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Propagation of Nonlinear Surface Waves in Relativistically Degenerate Quantum Plasma Half-Space

Swarniv Chandra, Parthasona Maji, Basudev Ghosh

Abstract**—**The nonlinear self-interaction of an electrostatic surface wave on a semibounded quantum plasma with relativistic degeneracy is investigated by using quantum hydrodynamic (QHD) model and the Poisson's equation with appropriate boundary conditions. It is shown that a part of the second harmonic generated through self-interaction does not have a true surface wave character but propagates obliquely away from the plasma-vacuum interface into the bulk of plasma.

Keywords**—**Harmonic Generation, Quantum Plasma, Quantum Hydrodynamic Model, Relativistic Degeneracy, Surface waves.

I. Introduction

OVER the past few years quantum effects in plasmas have attracted considerable interest due to its relevance in super dense astrophysical plasmas, nanodevices and intense laser-solid plasma experiments [1], [2]. The surface waves propagating along the interface between a vacuum and a quantum plasma has attracted much attention because of its wide applications in many areas such as laser-plasma interaction, plasma technology, plasma diagnostics, plasma heating by large amplitude waves, microwave electronics, plasma spectroscopy, nanotechnology, surface sciences etc. Some authors have investigated the nonlinear surface wave propagation on a classical plasma half-space [3], [4]. Its frequency ranges from $\omega=0$ to $\omega_e/\sqrt{2}$, where ω_e is the electron plasma frequency. However, to the best of our knowledge the works done so far on surface wave phenomena in quantum plasma is mostly limited to linear theory [5]-[7]. Quantum effects on the linear dispersion relation of surface Langmuir oscillations have been investigated by Chang and Jung [3]. Using QHD model along with Maxwell's equations the linear dispersion character of transverse electric (TE) surface modes on a semibounded quantum plasma has been examined by Mohamed and Aziz [6]. Lazar et al. [7] have shown that the quantum effects are mainly relevant for the electrostatic surface waves in a dense gold metallic plasma. Dispersion properties of electrostatic surface modes at the interface between magnetized electron-positron quantum plasma and the vacuum have been studied by Misra et al. [8]. So far no one has investigated the nonlinear propagation of surface waves along a quantum plasma half-space with arbitrary temperature. Also most of the work in quantum plasma assumes the particle velocities are far less than the speed of light. But in cases especially where relativistic degeneracy plays an important role these considerations must be taken into account. In this paper we have investigated the nonlinear self-interaction of electrostatic surface waves on a quantum plasma half-space bounded by vacuum by using the standard perturbation technique and employing the relativistic degeneracy pressure given by Chandrasekhar [9]..

II. Basic Formulation

We consider a homogeneous unmagnetized two-component electron-ion quantum plasma occupying the half-space x>0 and bounded by vacuum (x<0). The wave is supposed to propagate parallel to the interface (x=0) along the z-direction. We are interested only in the processes where all characteristic times are much smaller than ion plasma period. Electrostatic surface waves are considered to propagate in completely degenerate dense plasma consisting of mobile electrons and stationary cold ions forming a uniform neutralizing background. In degenerate plasmas the rate of electron-ion collisions is limited due to the Pauli blocking mechanism which allows only degenerate particles with energies limited to a narrow range around the Fermi energy to interact, hence the plasma may be considered to be almost collisionless. For electrons the thermal pressure is assumed to be negligible as compared to the degeneracy pressure which arises due to the implications of Pauli's exclusion principle. The dynamics of such a plasma is governed by the following normalized quantum hydrodynamic equations:

$$\frac{\partial(n_e)}{\partial t}+\frac{\partial(n_e u_e)}{\partial x}=0 \tag{1}$$

$$\left(\frac{\partial}{\partial t}+u_e\frac{\partial}{\partial x}\right)u_e=\frac{1}{m_e}\left[e\frac{\phi}{x}\quad\frac{1}{n_e}\frac{\partial P_e}{x}\,\partial\,\frac{\hbar^2}{2m_e}\,\frac{}{x}\left[\frac{1}{\sqrt{n_e}\,\partial}\frac{\partial^2\sqrt{n_e}}{x^2}\right]\right]\partial \quad (2) \; + \; \partial$$

$$\frac{\partial^2\phi}{\partial x^2}=\left(n_e-Z_i n_i\right) \tag{3}$$

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For vacuum (x<0) the Laplace's equation is:

$$\frac{\partial^2 \phi_v}{\partial x^2} = 0 \qquad (4)$$

Here we add a subscript 'v' to indicate vacuum field quantity. We now apply the following normalization scheme Here u_e and p_e are respectively the fluid velocity and degeneracy pressure of the electrons, ħ is the Planck's constant divided by 2π, ϕ is the electrostatic wave potential and $Z_i e$ is the charge of an ion. Following Chandrasekhar [9] the electron degeneracy pressure in fully degenerate and relativistic configuration can be expressed in the following form:

$$P_e = \left(\pi m_e^4 c^5 / 3h^3\right)\left[R_e(2R_e^2 - 3)\sqrt{1+R_e^2} + 3\sinh^{-1} R_e\right] \qquad (5)$$

in which

$$R_e = p_{Fe}/m_e c = \left[3h^3 n_e / 8\pi m_e^3 c^3\right]^{1/3} = R_{e0} n_e^{1/3} \qquad (6)$$

where

$R_{je} = \left(n_{e0}/n_0\right)^{1/3}$ with $n_0 = 8\pi m_e^3 c^3 / 3h^3 \approx 5.9\times 10^{29}\, cm^{-3}$,

'c' being the speed of light in vacuum. p_{Fe} is the electron Fermi relativistic momentum. It is to be noted that in the limits of very small and very large values of relativity parameter R_e, we obtain:

$$P_e = \left(3/\pi\right)^{2/3}\left(h^2/20m_e\right) n_e^{5/3} \quad \text{(For Re} \to 0) \qquad (7a)$$

$$P_e = \left(3/\pi\right)^{1/3}\left(hc/8\right) n_e^{4/3} \quad \text{(For Re} \to \infty) \qquad (7b)$$

Note that the degenerate electron pressure depends only on the electron number density but not on the electron temperature. Now considering the fact that $\frac{1}{n_e}\frac{\partial P_e}{\partial x} = \frac{\partial\sqrt{1+R_e^2}}{\partial x}$ the basic equations (1)-(4) can be rewritten in the following normalized form:

$$\frac{\partial(n_e)}{\partial t} + \frac{\partial(n_e u_e)}{\partial x} = 0 \qquad (8)$$

$$\left(\frac{\partial}{\partial t} + u_e\frac{\partial}{\partial x}\right)u_e = \frac{\partial\phi}{\partial x} - F_e\frac{\partial n_e}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right] \qquad (9)$$

$$\frac{\partial^2\phi}{\partial x^2} = \left(n_e - n_i\right) \qquad (10)$$

$$\frac{\partial^2\phi_v}{\partial x^2} = 0 \qquad (11)$$

where $F_e = \left(\Lambda/3\right)\left(\delta_e^{2/3} R_{e0}^2 / \sqrt{1+R_{e0}^2}\right)$ is the term arising from relativistic pressure in weakly relativistic case, whereas for ultra relativistic case $F_e = \delta_e^{1/3}\Lambda R_{e0}/3$ where $\Lambda = m_e c^2 / 2k_B T_{Fe}$; H is the non-dimensional quantum diffraction parameter defined as $H = \hbar\omega_e / 2k_B T_{Fe}$, where T_{Fe} is the Fermi temperature for electrons; n_{e0} and n_{i0} are the equilibrium number densities of electrons and ions respectively. The normalization is carried out in the following manner:

$$x \to \frac{x\omega_e}{c_s}, t \to t\omega_e,\ \phi \to \frac{e\phi}{2k_B T_F},\ n_e \to \frac{n_e}{n_{e0}},\ n_i \to \frac{n_i}{n_{i0}} \text{ and } u_e \to \frac{u_e}{c_s}$$

in which $\omega_e = \sqrt{4\pi n_{e0} e^2 / m_e}$ is the electron plasma frequency, $c_s = \sqrt{2k_B T_{Fe} / m_e}$ is the quantum electron-acoustic speed.

It is to be noted that the parameter R_{e0} is a measure of the relativistic effects and may be called relativistic degeneracy parameter. For ultra relativistic case $R_{e0} >> 1$ and for weakly relativistic case $R_{e0} << 1$. The parameter R_{e0} can also be related to mass density as $\rho(gr/cm^3) = 1.97\times 10^6 \cdot R_{e0}^3$. The density of white dwarfs can be in the range $10^5 < \rho < 10^9$. So, in this case, the relativity parameter R_{e0} can be in the range $0.37 < R_{e0} < 8$. The boundary conditions to be used are the continuity of electric potential and normal electric displacement across the plasma-vacuum interface. Also for plasma equilibrium we assume that the normal component of electron fluid velocity is zero on the interface.

Combining (8)-(10) we obtain:

$$\left(\frac{\partial^2}{\partial t^2} + 1 - \nabla^2 + \frac{H^2}{4}\nabla^4\right)\nabla^2\phi = \frac{\partial Q}{\partial t} + \vec{\nabla}\cdot\vec{T} + \nabla^2 R \qquad (12)$$

where

$$Q = -\vec{\nabla}\cdot\left(n\vec{u}\right),\ \vec{T} = \left(\vec{u}\cdot\vec{\nabla}\right)\vec{u} + F_e n\nabla n,\ R = \frac{H^2}{4}n\nabla^2 n \qquad (13)$$

III. Linear Analysis

We assume that every field quantity f (representing ϕ, ϕ_v, n, u_x, u_z) has the following form:

$$f = f_0 + f_1 \qquad (14)$$

where f_0 is the equilibrium value and f_1 is a small perturbation ($f_1 << f_0$) and

$$f_1 = f_1(x)\exp[i(kz - \omega t)] + cc. \qquad (15)$$

Now neglecting the nonlinear terms in (12) we find that the linear surface wave is associated with the following perturbation quantities which decays exponentially in the direction normal to the interface:

$$\phi_1 = \frac{A}{(\gamma - k)}\left[-2ke^{-\gamma x} + \left(\gamma + k\right)e^{-kx}\right] \qquad (16)$$

$$\phi_{v1} = Ae^{kx} \qquad (17)$$

$$n_1 = A\left(\gamma^2 - k^2\right)e^{-\gamma x} = \alpha e^{-\gamma x} \qquad (18)$$

$$u_{x1} = \frac{iA\gamma}{\omega}\left[\left(\gamma^2 - k^2\right)\left(1 - \frac{H^2\left(\gamma^2 - k^2\right)}{4}\right) - 1\right]\left(e^{-\gamma x} - e^{-kx}\right) = i\left(\psi_x e^{-\gamma x} + \theta_x e^{-kx}\right) \qquad (19)$$

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$$u_{z1} = \frac{A}{\omega}\left[\left(\gamma^2 - k^2\right)\left(1 - \frac{H^2\left(\gamma^2-k^2\right)}{4}\right) - 1\right]\left(\gamma e^{-\gamma x} + k e^{-kx}\right) \tag{20}$$
$$= \left(\psi_z e^{-\gamma x} + \theta_z e^{-kx}\right)$$

where

$$\gamma^2 = 1 + \left(\Lambda R_e/3\right)k^2 + \left(H^2k^4/2\right) - \omega^2 \tag{21}$$

Frequency (ω) and wavenumber (k) satisfy the following dispersion relation:

$$\omega = \frac{1}{\sqrt{2}}\left(1 + \frac{k}{\sqrt{2}}\sqrt{\frac{\Lambda R_e}{3} + \frac{H^2k^2}{4}}\right) \tag{22}$$

The plot of the linear dispersion relation (22) for different values of relativistic degeneracy parameter R_e is shown in Fig. 1. It is obvious that the dispersion characteristics become steeper with increase in the value of R_e.

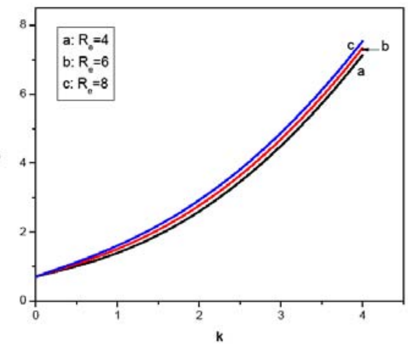

Fig. 1 Linear dispersion charateristics for different values of relativistic degeneracy parameter R_e

IV. Nonlinear analysis

To study the behavior of small but finite amplitude waves we express the field quantities as:

$$f = f_0 + f_1 + f_2 \tag{23}$$

where f_1 is the known perturbed quantities associated with linear effects and f_1 is the additional effect due to nonlinearity. Substituting the expansion (23) in (12) and keeping in mind that f_1 satisfy the linear equations we obtain:

$$\left(\frac{\partial^2}{\partial t^2} + 1 - \nabla^2 + \frac{H^2}{4}\nabla^4\right)\nabla^2\phi_2 = \frac{\partial Q}{\partial t} + \vec{\nabla}\cdot\vec{T} + \nabla^2 R \tag{24}$$

$$\nabla^2\phi_{v2} = 0 \tag{25}$$

We consider only weakly nonlinear situations i.e. $f_2 \ll f_1$; this allows us to evaluate the nonlinear terms on the right-hand side of (12) by using the known linear components (16)-(20). Since we are only interested in the oscillatory part of the second order perturbed quantities the nonoscillatory terms arising from the nonlinear terms on the RHS of (24) will be ignored. Thus using the linear components (16)-(20) and specified boundary conditions we finally obtain the second order oscillatory quantities as:

$$\phi_2 = \left[B_2 e^{-2kx} + D_2 e^{2i\beta x} + \sum_{j=1}^{3}\frac{F_j e^{-p_j x} p_j^{-2}}{\left(1 - p_j^2 + \frac{H^2 p_j^4}{4} - 4\omega^2\right)}\right] e^{2i(kz-\omega t)} \tag{26}$$

$$n_2 = \left[\begin{array}{l} 4k^2 B_2 e^{-2kx} - 4\beta^2 D_2 e^{2i\beta x} \\ + \displaystyle\sum_{j=1}^{3}\frac{F_j e^{-p_j x}}{\left(1 - p_j^2 + \left(H^2 p_j^4/4\right) - 4\omega^2\right)}\end{array}\right] e^{2i(kz-\omega t)} \tag{27}$$

$$u_{x2} = \frac{i}{2\omega}\left\{\begin{array}{l} e^{-2kx}\left[\begin{array}{l} k\left(\theta_x^2 + 2B_2\right) + 8k^3B_2 \\ + \dfrac{32H^2k^5B_2}{4} + 2ik\theta_x\theta_z\end{array}\right] \\ + e^{-2\gamma x}\left[\begin{array}{l}\gamma\left(\psi_x^2 + \alpha^2\right) + 8k^3B_2 + \\ H^2\alpha^2\gamma\dfrac{\left(\gamma^2 - k^2\right)}{4} + 2ik\psi_x\psi_z\end{array}\right] \\ + e^{-(\gamma+k)x}\left(\left(\gamma+k\right)\theta_x\psi_x + 2ik\left(\theta_x\psi_z + \theta_z\psi_x\right)\right) \\ - e^{2i\beta x}\left(2i\beta D_2 - 8i\beta^3 D_2 + 32\beta^5 D_2\right) \\ + \displaystyle\sum_{j=1}^{3}\frac{F_j e^{-p_j x}\left(1 + p_j^2 + \left(H^2 p_j^4/4\right)\right)}{p_j\left(1 - p_j^2 + \left(H^2 p_j^4/4\right) - 4\omega^2\right)}\end{array}\right\} e^{2i(kz-\omega t)} \tag{28}$$

$$u_{z2} = \frac{i}{2\omega}\left\{\begin{array}{l} e^{-2kx}\left(\begin{array}{l} k\theta_x\theta_z + 2H^2k^5B_2 \\ + 2ik\left(\theta_z^2 - B_2 - 4k^2B_2\right)\end{array}\right) \\ + e^{-2\gamma x}\left(ik\left(2\psi_z^2 - \alpha^2 k - \dfrac{H^2\alpha^2\left(\gamma^2 - k^2\right)}{4}\right)\right) \\ + e^{-2\gamma x}\gamma\psi_x\psi_z \\ + e^{-(\gamma+k)x}\left(\theta_x\psi_z\gamma + \theta_z\psi_x k + i4k\theta_x\psi_z\right) \\ - e^{2i\beta x} D_2\left(2ik\left(1 - 4\beta^2\right) + 8k^3\beta^2H^2\right) \\ + \displaystyle\sum_{j=1}^{3}\frac{F_j e^{-p_j x}\left(2ik\left(1 + p_j^2\right) - 2H^2k^3p_j^2\right)}{p_j^2\left(1 - p_j^2 + \left(H^2 p_j^4/4\right) - 4\omega^2\right)}\end{array}\right\} e^{2i(kz-\omega t)} \tag{29}$$

where

$$4\beta^2 = 4\omega^2 - 1 - 4k^2 - 4H^2k^4 \tag{30}$$

The constants B_2, D_2, p_j's and F_j's are given as:

$$D_2 = \frac{4kE_2}{\left(2i\beta + 2k\right)} - \sum_{j=1}^{3} F_j \frac{\left(2k - p_j\right)}{\left(2i\beta + 2k\right)p_j^2\left(1 - p_j^2 + \left(H^2 p_j^4/4\right) - 4\omega^2\right)^1} \tag{31}$$

$$B_2 = \frac{\left(2i\beta - 2k\right)E_2}{\left(2i\beta + 2k\right)} - \sum_{j=1}^{3}\frac{F_j\left(2i\beta + p_j\right)}{\left(2i\beta + 2k\right)p_j^2\left(1 - p_j^2 + \left(H^2 p_j^4/4\right) - 4\omega^2\right)} \tag{32}$$

$$p_1 = 2k \tag{33}$$

$$p_2 = 2\gamma \tag{34}$$

$$p_3 = k + \gamma \tag{35}$$

$$F_1 = 2k^2\left(\theta_x^2 - 2\theta_z^2\right) \tag{36}$$

$$F_2 = 2\alpha^2\left(\gamma^2 - k^2\right) - 2\omega\alpha\left(2k\psi_z - 2\gamma\psi_x\right) - \left(2\gamma^2\psi_x^2 + 4k^2\psi_z^2\right) + H^2\alpha^2\left(\gamma^2 - k^2\right)^2/4 \tag{37}$$

$$F_3 = 2\omega\alpha\left\{(\gamma+k)\theta_x - 2k\theta_z\right\} - \left\{(\gamma+k)^2\theta_x\psi_x + 8k^2\theta_z\psi_z\right\} \tag{38}$$

V. Results and Discussion

The dispersion relation (22) shows that the effect of the relativistic degeneracy of the electrons is to broaden the frequency spectrum of high frequency surface waves in a semi-infinite plasma. The solutions (26-29) for the second harmonics generated through quadratic nonlinearities show that a part of it decays in the usual way away from the interface while the other part represents a wave propagating obliquely away from the interface into the bulk of the plasma. It will continuously draw a fraction of wave energy away from the interface. The results presented in the paper may be useful to understand surface physics of semiconductors and metals where ions are almost immobile and the boundary is almost rigid.

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