subjected to a bending moment as well as to an axial force as shown in Figs. 3 (b) and (c). Realistically, concentrically loaded column are almost nonexistent. Practically, all columns have to be designed for some unforeseen eccentricity and can be considered in most cases as beams with relatively higher axial loading. The moment can be converted to a load P and eccentricity e, as shown in Fig. 3.

Over the years, resistance formulas for columns have been developed by researchers based on the equilibrium of the external forces in the section. The statistical parameters and characteristics of resistance, M_n, were calculated using Monte Carlo simulations and formulas. The variables that were considered as random variables include: strength of concrete f_c', yield strength of reinforcing steel f_y, dimension of the cross section, and area of reinforcing steel. Table VIII shows the geometrical properties (b and h) and reinforcement (A_s) for three selected weakened columns from the office building. The deterministic analysis results for these columns showed that their combined flexural and axial capacities were inadequate.

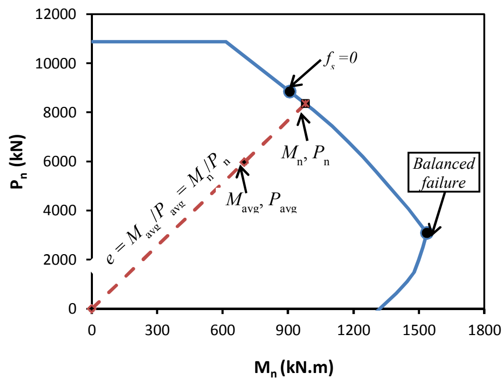

Fig. 2 Typical interaction diagram for an eccentrically loaded short column

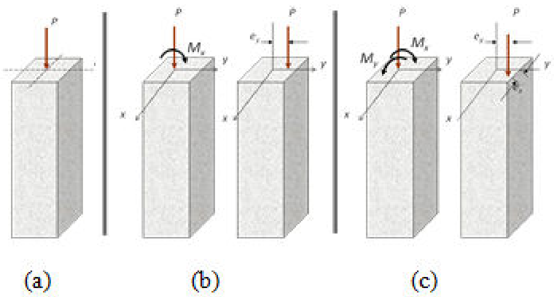

Fig. 3 Axially loaded column with (a) no eccentricity, (b) uniaxial moment, and (c) biaxial moment [13]

VII. Reliability Analysis

The purpose of this analysis is to estimate the reliability indices βs and/or failure probabilities *PF*s for selected weakened and/or damaged structural elements. The computed values of βs were compared with the corresponding target values of these indices as adopted by international code standards. The partial safety factors (PSFs) were also calculated based on the currently used reliability indices according to international standards, or on sound engineering judgment. These PSFs could be used should there be any remedial structural measures to fix the weakened structural elements of the buildings.

In this study, FORM and @RISK were used to compute the reliability indices and failure probabilities for the selected R/C structural elements. FORM was used to generate the partial safety factors for specified and recommended target reliability indices β_Ts, based on international standards and/or sound engineering judgment.

TABLE VII
DIMENSIONS, REINFORCEMENT, AND STRAIN IN EXTREME TENSION STEEL LAYER OF SELECTED BEAMS IN THE RECREATIONAL BUILDING

Beam	Width (mm)	Depth (mm)	Reinforcement				
			Longitudinal			Transverse	
			Bars	Layers	ε_t	No. of Stirrups and Size	s (mm)
B1	1000	1000	30 No. 32	3	0.00246	2 No. 16	150
B2	1000	1000	33 No. 32	3	0.00223	2 No. 16	125
B3	1200	1000	38 No. 32	3	0.00237	3 No. 13	100
B4	1000	1000	24 No. 32	3	0.00297	2 No. 16	150
B5	800	1000	21 No. 32	3	0.00268	2 No. 16	150

TABLE VIII
DIMENSIONS AND REINFORCEMENT OF SELECTED COLUMNS IN THE OFFICE BUILDING

Column	b (mm)	h (mm)	Reinforcement				
			Longitudinal		Transverse		ρ
			Bars	Distribution	Tie Size	s (mm)	
C1	600	600	16 No. 32	All Sides equal	3 No. 8	200	3.64%
C2	400	400	8 No. 19	All Sides equal	2 No. 6	200	1.42%

TABLE IX
STATISTICAL CHARACTERISTICS OF LOAD AND RESISTANCE VARIABLES GENERATED BY @RISK

Beam	M_D (kN.m)		M_L (kN.m)		M_{ss}	M_n (kN.m)	
	μ	σ	μ	σ	(kN.m)	μ	σ
B1	3,246	325	1,259	365	-	7,684	834
B2	3,569	357	1,430	415	-	8,355	908
B3	3,877	388	1,554	451	-	9,729	1,052
B4	2,721	272	1,106	321	993.4	6,347	691
B5	2,571	257	1,098	318	-	5,473	594

FORM was designed to handle simple to moderate limit-state or performance functions. So it has limitations when dealing with complex performance functions that consist of multi and numerous random variables. On the other hand, @RISK is a spreadsheet-based commercial computer software developed by Palisade Corporation. This software can perform reliability analysis based on Monte Carlo simulation without any limits regarding the formulation of a performance function under study. The only limitation with this program is its inability to compute the partial safety factors for both analysis and design procedures. In this study, both programs were used to perform the reliability-based analyses; in other words, they provided the advantage of the better of the two worlds. @RISK was used to quantify the statistical characteristics of strength for the selected beams and columns, which are functions of many random variables.

TABLE X
VALUES OF β AND P_F FOR SELECTED BEAMS GENERATED BY @RISK

Beam	β	P_F
B1	3.63	1.41E-04
B2	3.42	3.18E-04
B3	3.90	4.85E-05
B4	1.97	2.40E-02
B5	2.67	3.81E-03

TABLE XI
VALUES OF β AND PSFS FOR STRENGTH AND LOADS FOR SELECTED BEAMS GENERATED BY FORM

Beam	β	ϕ	γ_L	γ_D
B1	3.636	0.73	1.49	1.15
B2	3.446	0.74	1.48	1.14
B3	3.943	0.71	1.55	1.16
B4	1.970	0.84	1.26	1.08
B5	2.660	0.80	1.39	1.11

TABLE XII
AVERAGE VALUES OF RELIABILITY INDEX β AND CORRESPONDING FAILURE PROBABILITY P_F FOR R/C BEAMS IN THE RECREATIONAL BUILDING

Beam	β_{Avg}	P_F
B1	3.636	1.39E-04
B2	3.446	2.84E-04
B3	3.943	4.03E-05
B4	1.970	2.44E-02
B5	2.660	3.91E-03

A. Results of Reliability Analysis for the Beams

@RISK was employed to calculate and compute the statistical characteristics of loads (M_D, M_L & M_{ss}), bending moment resistance (M_n), reliability indices, probabilities of failure, and coefficients of variation for selected beams. These results are listed in Tables IX and X. The load and resistance statistical variables (listed in Table IX) were used in utilizing FORM to compute the reliability indices and the PSFs for the R/C beams. The results of FORM are listed in Table XI. The results listed in Tables X and XI show good agreements with the results obtained from Monte Carlo simulation, as implemented in @Risk and FORM. Table XII lists the average or mean values of the reliability indices for selected R/C beams and the associated probabilities of failure.

In order to compute the partial safety factors using FORM, it was necessary to convert the performance function of (2) to the following simplified version:

$$g = M_n - M_D - M_L - M_{ss} = 0 \quad (5)$$

where M_n as defined by (3), and M_D, M_L and M_{ss} are the moments due to dead, live, and support settlement loads, respectively. This action was necessary because, as was noted earlier, FORM has some limitations in handling complex performance function. Now, with the performance function of (3) and noting that M_n possesses all the statistical characteristics that were quantified by @RISK, which are shown in Table IX, FORM at this point can compute the partial safety factors.

These partial safety factors can be used in the design equation when redesigning the structural R/C beams that were of insufficient strength. Using the PSFs of Table XI, the performance function of (3) can be converted to design equations for beams B4 and B5, respectively, as follows:

$$0.84M_n \geq 1.26M_L - 1.08M_D - M_{ss} \quad (6)$$
$$0.84M_n \geq 1.26M_L - 1.08M_D - M_{ss} \quad (7)$$

Existing design codes are based on certain target reliability indices. The target reliability indices for beams in flexure (tension-controlled failure) and for axially loaded columns (compression-controlled failure) are 3.5 and 4.0, respectively [12]. The strain in the extreme tension layer of steel (ε_t) in all selected beams have values that are less than the tension-controlled limit of 0.005 and higher than the compression-controlled limit of 0.002. This means that all these beams are in the linear transition zone. Therefore, the target reliability index for each beam is calculated linearly based on ε_t that is listed in Table VII.

Based on the target reliability index, FORM was used to calculate the required strength and the revised partial safety factors for each beam, as listed in Fig. 3.

B. Results of Reliability Analysis for the Columns

Similar to the analysis done for beams columns C1 and C2 were analyzed using @RISK and FORM. The final results are listed in Tables XIV to XVIII. The analysis has demonstrated that column C1 was in the compression-controlled zone, whereas column C2 was in the tension-controlled zone.

TABLE XIII
PARTIAL SAFETY FACTORS BASED ON TARGET RELIABILITY INDEX β_T

Beam	β_T	Required Strength M_n (kN.m)	ϕ	γ_L	γ_D
B1	3.924	7,908	0.723	1.538	1.164
B2	3.949	8,781	0.727	1.554	1.164
B3	3.939	-	-	-	-
B4	3.838	7,667	0.760	1.531	1.155
B5	3.888	6,254	0.758	1.590	1.164

Fig. 4 shows the probability distribution function of the total average acting axial load on column C1 versus the probability distribution function of the axial load resistance for the same column. The common area between the two curves represents the probability of failure of C1. Fig. 5 shows the interaction diagram for column C1.

TABLE XIV
STATISTICAL CHARACTERISTICS OF LOAD AND RESISTANCE VARIABLES GENERATED BY @RISK

Column			C1 Moment (kN.m)	C1 Axial Load (kN)	C2 Moment (kN.m)	C2 Axial Load (kN)
Loads	Dead	μ	99.8	4,350	115	473
		σ	9.65	417	10.9	45.3
	Live	μ	53.5	799	53.6	68.5
		σ	12.5	185	12.6	15.8
	Wind	μ	10.6	70.1	-	-
		σ	4.79	32.1	-	-
Resistance (M_n, P_n)		μ	238	7,570	176	564
		σ	39.7	763	23.7	107

TABLE XV
VALUES OF β AND P_F FOR SELECTED BEAMS GENERATED BY @RISK

Beam	β	P_F
C1	3.26	5.61E-03
C2	0.137	0.446

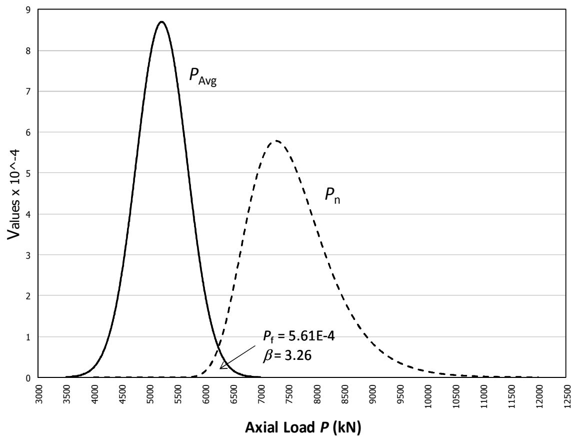

Fig. 4 The probability of axial load resistance versus average applied axial load for column C1 obtained from @RISK

TABLE XVI
VALUES OF β AND PSFS FOR STRENGTH AND LOADS FOR SELECTED BEAMS GENERATED BY FORM

Beam	β	ϕ	γ_D	γ_L	γ_w
C1	3.29	0.736	1.04	1.22	0.99
C2	0.145	0.96	1.00	1.00	-

TABLE XVII
AVERAGE VALUES OF RELIABILITY INDEX β AND CORRESPONDING FAILURE PROBABILITY P_F FOR SELECTED R/C BEAMS OF THE STADIUM BUILDING

Beam	β_{Avg}	P_F
C1	3.275	5.28E-04
C2	0.141	0.444

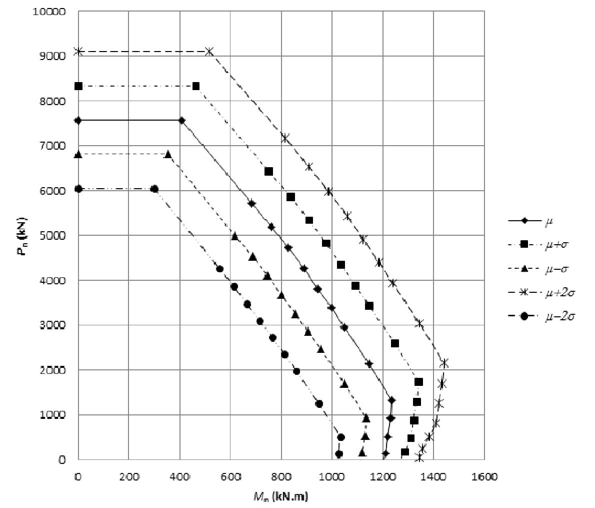

Fig. 5 Interaction diagram for column C1

TABLE XVIII
REQUIRED STRENGTH BASED ON TARGET RELIABILITY INDEX β_T

Column	β_T	$P_{n\,(req.)}$ (kN)	$M_{n\,(req.)}$ (kN.m)
C1	4.0	8,025	-
C2	4.0	950	276

VIII. DISCUSSION OF RELIABILITY ANALYSIS RESULTS AND RECOMMENDATIONS

FORM is a powerful tool that can be employed to assess and evaluate the reliability of a structural component or system as well as to develop and establish partial safety factors. In this study, FORM and @RISK were utilized, in combination and sometimes independently, to compute the average reliability indices and average failure probabilities of weakened or damaged reinforced concrete beams of the recreational building. It was obvious from the relatively low values of βs for some of the R/C beams and columns of Tables XII and XVII that if these structural elements were not promptly remedied, they could create risk and chances for both local collateral damage and possibly human loss. Local damage is emphasized herein because component reliability was utilized in the analysis rather than system reliability, which basically considers the whole building as a unit or a system, and consequently could have provided more rigorous and accurate results. In addition, local damage may or may not occur because of the redundancy of structural elements and components of the building.

The program @RISK was employed to quantify the statistical characteristic of the resistance variables M_n and P_n, which are functions of many random variables. Tables IX and XIV show the statistical characteristics of axial and moment resistance variables as were quantified using @RISK. These variables were also determined to have a Lognormal distribution based on the statistical best-fit analysis conducted by @RISK.

The specialized program FORM was used to develop the needed partial safety factors for the critical R/C beams and columns for specified and recommended reliability indices according to the international standards, and sometimes on sound engineering judgment. The partial safety factors were calculated for several randomly selected cases that cover a wide range of weakened or damaged beams and columns of the stadium and the recreational buildings.

For future studies on cases similar to this one, it is recommended that other structural components such as joints, one-way slabs, two-way slabs, and other structural components should be considered in reliability analysis. Also, other types of loading, if applicable, such as wind, seismic, dynamic, etc., should be taken under consideration.

Although system reliability has its own merit in producing more accurate results and fairly predicting the overall reliability index of the building, it requires more rigorous and detailed analysis and it can be costly and time consuming. Also, it requires more probabilistic information and statistics on strength, loads, materials, method of construction, etc., that might not be available for performing such an analysis in a proper and straightforward manner. System reliability involves evaluating and assessing the whole building as a unit rather than individual structural components and elements. However, structural components or elements should be first analyzed probabilistically by component reliability methods for assessing system reliability to complete the overall analysis of the whole engineering system, i.e., the building. This method is recommended and justified if money and time are not a problem, and if all the aforementioned needed information and data for this analysis are made available.

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