

# About the Instability Modes of Current Sheet in Wide Range of Frequencies

V. V. Lyahov, V. M. Neshchadim

**Abstract**—We offer a new technique for research of stability of current sheaths in space plasma taking into account the effect of polarization. At the beginning, the found perturbation of the distribution function is used for calculation of the dielectric permeability tensor, which simulates inhomogeneous medium of a current sheath. Further, we in the usual manner solve the system of Maxwell's equations closed with the material equation. The amplitudes of Fourier perturbations are considered to be exponentially decaying through the current sheath thickness. The dispersion equation follows from the nontrivial solution requirement for perturbations of the electromagnetic field. The resulting dispersion equation allows one to study the temporal and spatial characteristics of instability modes of the current sheath (within the limits of the proposed model) over a wide frequency range, including low frequencies.

**Keywords**—Current sheath, distribution function, effect of polarization, instability modes, low frequencies, perturbation of electromagnetic field dispersion equation, space plasma, tensor of dielectric permeability.

## I. INTRODUCTION

THE present paper is a continuation of the study of current sheaths started in papers [1]-[3]. These papers developed the kinetic theory of equilibrium and stability of the non-electroneutral current sheaths, which formed at arbitrary values of the medium parameters.

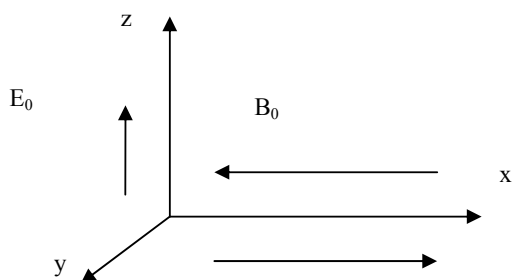


Fig. 1 The coordinate system of the current sheath

Assume that the plasma is concentrated in the  $z=0$  plane (see Fig. 1), and that anti-parallel magnetic fields are present in the top and bottom half-spaces. In this plane, a self-consistent current sheath dividing the areas with oppositely directed magnetic fields forms.

V. V. Lyahov is with the Institute of Ionosphere, 050020 Almaty, Kazakhstan (e-mail: v\_lyahov@rambler.ru).

V. M. Neshchadim is with the Institute of Ionosphere, 050020 Almaty, Kazakhstan.

In order to describe the equilibrium current sheath we generalized the Harris [4] distribution function and introduced the anisotropy of the temperatures of the plasma components along and across the sheath, namely:

$$f_{0\alpha} = \frac{m_\alpha}{2\pi\theta_{\alpha z}} n_0 (1 + \alpha_\alpha)^{\frac{1}{2}} \exp \left[ \frac{m_\alpha}{2\theta_{\alpha z}} (1 + \alpha_\alpha) U_\alpha^2 \right] \cdot \exp \left[ -\frac{W_\alpha}{\theta_{\alpha z}} - \frac{\alpha_\alpha P_{y\alpha}^2}{2\theta_{\alpha z} m_\alpha} + U_\alpha (1 + \alpha_\alpha) \frac{P_{y\alpha}}{\theta_{\alpha z}} \right]. \quad (1)$$

Here

$$\alpha_\alpha = \frac{\theta_{\alpha z}}{\theta_{\alpha y}} - 1 \quad (2)$$

is a degree of the temperature anisotropy, and

$$U_\alpha \quad (3)$$

is macroscopic velocity directed along the y-axis.

Study of the current-sheath equilibrium is presented in the paper [1].

## II. DERIVATION OF THE DISPERSION EQUATION

Investigation of the amplitude–frequency characteristics of the perturbations of a current sheath is based on the solution of Maxwell's equations for the perturbations of the electromagnetic field closed by a material equation. After substituting the expansions of the electromagnetic field

$$\begin{aligned} f_\alpha(\vec{P}, \vec{r}, z, t) &= f_{0\alpha}(\vec{P}) + \delta f_\alpha(\vec{P}, \vec{r}, z, t); \\ \vec{E}(\vec{r}, z, t) &= \vec{E}_0(z) + \delta \vec{E}(\vec{r}, z, t); \\ \vec{B}(\vec{r}, z, t) &= \vec{B}_0(z) + \delta \vec{B}(\vec{r}, z, t). \end{aligned} \quad (4)$$

into Maxwell's equations

$$\begin{aligned} \text{rot} \vec{B} &= \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + \frac{1}{\varepsilon_0 c^2} \vec{j}, \text{div} \vec{B} = 0, \\ \text{rot} \vec{E} &= -\frac{\partial \vec{B}}{\partial t}, \text{div} \vec{E} = \frac{\rho}{\varepsilon_0} \end{aligned} \quad (5)$$

where

$$\rho = \sum_\alpha e_\alpha \int \delta f_\alpha d\vec{P}; \quad \vec{j} = \sum_\alpha e_\alpha \int \vec{v} \delta f_\alpha d\vec{P} \quad (6)$$

we obtain the following system of equations:

$$\begin{cases} c^2 \varepsilon_0 \operatorname{rot} \delta \vec{B} = \frac{\partial \delta \vec{D}}{\partial t}, \operatorname{div} \delta \vec{B} = 0, \\ \operatorname{rot} \delta \vec{E} = -\frac{\partial \delta \vec{B}}{\partial t}, \operatorname{div} \delta \vec{D} = 0. \end{cases} \quad (7)$$

Here  $\delta \vec{B}(\vec{r}, t)$  and  $\delta \vec{E}(\vec{r}, t)$  are the perturbations of the magnetic induction and the electric intensity of the system in question, and  $\delta \vec{D}(\vec{r}, t)$  is the perturbation of the electric induction.

We use the following Fourier transforms:

$$\begin{aligned} \delta \vec{E}(\vec{r}, t) &= \delta \vec{E}(\omega, \vec{k}, z) \exp(-i\omega t + ik_x x + ik_y y), \\ \delta \vec{B}(\vec{r}, t) &= \delta \vec{B}(\omega, \vec{k}, z) \exp(-i\omega t + ik_x x + ik_y y), \\ \delta \vec{D}(\vec{r}, t) &= \delta \vec{D}(\omega, \vec{k}, z) \exp(-i\omega t + ik_x x + ik_y y). \end{aligned} \quad (8)$$

Then, taking into account the material equation for Fourier amplitudes,

$$\delta D_i(\omega, \vec{k}, z) = \varepsilon_0 \varepsilon_{ij}(\omega, \vec{k}, z) \delta E_j(\omega, \vec{k}, z) \quad (9)$$

from Maxwell's equations (7) we obtained the dispersion equation for perturbations of the electroneutral current sheath:

$$[k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, \vec{k}, z)] = 0. \quad (10)$$

Derivation of the permittivity tensor  $\varepsilon_{ij}(\omega, \vec{k}, z)$  is shown in a preceding paper [2].

In order to describe a tearing instability, along with the Cerenkov and cyclotron interactions of considered eigenmodes with particles of the magnetoactive plasma of the current sheath, we also consider adiabatic perturbations of the current and the magnetic field, respectively, as caused by adhesion of the current hairlines. In this case, the dispersion equation takes the form [2]:

$$[-Q \delta_{ix} \delta_{yy} + i\omega(k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, \vec{k}))] = 0 \quad (11)$$

The quantity  $Q = Q(\phi(z), A_y(z))$  is given in the Appendix.

The dispersion equation (11), was derived in the approximation of a weakly inhomogeneous medium, i.e. we ignored the dependence of the amplitudes of the Fourier-expansions in (8) on the thickness of the current sheath. Therefore, the obtained dispersion equation is correct for the short-wave instability modes  $k_x L > 1$  (here,  $L$  is the characteristic thickness of the sheath).

In this paper, we took into account the dependence of the

amplitudes of the Fourier expansions on the current-sheath thickness  $z$ . Since the theory of current sheaths is constructed for a collisionless plasma, the instability modes of such sheaths are developed in terms of a Landau antidamping mechanism for the Cherenkov, or synchrotron, channel. The intensity of the antidamping oscillations is naturally greater in the place where the density of particles is larger.

For our non-electroneutral model of a current sheath the analytical expression for the density profile of the plasma components is absent. However, in [3] it was found that the equilibrium solutions exist for the current layer in the neighborhood of points where the parameters of the problem provide the electroneutrality. Consequently, in the first approximation we can use the analytic electroneutral Harris expression for the plasma density [5]:

$$n_e(z) = n_i(z) = \frac{n_0}{\cosh^2(\frac{z}{L})} \quad (12)$$

where

$$L = \sqrt{\frac{2\theta_i}{\mu_0 e^2 n_0 U_i^2 (1 + \frac{\theta_e}{\theta_i})}} \quad (13)$$

It is natural to assume that the Fourier amplitudes of the swinging perturbations decrease with the sheath thickness in the same manner:

$$\begin{aligned} \delta E(\omega, \vec{k}, z) &= \delta E_0(\omega, \vec{k}) \frac{1}{\cosh^2(\frac{z}{L})}, \\ \delta A_y(\omega, \vec{k}, z) &= \delta A_{y0}(\omega, \vec{k}) \frac{1}{\cosh^2(\frac{z}{L})} \end{aligned} \quad (14)$$

In this case, the dispersion equation, i.e. (11), takes a more complex form:

$$\det(D_{ij}) = 0. \quad (15)$$

Here

$$\begin{aligned} D_{11} &= -\frac{i \cdot \varepsilon_0 (4 \frac{c^2}{L^2} - c^2 k_y^2 - \frac{6 \frac{c^2}{L^2}}{\cosh(\frac{z}{L})^2} - \omega^2 \varepsilon_{xx})}{\cosh(\frac{z}{L})^2} \\ D_{12} &= -\frac{i \cdot \varepsilon_0 (c^2 k_x k_y - \varepsilon_{yx} \omega^2)}{\cosh(\frac{z}{L})^2} \end{aligned}$$

$$\begin{aligned}
D_{13} &= \frac{\varepsilon_0 (2 \tanh(\frac{z}{L}) \frac{c^2}{L} \cdot k_x + i \cdot \varepsilon_{zx} \omega^2)}{\cosh(\frac{z}{L})^2} \\
D_{21} &= - \frac{i \cdot (c^2 \varepsilon_0 k_x k_y - \varepsilon_{xy} \omega^2)}{\cosh(\frac{z}{L})^2} \\
D_{22} &= - \frac{i \cdot (4 \frac{c^2}{L^2} \varepsilon_0 - c^2 k_x^2 \varepsilon_0 - \frac{6 c^2}{L^2} \varepsilon_0 - \omega^2 \varepsilon_{yy} - Q)}{\cosh(\frac{z}{L})^2} \\
D_{23} &= \frac{2 \tanh(\frac{z}{L}) \cdot \frac{c^2}{L} \cdot \varepsilon_0 k_y + i \cdot \varepsilon_{zy} \omega^2}{\cosh(\frac{z}{L})^2} \\
D_{31} &= \frac{2 \tanh(\frac{z}{L}) \cdot \frac{c^2}{L} \cdot \varepsilon_0 k_x + i \cdot \varepsilon_{xz} \omega^2}{\cosh(\frac{z}{L})^2} \\
D_{32} &= - \frac{i \cdot (2i \cdot \tanh(\frac{z}{L}) \cdot \frac{c^2}{L} \cdot \varepsilon_0 k_y - \varepsilon_{yz} \omega^2)}{\cosh(\frac{z}{L})^2} \\
D_{33} &= \frac{i \cdot (c^2 \varepsilon_0 k_x^2 + c^2 \varepsilon_0 k_y^2 + \varepsilon_{zz} \omega^2)}{\cosh(\frac{z}{L})^2}
\end{aligned}$$

Equation (17) is a transcendental equation.

### III. RESULTS OF THE STUDY OF THE TEARING INSTABILITY

Solution of the dispersion equation, i.e. (15), is implemented using the MAPLE Package. The calculations were realized for the following parameters of the current sheath in the magnetospheric tail:

$$\begin{aligned}
m_e &= 9.1 \cdot 10^{-31} \text{ kg}, m_i = 1836 \cdot 9.1 \cdot 10^{-31} \text{ kg}, \\
c &= 3 \cdot 10^8 \text{ m/s}, \theta_e = 1.6 \cdot 10^{-16} \text{ J}, \\
\theta_i &= 1.6 \cdot 10^{-15} \text{ J}, e_i = -e_e = 1.6 \cdot 10^{-19} \text{ C}, \\
\varepsilon_0 &= 8.85 \cdot 10^{-12} \text{ F/m}, \mu_0 = 1.26 \cdot 10^{-6} \text{ Gs/m}, \\
n_{oe} &= n_{oi} = 0.16 \cdot 10^6 \text{ m}^{-3}.
\end{aligned} \tag{16}$$

Values of the equilibrium magnetic field  $B_0(z)$  and equilibrium magnetic potential  $A_y(z)$  at any point in the current sheath were taken from the appropriate solution for the equilibrium current sheath [1]. The calculations were performed for the case of an absence of the polarization process in the cross-section  $z = -1 \cdot 10^3 \text{ m}$ . The macroscopic

velocity of the ion component was taken to be  $U_i = 3 \cdot 10^6 \text{ m/s}$ .

Solution of (15) has 108 complex roots, i.e. 108 possible eigenmodes of the current sheath instability. Among these modes there are also modes of tearing instability. Fig. 2 shows a dispersion curve for the imaginary part of frequency of the tearing mode.

Here

$$\begin{aligned}
\Omega_{0i} &= \frac{e_i B_0}{m_i} = 2.56 \text{ s}^{-1} \text{ and} \\
L &= \sqrt{\frac{2\theta_i}{\mu_0 e^2 n_0 U_i^2 (1 + \frac{\theta_e}{\theta_i})}} = 2.50 \cdot 10^5 \text{ m}
\end{aligned}$$

is the characteristic thickness of the current sheath.

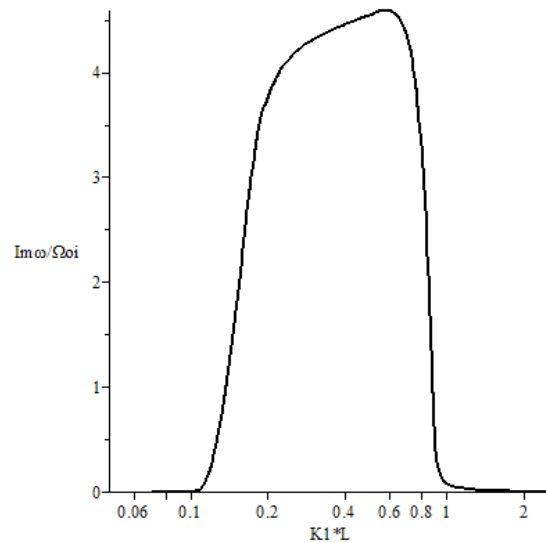


Fig. 2 Dispersion curve of the tearing-mode

### IV. CONCLUSION

We obtained a dispersion equation, which allowed us explore the temporal and spatial characteristics of all of the instability modes of the current sheath (within the limitations of the proposed model) over a wide range of frequencies, including low frequencies. The dispersion equation follows from the condition of nontrivial solutions for the perturbations of the electromagnetic field. The amplitudes of the Fourier perturbations are considered to be exponentially decreasing on the thickness of a current sheath. The tearing instability of the current sheath was explored.

By using this method, even as applied to the electroneutral current sheath, we revealed the existence of low-frequency tearing-like modes, which were essentially different from the earlier known tearing perturbations. The growth rate of these modes appears to be by 1–2 orders more than it was supposed earlier for tearing instabilities. In a sense, the tearing mode reappears as an initial stage of the process of reconnection of

magnetic fields in the current sheaths.

#### APPENDIX

$Q = jy1 + jy2$ ;

$$\begin{aligned}
 jy1 &= \sum_{\alpha} e_{\alpha} n_0 m_{\alpha}^3 M_{1\alpha} \left[ U_{\alpha} - \frac{\alpha_{\alpha}}{1 + \alpha_{\alpha}} \frac{e_{\alpha} A_y(z)}{m_{\alpha}} \right] \cdot Ey1; \\
 jy2 &= - \sum_{\alpha} \frac{\sqrt{\pi}}{4} m_{\alpha}^3 \frac{2\pi\theta_{z\alpha}}{m_{\alpha}} L_{\alpha} M_{2\alpha} N_{\alpha} \left( \frac{m_{\alpha}(1 + \alpha_{\alpha})}{2\theta_{z\alpha}} \right)^{\frac{5}{2}} \cdot \\
 &\cdot \left[ \left( - \frac{U_{\alpha}(1 + \alpha_{\alpha})m_{\alpha}}{\theta_{z\alpha}} + \frac{\alpha_{\alpha}e_{\alpha}A_y(z)}{\theta_{z\alpha}} \right)^2 + \frac{m_{\alpha}(1 + \alpha_{\alpha})}{\theta_{z\alpha}} \right] \cdot Ey2 \\
 M_{1\alpha} &= \frac{e_{\alpha}(1 + \alpha_{\alpha})U_{\alpha}}{\theta_{z\alpha}} - \frac{\alpha_{\alpha}e_{\alpha}^2 A_y(z)}{\theta_{z\alpha} m_{\alpha}}; \\
 M_{2\alpha} &= \frac{\alpha_{\alpha}e_{\alpha}}{\theta_{z\alpha}}; \\
 L_{\alpha} &= e_{\alpha} n_0 (1 + \alpha_{\alpha}) \left( \frac{m_{\alpha}}{2\pi\theta_{z\alpha}} \right)^{\frac{3}{2}} \exp \left[ \frac{m_{\alpha}(1 + \alpha_{\alpha})U_{\alpha}^2}{2\theta_{z\alpha}} \right]; \\
 N_{\alpha} &= \exp \left[ - \frac{e_{\alpha}\phi(z)}{\theta_{z\alpha}} + \frac{(1 + \alpha_{\alpha})e_{\alpha}U_{\alpha}A_y(z)}{\theta_{z\alpha}} - \frac{\alpha_{\alpha}e_{\alpha}^2 A_y^2(z)}{2\theta_{z\alpha}m_{\alpha}} \right]; \\
 Ey1 &= \exp \left[ - \frac{e_{\alpha}\phi(z)}{\theta_{z\alpha}} + \frac{e_{\alpha}U_{\alpha}A_y(z)}{\theta_{z\alpha}} - \frac{\alpha_{\alpha}}{(1 + \alpha_{\alpha})} \frac{e_{\alpha}^2 A_y^2(z)}{2\theta_{z\alpha}m_{\alpha}} \right]; \\
 Ey2 &= \exp \left\{ \frac{\left[ - \frac{(1 + \alpha_{\alpha})U_{\alpha}m_{\alpha}}{\theta_{z\alpha}} + \frac{\alpha_{\alpha}e_{\alpha}A_y(z)}{\theta_{z\alpha}} \right]^2}{\frac{2(1 + \alpha_{\alpha})m_{\alpha}}{\theta_{z\alpha}}} \right\}.
 \end{aligned}$$

#### REFERENCES

- [1] V.V. Lyahov, V.M. Neshchadim, "Kinetic theory of the current sheath. I. On polarization of an equilibrium current sheath," *Advances in Space Research*, vol. 50(5), pp. 318-326, 2012, DOI:10.1016/j.asr.2012.04.021.
- [2] V.V. Lyahov, V.M. Neshchadim, "Kinetic theory of the current sheath. II. Effect of polarization on the stability of a current sheath," *Advances in Space Research*, vol. 51(5), pp. 730-741, 2013, DOI:10.1016/j.asr.2012.10.006.
- [3] V.V. Lyahov, V.M. Neshchadim, "About the equilibrium and stability of non-electroneutral current sheaths," *Advances in Space Research*, 2014, DOI:10.1016/j.asr.2014.05.004.
- [4] E.G. Harris, "On a Plasma Sheath Separating Regions of Oppositely Directed Magnetic Field," *Nuovo Cim.* vol. 23(1), pp. 115-121, 1962.
- [5] M. Dobrovoly, "Instability of a Neutral Sheet," *Nuovo cimento.* vol. 55(2), pp 427-442, 1968.