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Evaluation of Dynamic Behavior of a Rotor-Bearing System in Operating Conditions

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***Abstract*—**Most flexible rotors can be considered as beam-like structures. In many cases, rotors are modeled as one-dimensional bodies, made basically of beam-like shafts with rigid bodies attached to them. This approach is typical of rotor dynamics, both analytical and numerical, and several rotor dynamic codes, based on the finite element method, follow this trend. In this paper, a finite element model based on Timoshenko beam elements is utilized to analyze the lateral dynamic behavior of a certain rotor-bearing system in operating conditions.

***Keywords*—**Finite element method, Operational deflection shape, Timoshenko beam elements, Unbalance response.

I. Introduction

ROTATING machines are extensively used in engineering applications. The demand for more powerful rotating machines has led to higher operating speeds resulting in the need for accurate prediction of the dynamic behavior of rotors. It is vital to precisely determine the dynamic characteristics of rotors in the design and development stages of engines in order to avoid resonant conditions. Thus, much research has been carried out in the field of rotor dynamics.

Chiu and Chen [1] analytically studied the shaft-torsion and blade-bending coupling vibrations in a multi-disk rotor system. They obtained the natural frequencies and mode shapes of the system for one to three-disk cases. Whalley and Abdul-Ameer [2] calculated the critical speed and rotational frequency of shaft-rotor systems where the shaft profile is contoured. Lazarus et al. [3] suggested a 3D finite element method based on the modal theory in order to analyze linear periodically time-varying systems. They also performed experimental investigations on a test rig composed of an asymmetric rotor running on non-isotropic supports. Al-bedoor [4] presented a dynamic model for a typical elastic blade attached to a disk driven by a shaft which is flexible in torsion. He employed the Lagrangian approach in conjunction with the finite element method in deriving the equations of motion, within the assumption of small deformation theory. Harsha et al. [5] investigated the effect of the speed of balanced rotor on the nonlinear vibrations of the rotor. They used a new reduction method to increase the numerical stability.

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Dynamic analysis of multi-stage cyclic structures was reported by Laxalde et al. [6], [7], Laxalde and Pierre [8] and Chatelet et al. [9]. Chatelet et al. [9] studied the modeling approaches for dynamic analyses of the rotating assemblies of turbo machines. They compared the results obtained from the 3D finite element modeling of two case studies with those obtained from a program named ROTORINSA which is based on beam type 1D finite elements.

Jalali et al. [10] predicted the dynamic behavior of a rotor-bearing system with a 1D finite element model, a 3D finite element model and experimental modal test. They obtained natural frequencies and mode shapes of the rotor at rest under free-free boundary condition using beam model, 3D FE model and modal test. Also, they performed a full rotor dynamic analysis for the rotor by the use of both FE models. Jeon et al. [11] performed a rotor dynamic analysis for a high thrust class liquid rocket engine turbo pump considering the dynamic force characteristics of ball bearings and pump impeller seals. Yu et al. [12] proposed a finite element model using a 3-node spatial element based on Timoshenko beam theory which provided by ANSYS package for modal analysis of crankshaft. They obtained the natural frequencies and mode shapes of the crankshaft with the proposed model.

In this paper, lateral dynamics of a rotor with certain geometrical and mechanical properties is analyzed using a Timoshenko beam finite element model. The bending critical speed, the Campbell diagram, the operational deflection shapes at the critical speeds, the mode shapes of the rotor in rotating condition and the unbalance response of the rotor to an imbalance are the results obtained from the beam FE model. The calculated critical speed is far from the operational speed range of the rotor, thus, the rotor would not experience resonance.

II. Theoretical Formulation

The equations describing the motion of even a simple rigid body with mass m and principal moments of inertia J_ξ, J_η, and J_ζ referred to a reference frame $\xi\eta\zeta$ fixed to it in the three dimensional space are actually complex, particularly when dealing with the rotational degrees of freedom, and they do not allow the direct use of any linear model. With reference to an inertial frame xyz and a rotating frame $\xi\eta\zeta$ fixed to the rigid body and coinciding with its principal axes of inertia, the six equations of motion under the action of the generic force $\vec{F}$ and moment $\vec{M}$ can be written in the form [13].

$$M\ddot{x} = F_x\,, \qquad M_\xi = \dot{\Omega}_\xi J_\xi + \Omega_\zeta \Omega_\eta (J_\zeta - J_\eta) \tag{1}$$

$$m\ddot{y} = F_y \ , \qquad M_\eta = \dot{\Omega}_\eta J_\eta + \Omega_\xi \Omega_\zeta (J_\xi - J_\zeta)$$
$$m\ddot{z} = F_z \ , \qquad M_\zeta = \dot{\Omega}_\zeta J_\zeta + \Omega_\xi \Omega_\eta (J_\eta - J_\xi)$$

The three equations for the rotational degrees of freedom, which are the well-known Euler equations, are clearly nonlinear in the angular velocity $\vec{\Omega}$.

However, a number of simplifications allow a linearized model to be obtained that retains the basic features of the dynamic behavior of rotating systems and allow us to describe it correctly, both in a qualitative and a quantitative manner.

The two assumptions of small unbalance and small displacements allow the linearization of the equations of motion in a way that is consistent with what is usually done in the dynamics of structures. However, even in the case of the discretized model of a linear rotor that is axially symmetrical about its spin axis and rotates at a constant spin speed Ω, the linearized equation of motion (dynamic equilibrium equation) is of the following general form [13]:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\mathbf{C} + \mathbf{G})\dot{\mathbf{q}}(t) + (\mathbf{K} + \mathbf{H})\mathbf{q}(t) = \mathbf{f}(t) \tag{2}$$

where $\mathbf{q}(t)$ is a vector containing the generalized coordinates, referred to an inertial frame, $\mathbf{M}$ is the symmetric mass matrix, $\mathbf{C}$ is the symmetric damping matrix, $\mathbf{G}$ is the skew-symmetric gyroscopic matrix, $\mathbf{K}$ is the symmetric stiffness matrix, $\mathbf{H}$ is the skew-symmetric circulatory matrix, and $\mathbf{f}(t)$ is a time-dependent vector in which all forcing functions are listed.

When dealing with rotating systems, one of the forcing functions is usually that caused by the residual unbalance that, although small, cannot nevertheless be neglected. Unbalance forces are harmonic functions of time, with an amplitude proportional to Ω^2 and a frequency equal to Ω.

The gyroscopic matrix contains inertial and hence conservative terms that, in the case of rotor dynamics, are strictly linked with the gyroscopic moments acting on the rotating parts of the machine. If the equation is written with reference to a non-inertial frame, terms linked with Coriolis acceleration also are present in the gyroscopic matrix. The circulatory matrix contains non-conservative terms linked with the internal damping of rotating elements and, when using a linearized model for fluid bearings or seals, with the damping of the fluid film surrounding the rotor. In this paper, the circulatory matrix is equal to zero because damping is neglected in the models.

Equation (2) is that of a non-natural, circulatory system and hence differs from the typical equations encountered in dynamics of structures, where all matrices are symmetric. It must be noted that in rotor dynamics, the gyroscopic and circulatory matrices $\mathbf{G}$ and $\mathbf{H}$ are proportional to the spin speed Ω, and when Ω tends to zero, the skew-symmetric terms vanish and the equation reduces to that of a still structure.

Also, if $(t) = 0$ as well as $\Omega = 0$, (2) can be used for modal analysis of the system in non-rotating situation. In addition, the damping and stiffness matrices $\mathbf{C}$ and $\mathbf{K}$ may depend on the spin speed, often on its square Ω^2, and $\mathbf{H}$ can be a more complex function of Ω.

Most flexible rotors can be considered as beam-like structures. Under fairly wide assumptions, the lateral behavior of a beam can be considered as uncoupled from its axial and torsional behavior. The same uncoupling is usually assumed in rotor dynamics, with the difference that no further uncoupling between bending in the principal planes is possible. When the flexural behavior can be uncoupled from the axial and torsional ones, (2) holds for the first one, and the torsional and axial equations of motion are usually those of a natural, non-circulatory system [13].

In this paper, the matrices of the beam and solid elements should be calculated from the formulation of the finite element method and the matrices in (2) should be obtained by assembling the global matrices for the whole finite element model [10]-[13]. When the natural frequencies of the rotor at various rotor speeds are calculated, the Campbell diagram can be plotted. The natural frequencies of the rotor at various rotor speeds can be calculated by solving the eigenvalue problem form of (2). In addition, The unbalance response of the rotor can be obtained by obtaining the solution of (2) when $f(t)$ is a harmonic function of time, with an amplitude proportional to Ω^2 and a frequency equal to Ω.

III. Finite Element Model

Fig. 1 shows the rotor and the model of the rotor with Timoshenko beam elements. The FE model consists of 22 Timoshenko beam elements.

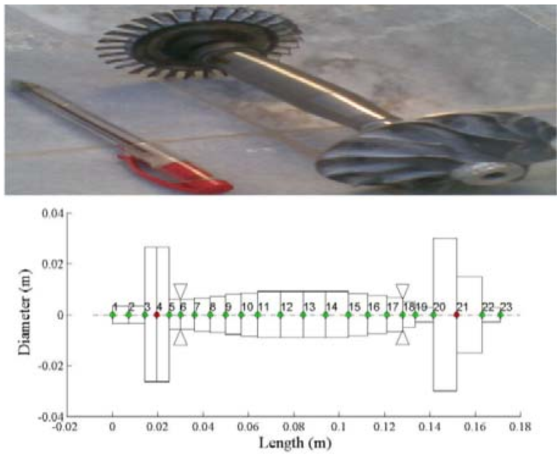

Fig. 1 Model of the system for dynamic analysis

The turbine’s disk is modeled using two large beams with the density of zero. A concentrated mass in node 4 is used to model the turbine’s inertial properties. The radial compressor’s disk is modeled using two massless beams and a concentrated mass (node 21). Two springs are used to model the bearings in nodes 6 and 18. Every node used in the system has 4 degrees of freedom. These include translations in the nodal directions and rotations about nodal axes. Table I presents the mechanical and geometric properties of the elements and Table II indicates the characteristics of concentrated masses.

TABLE I
MECHANICAL AND GEOMETRIC PROPERTIES OF THE ELEMENTS

Element Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
d_o(mm)	7	7	53	53	12	12	13.2	14.4	15.8	17	18	18	18	18	16.5	15	13.6	10	5.5	60	30	6.5
l (mm)	7.1	7.1	5.3	5.4	5	6.1	6.8	6.9	6.9	7.2	10	10	10	10	8.4	8.4	6.9	5.7	8	10.1	11.3	8
ρ ($^{kg}/_{m^3}$)	2770	2770	0	0	2770	2770	2770	2770	2770	2770	2770	2770	2770	2770	2770	2770	2770	2770	2770	0	0	2770
E (Gpa)	72	72	200	200	72	72	72	72	72	72	72	72	72	72	72	72	72	72	72	96	96	72

TABLE II
CHARACTERISTICS OF CONCENTRATED MASSES

Mass number	1	2
Node number	4	21
m (g)	74.79	84.42
I_p (kgm^2)	2.61×10^{-5}	1.91×10^{-5}
I_d (kgm^2)	1.32×10^{-5}	1.28×10^{-5}

IV. RESULTS

In order to investigate the dynamic behavior of the rotor at operational speeds, critical speeds and the Campbell diagram are obtained. The numerical analyses are performed considering speeds ranging from 0 to 150000 rpm. The Campbell diagram is shown in Fig. 2.

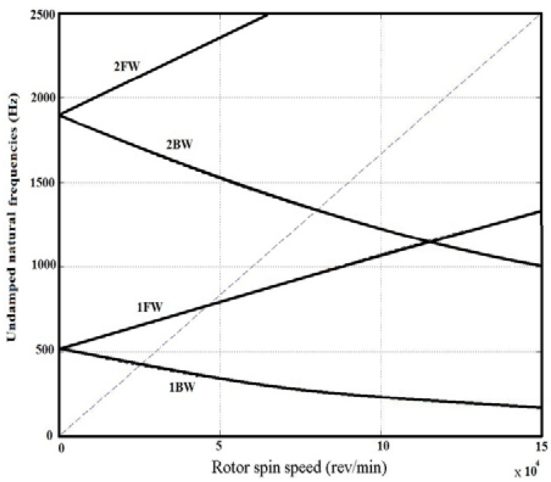

Fig. 2 Campbell diagram

The speeds which are the coincidence of the shaft rotating speed and the rotating natural frequencies of the rotor are 25041rpm, 45894 rpm and 79876 rpm which are from the first backward whirling, first forward whirling and second backward whirling, respectively. An imbalance of $0.07479e - 6$ kg m at the gravity center of the turbine is also considered. The unbalance response of the rotor evaluated at nodes is shown in Fig. 3. Also, the operational deflection shapes (ODS) at three mentioned speeds are obtained (Figs. 4-6). As seen in the figures, the deflection at the second speed, which is from forward whirl, is more than the others, which are from backward whirl modes. By the operational deflection shapes and the unbalance response, it is clear that the real critical speed of the rotor is the speed corresponding to the intersection of the $\omega = \Omega$ line with the forward whirl curve not the backward whirl curves.

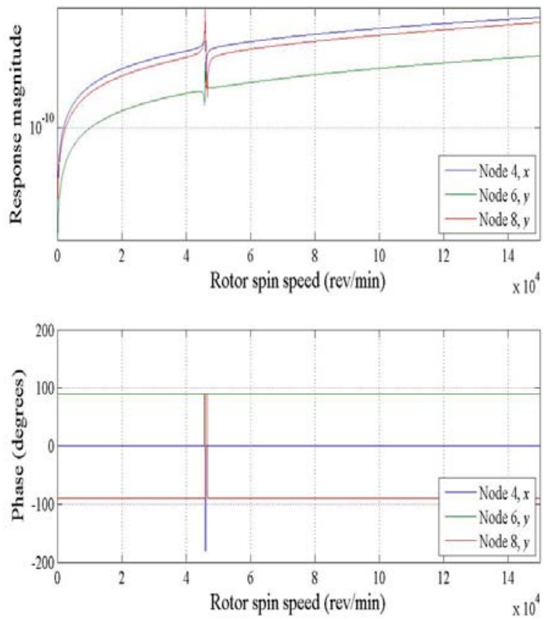

Fig. 3 Unbalance response

The speed in which response of the rotor peaks in Fig. 3, is the bending critical speed because the natural frequency of the rotor coincides with the excitation frequency.

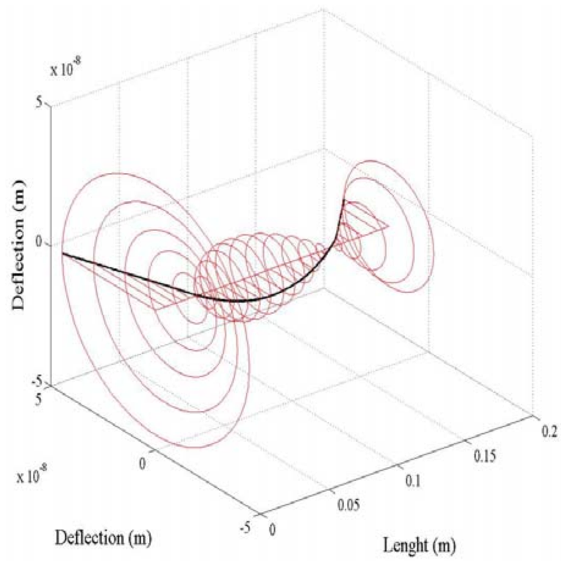

Fig. 4 The operational deflection shape at 1BW

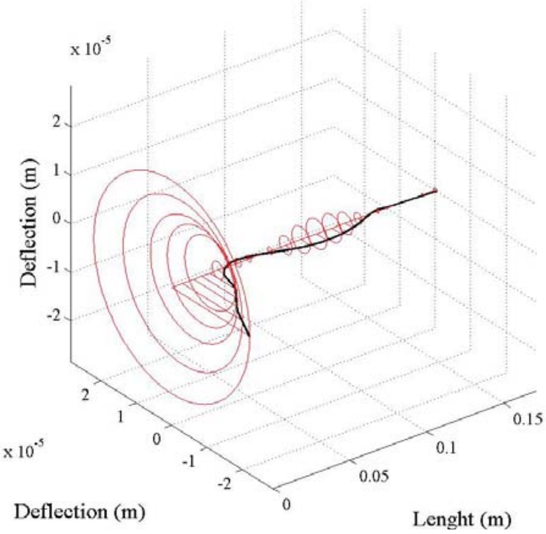

Fig. 5 The operational deflection shape at 1FW

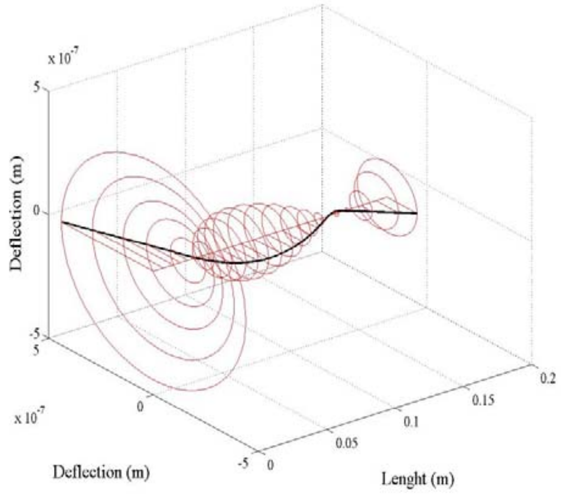

Fig. 6 The operational deflection shape at 2BW

It is obvious from Figs. 4-6 that displacement peaks at the critical speed point which is obtained from the Campbell diagram and unbalance response.

The mode shapes corresponding to these three speeds are also obtained. The mode shape of the rotor at the speed corresponding to 1BW, 1FW and 2BW are shown in Figs. 7-9.

The Campbell diagram of the rotor using a 3D finite element model is also obtained in [10] and good agreement between the results of the two FE models shows the accuracy of numerical analyses.

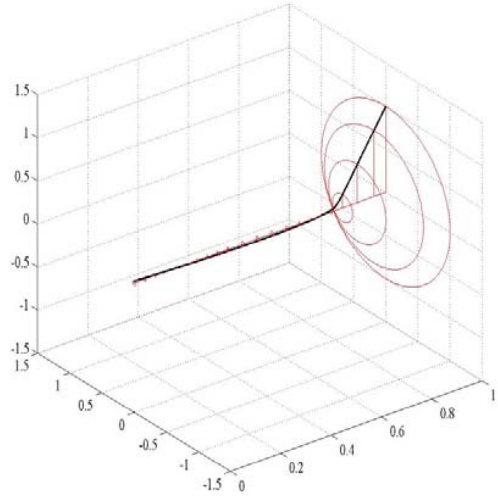

Fig. 7 Mode shape corresponding to 1BW

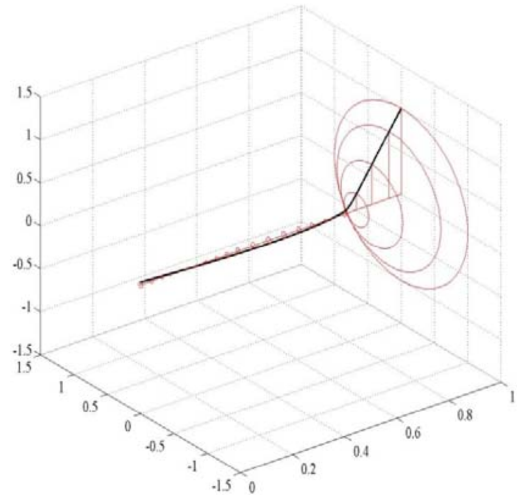

Fig. 8 Mode shape corresponding to 1FW

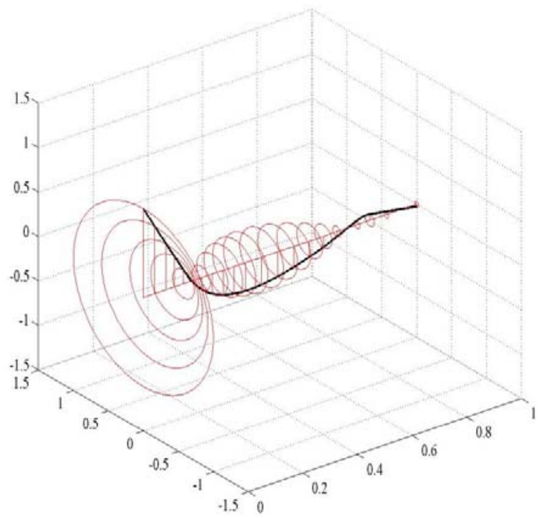

Fig. 9 Mode shape corresponding to 2BW

V. Conclusion

Rotors in general have complex geometries which make analytical modeling of the rotor to determine its dynamic behavior difficult. For this purpose, strong approaches such as finite element method are used to analyze the system dynamics. In this paper, the lateral dynamics of a certain rotor-bearing system was investigated using a finite element model based on Timoshenko beam elements. The Campbell diagram, operational deflection shapes, unbalance response of the rotor to a center of mass imbalance at the turbine and the mode shapes corresponding to speeds which were coincidences of the shaft rotating speed and the rotating natural frequencies in Campbell diagram were depicted. By the operational deflection shapes and the unbalance response, it was proven that the real critical speed of the rotor was the speed corresponding to the intersection of the $\omega = \Omega$ line with the forward whirl curve not the backward whirl curves.

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