

Defuzzification of Periodic Membership Function on Circular Coordinates

Takashi Mitsuishi, Koji Saigusa

Abstract—This paper presents circular polar coordinates transformation of periodic fuzzy membership function. The purpose is identification of domain of periodic membership functions in consequent part of IF-THEN rules. Proposed methods in this paper remove complicatedness concerning domain of periodic membership function from defuzzification in fuzzy approximate reasoning. Defuzzification on circular polar coordinates is also proposed.

Keywords—Defuzzification, periodic membership function, polar coordinates transformation.

I. INTRODUCTION

ZADEH brought in the notion of fuzziness in 1965 [1], [2]. And then Mamdani has applied it to the study of control theory. It is called Mamdani method [3]. This method is one of the ways to numerically represent the control given by human language and sensitivity, and it has been applied in various practical control plants. Nowadays fuzzy logic develops gradually to the neural network, expert systems, operation research, and others [7], [8]. The fuzzy approximate reasoning process involves IF-THEN rules constructed from membership functions (fuzzy sets), inference calculation, and defuzzification. Defuzzification is a conversion from fuzzy set as output of fuzzy inference to crisp quantity. Center of gravity method (centroid method), weighted average method, height method, center of sums method, etc., many methods have been proposed previously [4]-[6]. One of the most popular defuzzification methods is the center of gravity method [9]. It computes the center of gravity of an area under the membership function. However, in the case of a periodic membership function, an area may be divided according to configuration of domain of them. Therefore the interval, which is the domain of membership function needs to shift for avoiding division of the area. The interval should have both ends where the value of membership function is equal to 0 [10]-[12]. The setting of the interval for each inference increases computational complexity. Therefore the authors propose the method of circular polar coordinates transformation of periodic membership and its defuzzification method in this study [13]. The transformation to circular polar coordinates simplifies domain of the periodic membership function. The paper is organized as follows: In 2nd section, the periodic fuzzy membership function and some example are shown. And details of circular polar

coordinates transformation for periodic membership function are described. The discretization of membership function on circular polar coordinates for simplicity and its defuzzification method are discussed in 3rd section.

II. CIRCULAR POLAR COORDINATE TRANSFORMATION OF PERIODIC FUZZY MEMBERSHIP FUNCTION

A. Periodic Fuzzy Membership Function

Most of membership functions of fuzzy sets are defined on a fixed interval which is usually closed in practice usage. On the other in some of fuzzy approximate reasoning, time, season, direction, point of the compass, and hue of color may be inferred. The membership functions of those fuzzy sets are periodic functions. The fuzzy grades of them return to the same value at regular intervals, and a closed interval is not confirmed. A membership function is said to be periodic with period $\omega > 0$, if we have

$$\mu(v) = \mu(v + \omega)$$

for all variable v in carrier of the membership function.

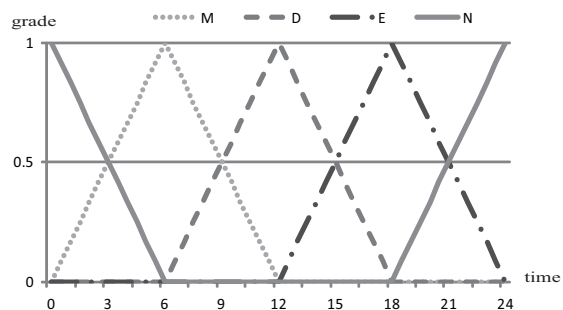


Fig. 1. Membership functions of time on Cartesian coordinates

Fig. 1 shows an example of membership functions of the fuzzy sets expressing morning (M), daytime (D), evening (E) and night (N) in twenty four hours. Assuming that the interval of them is $[0, 24]$, the membership function of the night is separated in front and rear of the interval. It has no influence that membership functions are in premise part of IF-THEN rules. It is inconvenience for defuzzification and the composition of the membership functions in consequent part of IF-THEN rules. In particular using the center of gravity method for defuzzification for the membership function representing night (N), the calculated value is incorrect value 12 o'clock against the better value 0 (24) o'clock.

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To avoid the problem in the defuzzification, the interval of the membership functions should be sufficiently wide. And they should be periodic. The following $\mu_M(t)$, $\mu_D(t)$, $\mu_E(t)$ and $\mu_N(t)$ are membership functions of them for time $t \in [0, \infty)$ respectively.

$$\mu_M(v) = \begin{cases} \frac{t-24n}{6}, & t \in [24n, 6+24n] \\ -\frac{t-24n}{6} + 2, & t \in [6+24n, 12+24n] \end{cases}$$

$$\mu_D(v) = \begin{cases} \frac{t-24n}{6} - 1, & t \in [6+24n, 12+24n] \\ -\frac{t-24n}{6} + 3, & t \in [12+24n, 18+24n] \end{cases}$$

$$\mu_E(t) = \begin{cases} \frac{t-24n}{6} - 2, & t \in [12+24n, 18+24n] \\ -\frac{t-24n}{6} + 3, & t \in [18+24n, 24+24n] \end{cases}$$

$$\mu_N(v) = \begin{cases} \frac{t-24n}{6} - 3, & t \in [18+24n, 24+24n] \\ -\frac{t-24n}{6} + 1, & t \in [24n, 6+24n] \end{cases}$$

Here $n = 0, 1, 2, \dots$ is period. Thus, we introduce transformation the periodic membership function on the Cartesian coordinates to the polar coordinates as Fig. 2.

B. Converting between Circular and Cartesian Coordinates

Let v_0 be a fixed real number and let $\mu(v) : [v_0, v_0 + \omega] \rightarrow [0, 1]$ be a periodic membership function. The circle polar coordinates $\mu(v)$ (the radial coordinate) and θ (the angular coordinate) can be converted to the Cartesian coordinates x and y as follows:

$$x = \mu(v) \cos \theta, \quad y = \mu(v) \sin \theta$$

where

$$\theta = \frac{2\pi}{\omega}(v - v_0).$$

In this work, we discuss about only the circular polar coordinates, although there are cylindrical polar coordinates and spherical polar coordinates. We can treat discontinuous periodic membership function on the interval $[0, 24]$ like night (N) as a closed plane figure in Fig. 2. Moreover the transformation is unique for each membership function. Based on these conversions, we can prevent the domain of membership function from separating on the periodic interval.

III. DEFUZZIFICATION METHOD

The output of approximate reasoning is a membership function (fuzzy set). Since it cannot be used as the kind of input, it should be converted to certain crisp value. The method which obtains a crisp value, which is a representative point from the resulting membership function, is called a defuzzification method.

It needs that a singleton is calculated from a closed plane figure which is a periodic function on the circular polar

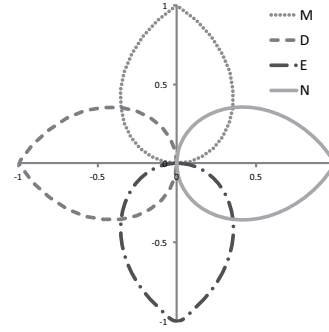


Fig. 2. Membership functions of time on the circular polar coordinates

coordinates. The center of gravity method is widely used as defuzzification method. However, if the closed plane figure is continuous, the computation will be large. Therefore for the practical example to be simple and a significant reduction in computational complexity, we discretize the continuous interval and the membership function. Put

$$v_1 = v_0, \quad v_2 = v_0 + \frac{\omega}{n-1}, \quad v_3 = v_0 + \frac{2\omega}{n-1}, \quad \dots,$$

$$v_{n-1} = v_0 + \frac{(n-2)\omega}{n-1}, \quad v_n = v_0 + \omega.$$

Then we have

$$x_i = \mu(v_i) \cos \left\{ \frac{2\pi}{\omega}(v_i - v_0) \right\},$$

$$y_i = \mu(v_i) \sin \left\{ \frac{2\pi}{\omega}(v_i - v_0) \right\}$$

where

$$v_i \in [v_0, v_0 + \omega], \quad i = 1, 2, \dots, n.$$

We approximate the closed plane figure to the polygon for simplification. From the points of the polygon (x_i, y_i) ($i = 1, 2, \dots, n$), the physical center of gravity is obtained as follows:

$$(x^*, y^*) = \left(\frac{1}{n} \sum_{i=1}^n x_i, \frac{1}{n} \sum_{i=1}^n y_i \right).$$

By using the physical center of gravity, we propose the definition of a defuzzified value of the periodic function on the circular polar coordinates.

$$v^* = \begin{cases} v_0 + \frac{\omega}{4}, & (x^* = 0, y^* > 0) \\ v_0 + \frac{3\omega}{4}, & (x^* = 0, y^* < 0) \\ v_0 + \frac{\omega}{2\pi} \arctan \frac{y^*}{x^*}, & (\text{otherwise}) \end{cases}$$

The argument of the physical center of gravity converted into the value on Cartesian coordinates which is domain of the primary periodic membership function $[v_0, v_0 + \omega]$.

IV. NUMERICAL VALIDATION

By way of example, Fig. 3 illustrates the aggregate graph which is composed from the membership functions of morning (M), day (D), evening (E) and night (N) in Fig. 2 scaled down by 0.2, 1, 0.7 and 0.8, respectively [3], [4].

$$\mu^*(t) = 0.2\mu_M(t) + \mu_D(t) + 0.7\mu_E(t) + 0.8\mu_N(t)$$

$$= \begin{cases} -\frac{t-24n}{10} + 0.8, & t \in [24n, 6+24n] \\ -\frac{t-24n}{30} + 0.4, & t \in [6+24n, 12+24n] \\ \frac{7(t-24n)}{60} - 1.4, & t \in [12+24n, 18+24n] \\ \frac{t-24n}{60} + 0.4, & t \in [18+24n, 24+24n] \end{cases}$$

If the function above is not periodic, defuzzified crisp value of the membership function is following:

$$t^* = \frac{\int_0^{24} t\mu^*(t)dt}{\int_0^{24} \mu^*(t)dt} = 14.24.$$

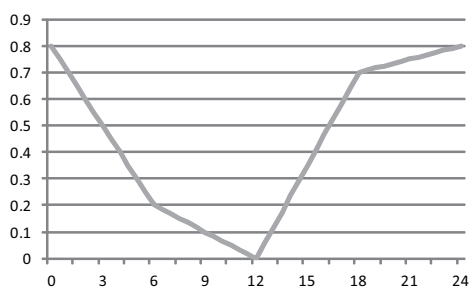


Fig. 3. Output of approximate reasoning on Cartesian coordinates

This is clearly inappropriate to defuzzified value. Then the membership function $\mu^*(t)$ is polar coordinates transformed and shown graphically in following Fig. 4.

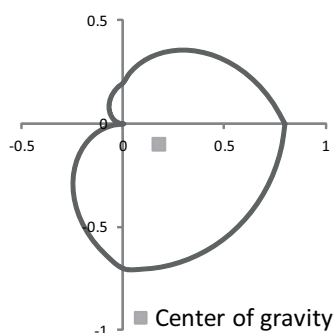


Fig. 4. Output of approximate reasoning on the circular polar coordinates

From this figure, we need to obtain crisp value on the interval $[0, 24]$ for defuzzification. The physical center of gravity of this figure in Fig. 4 is following:

$$(x^*, y^*) = (0.1754, -0.0994).$$

Here, this is approximate discretized value to simplify calculation. By using the defuzzification proposed, the center of gravity is

$$t^* = \frac{24}{2\pi} \arctan \frac{-0.0994}{0.1754} = 22.03.$$

On the other, the center of gravity with adjustments for the interval is

$$t^* = \frac{\int_{12}^{36} t\mu^*(t)dt}{\int_{12}^{36} \mu^*(t)dt} = 22.23.$$

Both results are closed enough. The difference of both is affected by the approximation. Then it can be considered that the proposed method in this study is one of the effective methods for periodic membership function.

V. CONCLUSIONS

In this study, to reduce computational complexity concerning defuzzification of periodic membership function, the one of the methods of its conversion from periodic interval in rectangle coordinates system to circular polar coordinates system is proposed. And the new defuzzification method is shown. These techniques will be applied to the simple color construction system using fuzzy logic control as our previous work. It can be consider that proposed methods for periodic membership function are effective for decision making support system in social science field. Moreover continuity defuzzification is useful tool for optimal fuzzy logic control.

The physical center of gravity is calculated as a defuzzification point in this paper. However there are some proposed method which defines geometric centroid and other one in a different meaning. In the future, other defuzzification method is discussed with development of this study. The distance of center of gravity from the origin of the coordinates system is considered as some indicator of approximate reasoning.

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