

Fast Accurate Detection of Frequency Jumps Using Kalman Filter with Non Linear Improvements

Mahmoud E. Mohamed, Ahmed F. Shalash, Hanan A. Kamal

Abstract—In communication systems, frequency jump is a serious problem caused by the oscillators used. Kalman filters are used to detect that jump, despite the tradeoff between the noise level and the speed of the detection. In this paper, an improvement is introduced in the Kalman filter, through a nonlinear change in the bandwidth of the filter. Simulation results show a considerable improvement in the filter speed with a very low noise level. Additionally, the effect on the response to false alarms is also presented and false alarm rate show improvement.

Keywords—Kalman Filter, Innovation, False Detection.

I. INTRODUCTION

ONE of the most widely used techniques in communication systems is the orthogonal frequency division multiplexing (OFDM) technique. Many standards have been implemented using OFDM, such as DVB-T, DVB-T2, Wi-Fi, WiMAX and LTE. One of the problems which OFDM suffers from is the frequency offset of the carrier, since the OFDM technique is very sensitive to any frequency offset. An additional problem is the sudden frequency jump in the frequency of the oscillator. It is necessary to detect this sudden frequency jump quickly and accurately. This sudden jump may be caused by humidity, temperature, or other mechanical problems due to aging. To measure the frequency of the oscillator, an analog sensor is assumed to be used, where the output of the sensor is a linear function of the frequency of the oscillator. This output is noisy and needs to be filtered to obtain the time and the value of the frequency jump.

One of the commonly used techniques is the dynamic Allan variance (DAVAR), but it has a high computational cost as in [1]. This problem has been solved in [2]. Another problem is the uncertainty of the solution surface, which was discussed and solved in [3]. However, DAVAR still suffers the problem of non-causality, where it uses data after the jump to detect the jump. In addition, it still has a high computational cost compared to other techniques. Another technique used in [4] is based on the Generalized Likelihood Ratio Test (GLRT) which detects the presence of the frequency jump. Unfortunately, this is a probabilistic technique which cannot detect the value of the jump accurately. Another technique is based on the Kalman filter algorithm [5], [6] where the sensor

and noiseless outputs are used as the observation and the state respectively. Accordingly, it has a low computational cost compared to the other techniques. It has the added advantage of causality. This technique requires a lot of time to detect the frequency jump if a low noise level is required.

In this paper, an improvement in the Kalman filter is introduced, where a nonlinear change in the bandwidth of the filter will be done. This change is made by increasing the Kalman filter gain upon detecting a high innovation value. The innovation is the difference between the measured observation and the estimated one. If this innovation rises above its covariance by a certain threshold, then an action is taken. This change can be made in four different ways. The first is affected by introducing a large momentary step directly after the detection of the innovation. The second is to make a relatively small increase and hold that value until innovation decreases below the threshold and then remove this increase. The third is to increase the Kalman gain gradually until the innovation falls below the threshold. The last is to increase the Kalman gain to a relatively big value and then decrease it gradually until the innovation gets back below the threshold after that bring it to the normal value. This change in the Kalman filter gain can be obtained by increasing the covariance process noise which represents the frequency jump. The results will be carried using the four methods and it will be shown that they are faster than the ordinary Kalman filter. Also, the false alarm tracking will be better in some cases but not in all. The remainder of the paper is organized as follows: Section II introduces the problem formulation and the system modeling. Section II introduces the Kalman filter technique with the improvement done on it. Section IV provides some simulation results for the improvements done versus the algorithm in [5], and Section V is the conclusion.

II. SYSTEM MODEL

At the receiver the oscillator used for the carrier demodulation has a model which is

$$V(t) = V_0 \sin(2\pi f_0 t) \quad (1)$$

where V_0 is the amplitude of the oscillator and f_0 is the carrier frequency. The frequency and the amplitude of the oscillator signal will change due to some imperfections in the oscillator circuits due to humidity, temperature, and the age of oscillator. In this paper, the change in the amplitude is not considered, only the change in the frequency will be considered, only the change in the frequency will be considered. The change in the

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frequency is considered a frequency jump, which is assumed to be a sudden jump. Accordingly, the frequency will be

$$f(t) = f_0 + f_1(t) \quad (2)$$

where,

$$f_1(t) = \Delta f U(t - t_0) \quad (3)$$

where Δf is the sudden change value and $U(t)$ is the unit step function, and t_0 is the time at which frequency jump occurs.

From (1) and (2)

$$V(t) = V_0 \sin(2\pi f_0 t + 2\pi f_1(t)t) \quad (4)$$

The frequency of the output signal is measured by an analog sensor, which consists of some opamps and resistors which introduce flicker noise and additive white Gaussian noise (AWGN). The flicker noise can be neglected, since the circuit is operating in a high frequency range. In this paper, only AWGN with zero mean, which will be added to our measured frequency, is considered. Therefore the output of the sensor is

$$x_i(t) = C_1 f_0 + C_1 f_1(t) + x_g(t) \quad (5)$$

where $x_g(t)$ is the additive white Gaussian noise introduced from the analog sensor, and C_1 is the sensor constant.

Defining,

$$x(t) = x_i(t) - C_1 f_0 \quad (6)$$

Then

$$x(t) = C_1 f_1(t) + x_g(t) \quad (7)$$

Defining

$$x_d(t) = C_1 f_1(t) \quad (8)$$

Then

$$x_d(t) = \Delta f U(t - t_0) \quad (9)$$

And (7) will be

$$x(t) = x_d(t) + x_g(t) \quad (10)$$

Therefore the output is the sudden jump in the frequency and the additive white Gaussian noise from the analog sensor.

In order to measure this jump accurately and fast, we use a discrete time filter, with sampling time T_s , then

$$t = kT_s \quad (11)$$

and (10) becomes

$$x(kT_s) = x_d(kT_s) + x_g(kT_s) \quad (12)$$

By eliminating the sampling time notation from the equation we get

$$x(k) = x_d(k) + x_g(k) \quad (13)$$

where,

$$x_d(k) = \Delta f U(k - k_0) \quad (14)$$

where

$$t_0 = k_0 T_s \quad (15)$$

In the next section, the Kalman filter will be used with the improvement using this model stated in this section. The Kalman filter will be discussed in the Z-domain to obtain its bandwidth. The four improvements will be listed. Also, these improvements will be discussed in the time domain to show their effect on the Kalman filter equations and how they can be modeled.

III. FREQUENCY JUMP DETECTION

The authors in [5] had proposed a technique to detect the jump using the Kalman filter. Because it is a recursive filter, it has a low computational cost and low complexity. In addition, we have only one state, so it will be solved easily by the Kalman filter. The oscillator frequency jump is the filter state $x_d(k)$, and the discretized output of the analog sensor $x(k)$ is the observation, so we can write the system model as

$$x_d(k) = A x_d(k-1) + w(k) \quad (16)$$

$$x(k) = C x_d(k) + v(k) \quad (17)$$

where, A is the state transition model, C is the observation model, $w(k)$ is the process noise which represents the sudden jump, and $v(k)$ which represents the additive white Gaussian noise added by the analog sensor.

The Kalman filter algorithm can be written as follow;

$$\hat{x}_d(k|k-1) = A \hat{x}_d(k-1|k-1) \quad (18)$$

where, $\hat{x}_d(k-1|k-1)$ is the updated (a posteriori) state estimate at time $k-1$ using measurements up to the same time instant $k-1$, and $\hat{x}_d(k|k-1)$ is the predicted (a priori) state estimate at time instant k using estimated states up to $k-1$.

$$P(k|k-1) = A^2 P(k-1|k-1) + Q \quad (19)$$

where $P(k|k-1)$ is the predicted (a priori) estimate covariance of the state at time instant k using the previous time instants up to $k-1$, and Q is the covariance of the process noise.

$$K(k) = \frac{P(k|k-1)C}{C^2 P(k|k-1) + R} \quad (20)$$

where, R is the covariance of observation noise, and $K(k)$ is the optimal Kalman filter gain.

$$\hat{x}_d(k|k) = \hat{x}_d(k|k-1) + K(k)[x(k) - C \hat{x}_d(k|k-1)] \quad (21)$$

where $\hat{x}_d(k|k)$ is the updated (a posteriori) state estimate at time instant k using measurements up to k instant. And finally

$$P(k|k) = [1 - K(k)C]P(k|k-1) \quad (22)$$

where, $P(k|k)$ is updated (a posteriori) estimate covariance of the sates at time instant k using measurements up to k instant. Using these equations we can investigate the bandwidth of the Kalman filter. Put $A = C = 1$, then take the Z-transform and assuming K constant, we can find that

$$\hat{X}_d(z) = \hat{X}_d(z) + K[X(z) - z^{-1}\hat{X}_d(z)] \quad (23)$$

Then

$$\frac{\hat{X}_d(z)}{X(z)} = \frac{K}{1 - (1-K)z^{-1}} = \frac{Kz}{z - (1-K)} \quad (24)$$

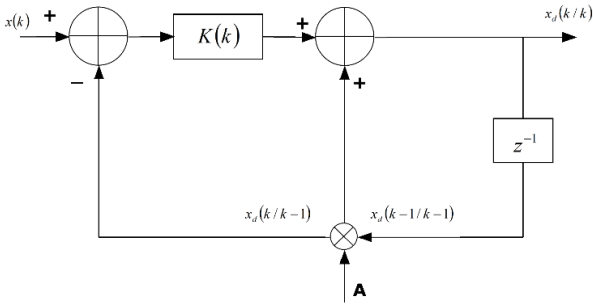


Fig. 1 Kalman filter model in solving frequency jump problem

So, Kalman filter is an IIR low pass filter where its bandwidth can be investigated as follows, put

$$z = e^{jw_c} \quad (25)$$

where w_c is the cut off frequency of the Kalman filter, then

$$\frac{K^2}{|(\cos(w_c) - (1-K) + j \sin(w_c))|^2} = \frac{1}{2} \quad (26)$$

Then

$$w_c = \cos^{-1} \left(\frac{2 - 2K - 2K^2}{2(1-K)} \right) \quad (27)$$

By evaluating (27), when K increase the bandwidth of the filter will decrease.

We want to filter a signal, which is constant most of the time with some noise with autocorrelation R . In addition, at some times the signal has sudden jump which can be modeled as $\frac{1}{w}$ in the continuous time domain. The improvement is to use the innovation and innovation covariance not only to detect the jump as in [6] but also to change the bandwidth of the Kalman filter. The bandwidth will be increased at this instant so the filter can respond to the jump faster then return it to the previous bandwidth. Using this non-linear change, the filter will keep the noise level small, and only the noise increase at the instant of the detection, but we will have the benefit of faster and accurate detection. So, to increase the bandwidth, the Kalman filter gain K at the jump instant will be increased, from (20) put $C = 1$, since, R is constant as it will be known from the analog sensor, then the only way is to increase the covariance process noise Q at this instant.

From (19) after putting $A = 1$, increasing Q will increase predicted (a priori) estimate covariance which will increase the optimal Kalman filter gain at this instant.

By using the innovation referred to in [5], which is defined as, the difference between the measured observation and the estimated one. This innovation will be compared with the innovation variance. If the ratio increases a certain threshold l then a sudden frequency jump occurs.

The innovation can be written as

$$e(k) = x(k) - A\hat{x}_d(k|k-1) \quad (28)$$

and the covariance of the innovation is

$$\sigma_e^2(k) = C^2 P(k|k-1) + R \quad (29)$$

If $e(k) > l\sigma_e(k)$, where l is a threshold ratio, this indicates that there's a sudden frequency jump, then we must take a nonlinear action.

In this paper, four different modifications to the Kalman filter gain, using the covariance of the process noise Q are introduced. The four forms can be listed as follows:

- 1) Upon detecting the innovation, make an increase in the covariance of the process noise Q which will be nearly equal to the value of the innovation as in Fig. 2 (a).
- 2) Upon detecting the innovation, increase the covariance of the process noise Q to a certain value Q_1 and check again if the innovation error is still over the threshold, the covariance of the process noise Q will be still equal to Q_1 until the innovation becomes below the threshold, then Q will get back to its value as in Fig. 2 (b).
- 3) Change the value of Q gradually from low to high upon detecting an innovation until it comes back below the threshold then we return Q to its initial value as in Fig. 2 (c).
- 4) Change the value of Q from high to low upon detecting an innovation until it comes back below the threshold then we return Q to its initial value as in Fig. 2 (d).

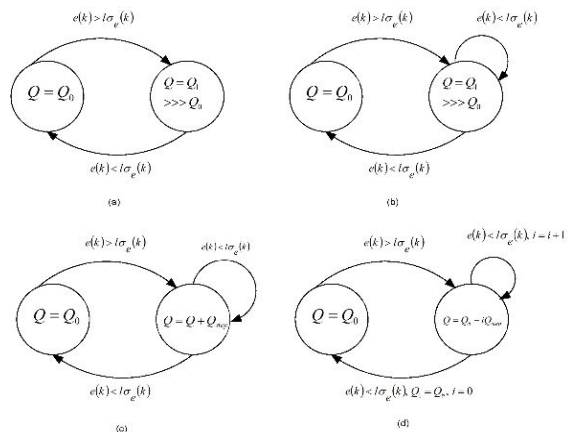


Fig. 2 State diagram representation for the non-linear change in the Kalman filter bandwidth using the innovation, where (a) is the first

case, (b) is the second case, (c) is the third case, and (d) is the fourth case

This nonlinear change can be expressed in time domain in the Kalman filter equations. Assuming first the optimal Kalman filter gain and covariance of the process noise are

$$K = K_0 \quad (30)$$

$$Q = Q_0 \quad (31)$$

where K_0 and Q_0 are constant values. Also, other values $K_{e,i}$ will be defined as the error in the steady state Kalman gain due to non-zero settling time of the Kalman filter.

So, the Kalman filter equations can be expressed in the four different cases as follows

The first case, the Kalman filter gain is increased by setting Q with a very large value nearly equal to the innovation for only one sample, then returning it to its default value Q_0 , so we can write the equation as:

$$Q(k) = Q_0 + Q_1 \delta(k - k_1) \quad (32)$$

$$K(k) = K_0 + K_1 \delta(k - k_1) + \sum_{i=2}^{m-1} K_{e,i} \delta(k - k_i) \quad (33)$$

where, k_1 is the sample at which the sudden frequency jump happens, and the term $K_1 \delta(k - k_1) (x(k_1) - \hat{x}_d(k_1 - 1))$ represents the nonlinear term in the Kalman filter equation. The k_e represents the non-zero settling time for the Kalman filter where it turns back at $k_m - 1$.

In the second case, upon detecting a frequency jump, the value of the Kalman gain is changed by setting Q to a certain value larger than Q_0 , until the value of the innovation becomes lower than the threshold, then return Q to its initial value Q_0 , so the equations can be expressed as

$$Q(k) = Q_0 + Q_1 \sum_{i=1}^n \delta(k - k_i) \quad (34)$$

$$K(k) = K_0 + K_1 \sum_{i=1}^n \delta(k - k_i) + \sum_{i=n+1}^{m-1} K_{e,i} \delta(k - k_i) \quad (35)$$

where, k_0 is the sample at which the step happens, and k_n is the last sample where the innovation exceeds the threshold, and the term $K_1 \left(\sum_{i=1}^n (x(k_i) - \hat{x}_d(k_i - 1)) \delta(k - k_i) \right)$ represents the nonlinear effect in Kalman filter.

In the third case, upon detecting the frequency jump, the Kalman gain is increased by gradually increasing Q by a constant step Q_{step} until the innovation becomes below the threshold then set Q to its initial value before the step happens Q_0 , the time domain equation can be written as

$$Q(k) = Q_0 + \sum_{i=1}^n Q_i \delta(k - k_i) \quad (36)$$

$$Q_i = Q_{i-1} + Q_{step} \quad (37)$$

$$K(k) = K_0 + \sum_{i=1}^n K_i \delta(k - k_i) + \sum_{i=n+1}^{m-1} K_{e,i} \delta(k - k_i) \quad (38)$$

where, k_0 is the sample where the step happens, and k_n is the last sample where the innovation exceeds the threshold, and the term $\left(\sum_{i=1}^n (K_i (x(k_i) - \hat{x}_d(k_i - 1)) \delta(k - k_i)) \right)$, represents

the nonlinear effect in Kalman filter, where $k_1 < k_2 < \dots < k_n$, where $k_{n-1} = k_n - 1$.

The last case, upon detecting the step we set Q with a relatively high value Q_b but not as in the first case and gradually decrease it until the innovation becomes below the threshold then set Q to its initial value Q_0 , the equations are the same like the last case except

$$Q_1 = Q_b \quad (39)$$

$$Q_i = Q_{i-1} - Q_{step} \quad (40)$$

$k_1 > k_2 > \dots > k_n$, where, $k_{n-1} = k_n - 1$.

In the next section, some simulation results will be listed to show the effectiveness of those improvements on the response to the jump and on the fault detection and tracking.

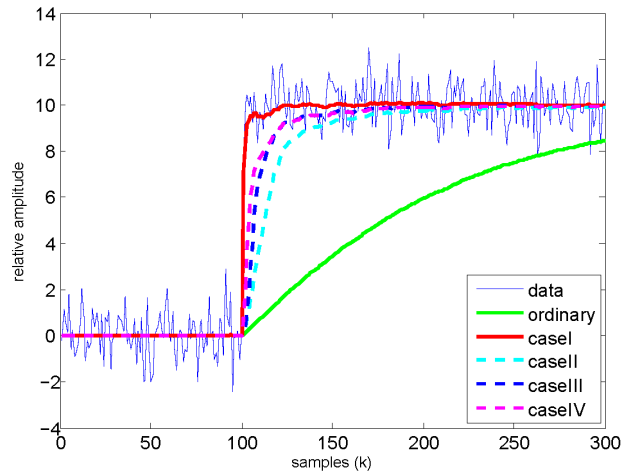


Fig. 3 Response of the Kalman filter in the ordinary case when no nonlinear change happens and the other cases stated in this paper, which shows a highly improvement in the transient time of the detection

IV. RESULTS

In the simulations, the problem was modeled as follows, the sensor output has been sampled by a sampling time $T_s = 1 \text{ sec}$. The oscillator used has a frequency jump at $k = k_1 = 101$, and the value of the jump measured by the sensor is 10 (relative units according to the sensor). The sensor adds AWGN with zero mean and unity variance (relative units according to the sensor), so the observation noise covariance will be equal to $R = 1$.

For the Kalman filter $Q = Q_0 = 0.0001$ has been used when there's no frequency jump. When a frequency jump is detected, in the first case, put $Q_1 = 10$, which is nearly the innovation value assumed to be known. In the second case, put $Q_1 = 0.01$, and wait until the innovation gets below the threshold again. In the third case a step $Q_{step} = 0.01$ has been

used. And finally at the fourth case, first put $Q_b = 3$, and use step $Q_{step} = 0.01$.

In Fig. 3, the sampled output of the sensor before using the Kalman filter $x(k)$, the output of the ordinary Kalman filter, and the output of the Kalman filter in each of the four cases mentioned in Section III have been plotted.

In this figure a high threshold $l = 5$ has been used in order to focus on the transient time only. In Fig. 3, all the cases have the same output when no jump has been detected as they all use the same $= Q_0$. Also, the ordinary Kalman filter has the worst transient time which can be defined at 95% from the final value (i.e. error=5%), and the first case has the best transient time.

The second case is the worst one among the four cases but it is better than the ordinary Kalman filter. Also the third and the fourth cases are very close to each other. The transient time in each case is listed in Table I. where the time instants are in second. In Figs. 3 and 5, the threshold has been changed to $l = 2$.

Fig. 4 shows the main problem associated with the first case when a false alarm happens. The filter in this case will respond very fast to this fault and detects it leading to a high probability of false detection. This problem is not severe in the other cases as shown in Fig. 5, as the filter in those cases has a slow response to those false alarms, which leads to a low probability of false detection. In both figures the response to this false alarms are marked.

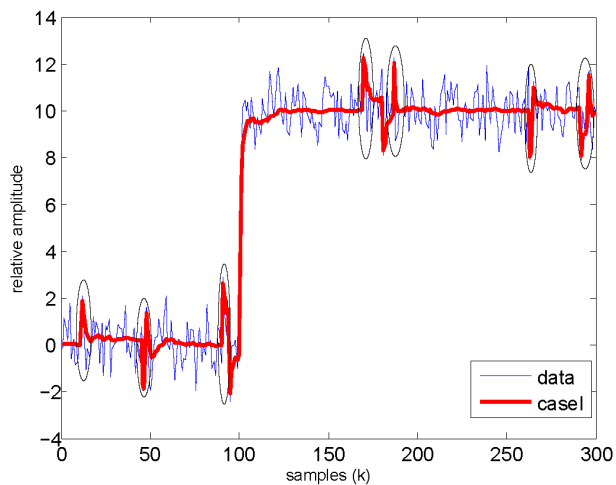


Fig. 4 The false alarms from using the first case which is marked by the ellipsoid shape

V. CONCLUSION

In this paper, the frequency jump problem in the oscillator has been considered. Four different ways have been introduced to improve the performance of the Kalman filter in detecting the sudden frequency jumps quickly and accurately. These ways are based on increasing bandwidth of the Kalman filter at the jump instant by increasing the Kalman filter gain using the covariance of the process noise. Some simulation

results have shown a considerable improvement happened in the output of the Kalman filter. It was founded that case I described in the paper show the best frequency jump tracking, however, it suffers from a false alarm problem. While cases III and IV alleviate the false alarm problem and show tracking properties that is considerably faster than the ordinary Kalman filter.

TABLE I
TRANSIENT TIME OF THE ORDINARY KALMAN FILTER AND THE OTHER FOUR WAYS, WHERE WE CAN CONSIDER THIS TIME WHEN THE OUTPUT IS 5% FROM THE FINAL RESULT, AND THE TIME INSTANTS ARE IN SECOND

Type	Transient time
Ordinary Kalman	400
First Case	120
Second Case	170
Third Case	150
Fourth Case	148

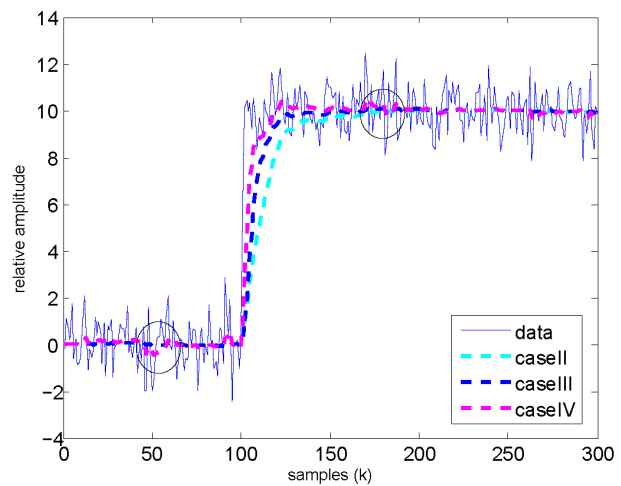


Fig. 5 The response of the other case to the false alarms, the response has been marked by a circle

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