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Residual Life Prediction for a System Subject to Condition Monitoring and Two Failure Modes

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Abstract—In this paper, we investigate the residual life prediction problem for a partially observable system subject to two failure modes, namely a catastrophic failure and a failure due to the system degradation. The system is subject to condition monitoring and the degradation process is described by a hidden Markov model with unknown parameters. The parameter estimation procedure based on an EM algorithm is developed and the formulas for the conditional reliability function and the mean residual life are derived, illustrated by a numerical example.

Keywords—Partially observable system, hidden Markov model, competing risks, residual life prediction.

I. Introduction

RECENTLY, due to the advances in sensor development, data measurement technology, and computer technology, it became possible to implement effective condition monitoring systems for critical equipment in many companies' information systems. This information can be utilized for the assessment of the actual condition of the operating equipment without any unwanted disruption or unplanned stopping of the operation, which usually result in a high cost due to lost production. A maintenance strategy referred to as condition-based maintenance (CBM) can then be developed and comparing with the traditional maintenance techniques, CBM reduces the risk of catastrophic system failure as well as the maintenance cost. It is obvious that the collected data carries only partial information about the unknown, hidden state of the equipment and the dimensionality of such data is typically very large, with lots of redundancy, noise, and substantial cross and auto correlation present.

Various approaches for processing and modeling of such information have been proposed in the literature which can be generally classified as nonparametric and parametric techniques (see e.g. [1], [2], [3], [4], [5]). Although systems with two failure modes appear in a variety of technical applications, majority of existing CBM models consider only single failure mode. In fact, [6] is the only reference where CBM with multiple failure modes was developed for continuously monitored degrading systems. However, this assumption is no longer valid when the system state is monitored at discrete times, which is the usual practice. Such a drawback of existing models motivates us to consider a CBM model with two modes of failures (competing risks) i.e., a catastrophic and degradation failures which arise quite naturally and are of much interest in the reliability area.

In this paper, we focus on the application of a parametric technique which can be used to extract useful information for early fault detection and reliability estimation of a technical system subject to both deterioration and sudden failures. The system is subject to condition monitoring and data collection at regular times. We assume that three types of data histories are available: histories that end with observable system failure caused by degradation, histories that end with observable sudden failure, and suspension histories.

The evolution of the actual state of the monitored equipment can be modeled in several ways, for example using the proportional hazards model [7], [2], hidden Markov model [8], [4] or hidden semi-Markov model [9], [10]. We assume that the degradation process evolves as a continuous-time homogeneous Markov chain $(Z_t : t \in \mathbb{R}^+)$ with state space $\mathcal{Z} = \{1, 2, 3\}$, where states 1 and 2 are unobservable, representing the healthy and unhealthy operational states respectively, and state 3 represents the observable failure state.

There have been two approaches for a joint parameter estimation of the hidden Markov model using the expectation-maximization (EM) algorithm. The first approach uses the pre-processed observation data directly and applies a state-space representation of the observation process model and Kalman filtering [11], which is computationally very intensive. The second approach focuses on fitting a vector autoregressive model to the pre-processed observation data, then calculating the residuals using the fitted model for the complete data histories and defining the observation process as the residual process. This approach utilizes the results in [12], where it was proved that such residuals are independent and normally distributed, which simplifies the application of EM algorithm for the joint HMM and residual process parameter estimation [13].

In this paper, we develop a new model for a system with two failure modes in Section II. The estimation procedure based on the EM algorithm is developed in Section III, where the observation process is defined as the residual process obtained after data pre-processing and fitting the reference model to the in-control portion of data histories. The formulas for the conditional reliability (RF) and mean residual life (MRL) function are presented in Section IV. The whole procedure is illustrated by an example in Section V, followed by conclusions in Section VI.

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II. Model Formulation

Assume that the degradation process of the system evolves as a continuous-time homogeneous Markov chain $(Z_t : t \in \mathbb{R}^+)$ with state space $\mathcal{Z} = \{1,2\}\bigcup\{3\}$, where states 1 and 2 are unobservable, representing the healthy and unhealthy operational states respectively, and state 3 represents the observable failure state. The system is assumed to start in a healthy state and the sojourn times in states 1 and 2 have exponential distributions with parameters λ_1 and λ_2, respectively. The transitions can only take place from each state to the next higher state. We further assume that the sudden failures may also occur during the system's operational time even when the system is working in a good condition. Let the random variable $\xi_1 \geq 0$ denote the time to failure of a system if the sudden failure occurs, and $\xi_2 \geq 0$ represents the observable time to failure if the system fails due to degradation. We note that only the smaller of the ξ_l, for $l = 1,2$ is in fact observable, together with the actual cause of failure. Suppose that at equidistance sampling times $\Delta, 2\Delta, \ldots$ for $\Delta \in (0,+\infty)$, vector data $X_1, X_2, \ldots \in \mathbb{R}^d$ is collected through condition monitoring, which gives partial information about the system state. We first identify the healthy portions of the data histories. There exists a variety of segmentation methods in the literature (see e.g.[14] and [15] for segmentation of short and long nonstationary time series). Next, a vector autoregressive (VAR) time series model is fitted using the healthy portion of the data to capture any dependencies among monitored variables. When vector AR model is identified, parameters are estimated, and the model adequacy is verified, the residuals of VAR model are calculated using all data histories and utilized to detect an early fault occurrence of an operating system. Residual monitoring has been proposed and studied by several authors (see e.g. [12], [16], [17]). The main advantage of the proposed approach is that the residuals of the fitted model are conditionally independent and normally distributed [12] which are essential properties for tractable maintenance modeling and fast parameter estimation. For successful application of the residual approach using vibration data see e.g. [3], paper [4] used spectrometric oil analysis data. Therefore the observation process, residuals $Y_1, Y_2, \ldots$ are assumed to be conditionally independent given the state of the system, and for each $n \in \mathbb{N}$, we assume that Y_n given $Z_{n\Delta} = i, i = 1,2$, has d-dimensional normal distribution $N_d(\mu_i, \Sigma_i)$ where $\mu_i \in \mathbb{R}^d$ and $\Sigma_i \in \mathbb{R}^{d\times d}$ are unknown observation parameters.

III. Parameter Estimation

Suppose we have collected N_1 sudden failure and N_2 degradation failure histories and we denote them as $F_1^l, \ldots, F_{N_l}^l$ for $l = 1,2$. Failure history F_i^l for $i = 1, \ldots, N_l$ is assumed to be of the form $\vec{Y}_i = (y_1^i, \ldots, y_{T_i}^i)$ and $\xi_l^i = t_i$, where $T_i\Delta < t_i \leq (T_i+1)\Delta$. The history $\vec{Y}_i$ represents the collection of all vector data $y_j^i \in \mathbb{R}^d$, $j \leq T_i$ which was obtained through condition monitoring until system failure at time t_i. Suppose further that we have collected M suspension histories, which we denote as $S_1, \ldots, S_M$. Suspension history S_j is assumed to be of the form $\vec{Y}_j = (y_1^j, \ldots, y_{T_j}^j)$ and $\xi_l^j > T_j\Delta$ for both $l = 1,2$.

Let $A = \{F_1^1, \ldots, F_{N_1}^1, F_1^2, \ldots, F_{N_2}^2, S_1, \ldots, S_M\}$ represent all observable data and $L(\Lambda, \Psi|A)$ be the associated likelihood function, where Λ and Ψ are the set of unknown state and observation parameters, respectively. Because the sample paths $(Z_t, t \in \mathbb{R}^+)$ of the state process are not observable, maximizing $L(\Lambda, \Psi|A)$ analytically is not possible. The EM algorithm resolves this difficulty by iteratively maximizing the so-called pseudo likelihood function [18].

A. Formula for the Likelihood Function

Let $\bar{A} = \{\bar{F}_1^1, \ldots, \bar{F}_{N_1}^1, \bar{F}_1^2, \ldots, \bar{F}_{N_2}^2, \bar{S}_1, \ldots, \bar{S}_M\}$ represent the complete data set, in which each observable data history for A has been augmented with the unobservable sample path information of the state process. Before we derive the formula for the complete likelihood function $L(\Lambda, \Psi|\bar{A})$ for N_1 and N_2 observed failure and M suspension histories, we first consider the case with a single sudden failure history for which $A = \{F^1\}$. Let τ_1 denote the unobservable sojourn time in the healthy state and $f_{\xi_l}(t)$ represents the probability density function of random variable ξ_l for $l = 1,2$. The complete likelihood function for the single sudden failure history is given by,

$$\begin{aligned} L_{\bar{F}^1}(\Lambda, \Psi|\bar{A}) &= L_{\bar{F}^1}(\Lambda, \Psi|\vec{Y} = \vec{y}, \xi_1 = t, \xi_2 > t, \tau_1 = w) \\ &= f_{\vec{Y}|\tau_1,\xi_1}(\vec{y}|w,t)P(\xi_2 > t|\tau_1 = w)f_{\tau_1}(w)f_{\xi_1}(t) \end{aligned}$$

and for each $w, t \in \mathbb{R}^+$, $p(\xi_2 > t|\tau_1 = w)$ represents the conditional reliability function of ξ_2 given τ_1 which is given by:

$$p(\xi_2 > t|\tau_1 = w) = \begin{cases} e^{-\lambda_2(t-w)} & t \geq w \\ 1 & t < w \end{cases} \quad (1)$$

We next consider the case where we have observed only a single degradation failure history for which $A = \{F^2\}$, i.e. we have collected data $\vec{y} = (y_\Delta, \ldots, y_{\Delta T})$ and observed failure time $\xi_1 > t, \xi_2 = t$. The complete likelihood function for the observed single degradation failure history is given by,

$$\begin{aligned} L_{\bar{F}^2}(\Lambda, \Psi|\bar{A}) &= L_{\bar{F}^2}(\Lambda, \Psi|\vec{Y} = \vec{y}, \xi_1 > t, \xi_2 = t, \tau_1 = w) \\ &= f_{\vec{Y}|\tau_1,\xi_2}(\vec{y}|w,t)f_{\tau_1|\xi_2}(w|t)f_{\xi_2}(t)P(\xi_1 > t) \end{aligned}$$

where for any $w \in ((k-1)\Delta, k\Delta], k = 1,2,\ldots,T$, $f_{\vec{Y}|\tau_1,\xi_2}(\vec{y}|w,t) = g(\vec{y}|w,t)$ (see 6). In order to be able to compute the likelihood function, the distributional properties of the sojourn time τ_1 and failure time ξ_2 are given by the following lemma.

Lemma 1. *For each $t \in \mathbb{R}^+$, the density function of ξ_2 is given by:*

$$f_{\xi_2}(t) = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}(e^{-\lambda_2 t} - e^{-\lambda_1 t})$$

and for all non-negative $w < t$, the conditional density function of τ_1 given ξ_2 is:

$$f_{\tau_1|\xi_2}(w|t) = \frac{(\lambda_1 - \lambda_2)e^{-\lambda_2 t - (\lambda_1 - \lambda_2)w}}{-e^{-\lambda_1 t} + e^{-\lambda_2 t}}$$

Proof: See Appendix A. ■

We next consider the case where we have observed only a single suspension history S, i.e. we have collected data $\vec{y} = (y_\Delta, \ldots, y_{\Delta T})$ and stopped observing the operating system at time $\Delta T \leq t < \Delta(T+1)$, $\xi_1 > t$ and $\xi_2 > t$. Given the observable data set $A = \{S\}$, the complete likelihood function can be derived as:

$$\begin{aligned} L_{\bar{S}}(\Lambda, \Psi|\vec{Y} = \vec{y}, \xi_1 > t, \xi_2 > t, \tau_1 = w) \\ = P(\vec{y}|\xi_1 > t, \xi_2 > t, \tau_1 = w) P(\xi_2 > t|\tau_1 = w) \\ f_{\tau_1}(w) P(\xi_1 > t), \end{aligned}$$

where for any $w \in ((k-1)\Delta, k\Delta], k = 1, 2, \ldots, T$ and $P(\vec{y}|\xi_1 > t, \xi_2 > t, \tau_1 = w) = g(\vec{y}|w, t)$ and for $w > \Delta T$, $P(\vec{y}|\xi_1 > t, \xi_2 > t, \tau_1 = w) = g(\vec{y}|t, t)$ (see 6 and 7).

In general case when we observe N_1 sudden failure, N_2 degradation failure, and M suspension histories, the associated complete likelihood function $L(\Lambda, \Psi|\bar{A})$ is given by:

$$L(\Psi, \Lambda|\bar{A}) = \prod_{i=1}^{N_1} L_{\bar{F}_i^1}(\Psi, \Lambda|\bar{A}) \prod_{j=1}^{N_2} L_{\bar{F}_j^1}(\Psi, \Lambda|\bar{A}) \prod_{k=1}^{M} L_{\bar{S}_k}(\Psi, \Lambda|\bar{A})$$

B. Formula for the Pseudo log-Likelihood Function

For any fixed estimates $\hat{\Lambda}, \hat{\Psi}$ of the state and observation parameters, and given observed failures and suspension histories the formula for the pseudo log-likelihood function $Q(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi}) = E_{\hat{\Lambda}, \hat{\Psi}}\Big(\ln L(\Lambda, \Psi|\bar{A})|A \Big)$ is given by:

$$\begin{aligned} Q(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi}) = \sum_{i=1}^{N_1} Q_{\bar{F}_i^1}(\Lambda.\Psi|\hat{\Psi}, \hat{\Lambda}) \\ + \sum_{i=1}^{N_2} Q_{\bar{F}_i^2}(\Lambda.\Psi|\hat{\Psi}, \hat{\Lambda}) + \sum_{k=1}^{M} Q_{\bar{S}_k}(\Lambda.\Psi|\hat{\Psi}, \hat{\Lambda}) \end{aligned} \quad (2)$$

Thus, to evaluate the pseudo log-likelihood function for all available histories, it suffices to evaluate the pseudo log-likelihood function for each individual history separately.

Lemma 2. *Given a single sudden failure history F^1, the pseudo log-likelihood function is decomposed as:*

$$Q_{\bar{F}^1}(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi}) = Q^{state}_{\bar{F}^1}(\Lambda|\hat{\Lambda}, \hat{\Psi}) + Q^{obs}_{\bar{F}^1}(\Psi|\hat{\Lambda}, \hat{\Psi})$$

Proof: See Appendix B. ■

Lemma 3. *Given a single degradation failure history F^2, the pseudo log-likelihood function is decomposed as:*

$$Q_{\bar{F}^2}(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi}) = Q^{state}_{\bar{F}^2}(\Lambda|\hat{\Lambda}, \hat{\Psi}) + Q^{obs}_{\bar{F}^2}(\Psi|\hat{\Lambda}, \hat{\Psi})$$

Proof: See Appendix C. ■

Lemma 4. *Given a single suspension history S, the pseudo log-likelihood function is decomposed as:*

$$Q_{\bar{S}}(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi}) = Q^{state}_{\bar{S}}(\Lambda|\hat{\Lambda}, \hat{\Psi}) + Q^{obs}_{\bar{S}}(\Psi|\hat{\Lambda}, \hat{\Psi})$$

Proof: See Appendix D. ■

Using Lemmas 2, 3, and 4, the pseudo log-likelihood function can be decomposed as,

$$Q(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi}) = Q^{state}(\Lambda|\hat{\Lambda}, \hat{\Psi}) + Q^{obs}(\Psi|\hat{\Lambda}, \hat{\Psi}) \quad (3)$$

where $Q^{state}(\Lambda|\hat{\Lambda}, \hat{\Psi})$ is only a function of the state parameter and $Q^{obs}(\Psi|\hat{\Lambda}, \hat{\Psi})$ is only a function of the observation parameter. This means that the M-step can be carried out separately for the state and residual observation parameters. Using Eqs. 6 and 7 and Lemmas 2, 3, and 4 we solve for the stationary point of the observation parameter by setting:

$$\frac{\partial Q(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi})}{\partial \mu_1} = \frac{\partial Q(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi})}{\partial \mu_2} = \mathbf{0}$$

$$\frac{\partial Q(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi})}{\partial \Sigma_1^{-1}} = \frac{\partial Q(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi})}{\partial \Sigma_2^{-2}} = \mathbf{0}$$

After some algebra, it is not difficult to check that there is a unique stationary point $\hat{\Psi} = (\hat{\mu}_1, \hat{\mu}_2, \hat{\Sigma}_1, \hat{\Sigma}_2)$ of the pseudo log-likelihood function given by,

$$\hat{\mu}_1 = \frac{\sum_{i=1}^{N_1} n_1^i \hat{c}^i + \sum_{j=1}^{N_2} n_1^j \hat{h}^j + \sum_{k=1}^{M} n_1^k \hat{r}^k}{\sum_{i=1}^{N_1} \langle \hat{c}^i, s_1^i \rangle + \sum_{j=1}^{N_2} \langle \hat{h}^j, s_1^j \rangle + \sum_{k=1}^{M} \langle \hat{r}^j, s_1^j \rangle}$$

$$\hat{\Sigma}_1 = \frac{\sum_{i=1}^{N_1} n_3^i \hat{c}^i + \sum_{j=1}^{N_2} n_3^j \hat{h}^j + \sum_{k=1}^{M} n_3^k \hat{r}^k}{\sum_{i=1}^{N_1} \langle \hat{c}^i, s_1^i \rangle + \sum_{j=1}^{N_2} \langle \hat{h}^j, s_1^j \rangle + \sum_{k=1}^{M} \langle \hat{r}^k, s_1^k \rangle}$$

$$\hat{\mu}_2 = \frac{\sum_{i=1}^{N_1} n_2^i \hat{c}^i + \sum_{j=1}^{N_2} n_2^j \hat{h}^j + \sum_{k=1}^{M} n_2^k \hat{r}^k}{\sum_{i=1}^{N_1} \langle \hat{c}^i, s_2^i \rangle + \sum_{j=1}^{N_2} \langle \hat{h}^j, s_2^j \rangle + \sum_{k=1}^{M} \langle \hat{r}^k, s_2^k \rangle}$$

$$\hat{\Sigma}_2 = \frac{\sum_{i=1}^{N_1} n_4^i \hat{c}^i + \sum_{j=1}^{N_2} n_4^j \hat{h}^j + \sum_{k=1}^{M} n_4^k \hat{r}^k}{\sum_{i=1}^{N_1} \langle \hat{c}^i, s_2^i \rangle + \sum_{j=1}^{N_2} \langle \hat{h}^j, s_2^j \rangle + \sum_{k=1}^{M} \langle \hat{r}^k, s_2^k \rangle}.$$

For definition of $s_1^i, s_2^i, n_1^i, n_2^i, n_3^i, n_4^i$ see [19].

The state parameters λ_1 and λ_2 can be also estimated by setting:

$$\frac{\partial Q(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi})}{\partial \lambda_1} = \frac{\partial Q(\Lambda, \Psi|\hat{\Lambda}, \hat{\Psi})}{\partial \lambda_2} = \mathbf{0}$$

The unique stationary points $\hat{\Lambda} = (\hat{\lambda}_1, \hat{\lambda}_2)$ of the pseudo likelihood function given explicitly by:

$$\begin{aligned} \hat{\lambda}_1 &= \frac{\sum_{i=1}^{N_1} \left(\hat{\alpha}_1^i + \hat{\nu}\right) + \sum_{j=1}^{N_2} \hat{\beta}_1^j + \sum_{k=1}^{M} \left(\hat{\gamma}_1^k + \hat{\nu}\right)}{\sum_{i=1}^{N_1} \left(\hat{\alpha}_2^i + \hat{\eta}\right) + \sum_{j=1}^{N_2} \hat{\beta}_2^j + \sum_{k=1}^{M} \left(\hat{\gamma}_2^k + \hat{\eta}\right)} \\ \hat{\lambda}_2 &= \frac{-\sum_{j=1}^{N_2} \hat{\beta}_1^j}{\sum_{i=1}^{N_1} (\hat{\alpha}_2^i - t\hat{\alpha}_1^i) + \sum_{j=1}^{N_2} (\hat{\beta}_2^j - t\hat{\beta}_1^j) + \sum_{k=1}^{M} (\hat{\gamma}_2^k - t\hat{\gamma}_1^k)} \end{aligned}$$

where $\hat{\nu} = \frac{e^{-\hat{\lambda}_1 t} \hat{g}(\vec{y}|t,t)}{\hat{d}}$.

Since sudden failure and degradation failure occur independently, and the collected data does not provide any information about the system's sudden failure, parameters of the distribution of time to sudden failure can be estimated independently by maximizing the corresponding likelihood function.

IV. Conditional Reliability Function prediction

Suppose sigma-algebra $\mathbf{F}_n = \sigma\big(Y_1,\ldots,Y_{n\wedge\lfloor\xi/\Delta\rfloor}, \xi \cdot I_{\{\xi\le n\Delta\}}\big)$ represents the information collected until sampling epoch $n\Delta$ where $\xi=\min(\xi_1,\xi_2)$ and $I_{\{\xi\le n\Delta\}}$ is the indicator random variable defined as

$$I_{\{\xi\le n\Delta\}} = \begin{cases} 1 & \xi \le n\Delta \\ 0 & \xi > n\Delta \end{cases} \tag{4}$$

Further assume that Π_n denotes the posterior probability that system is in the unhealthy state at nth sampling epoch. Using Bayes' rule for $n \ge 1$, the evolution of the posterior probability Π_n is given by:

$$\begin{aligned} \Pi_n &= P(Z_{n\Delta}=2|\mathbf{F}_n) \\ &= \frac{f(Y_n|\mu_2,\Sigma_2)U}{f(Y_n|\mu_2,\Sigma_2)U + f(Y_n|\mu_1,\Sigma_1)D} \end{aligned} \tag{5}$$

where $U = (P_{12}(\Delta)(1-\Pi_{n-1}) + P_{22}(\Delta)\Pi_{n-1}), D = P_{11}(\Delta)(1-\Pi_{n-1})$. where transition probabilities matrix $\mathbb{P}(t) = \big(P_{ij}(t)\big)_{i,j\in\mathcal{Z}}$ can be obtained by solving the Kolmogorov backward differential equations.

Lemma 5. *Let $R_{\xi_1}(t)$ be the reliability function for the sudden failure. For any $t \ge 0$, the conditional reliability and the mean residual life function at n^{th} decision epoch is given by,*

$$R(t|\Pi_n) = R_{\xi_1}(t|\xi_1 > n\Delta)R_{\xi_2}(t|\xi_2 > n\Delta)$$
$$\mu_{n\Delta} = \int_0^\infty R_{\xi_1}(t|\xi_1 > n\Delta)R_{\xi_2}(t|\xi_2 > n\Delta)dt$$

where $R_{\xi_1}(t|\xi_1 > n\Delta) = \frac{R_{\xi_1}(n\Delta+t)}{R_{\xi_1}(n\Delta)}$ *and* $R_{\xi_2}(t|\xi_2 > n\Delta) = \frac{\lambda_1 e^{-\lambda_1 t} - \lambda_2 e^{-\lambda_2 t} + (\lambda_2+\lambda_1-\Pi_n\lambda_2)(e^{-\lambda_2 t}-e^{-\lambda_1 t})}{\lambda_1-\lambda_2}$.

Proof: See Appendix E. ∎

V. Numerical example

Assume that the system deterioration follows a continuous-time homogenous Markov chain $Z_t : t \in \mathbb{R}^+)$ with the state space $S = \{1,2,3\}$. For the simulation purposes, $\lambda_1 = .15, \lambda_2 = .35$ are considered. We also assume that the system may fail due to sudden failure, and the corresponding failure time follows Weibull distribution with scale parameter $\beta = 1.5$ and shape parameter $\alpha = 5.75$. At equidistant sampling times for $\Delta = 1$, the observations are collected through condition monitoring and they are assumed to follow 2-dimensional normal distribution $N_2(\mu_1,\Sigma_1)$ or $N_2(\mu_2,\Sigma_2)$, depending on whether the system is in the healthy or unhealthy state, where

$$\mu_1 = \begin{pmatrix} .2 \\ -.1 \end{pmatrix}, \Sigma_1 = \begin{pmatrix} 1.5 & .5 \\ .5 & 1.5 \end{pmatrix}$$
$$\mu_2 = \begin{pmatrix} .8 \\ .6 \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} 2.5 & 2.5 \\ 2.5 & 3 \end{pmatrix}$$

Using these parameters, 100 degradation failure, 100 sudden failure, and 100 suspension histories were generated. Applying the estimation procedure developed in Section II to the generated data, the following parameter estimates were obtained:

$$\hat\lambda_1 = .13,\ \hat\lambda_2 = .31,\ \hat\alpha = 1.34,\ \hat\beta = 5.43$$

$$\hat\mu_1 = \begin{pmatrix} .20 \\ -.11 \end{pmatrix}, \hat\Sigma_1 = \begin{pmatrix} 1.43 & .36 \\ .36 & 1.33 \end{pmatrix}$$

$$\hat\mu_2 = \begin{pmatrix} .67 \\ .45 \end{pmatrix}, \quad \hat\Sigma_2 = \begin{pmatrix} 2.40 & 2.26 \\ 2.26 & 2.83 \end{pmatrix}$$

The iterations of the EM algorithm took on average 15.24 seconds, which is extremely fast for the off-line computations. This high computational speed is an attractive feature for real applications. After obtaining the parameter estimates, the model can be used for early fault detection based on conditional RF and MRL.

VI. Conclusions

In this paper, we have applied a parametric signal processing technique and a hidden Markov modeling for residual life prediction of a system subject to two failure modes which appear in a variety of technical applications. We have assumed that vector observations are available at regular sampling times through system condition monitoring. The observations are related to the true underlying state of the system which is unobservable. Three types of data histories have been considered: data histories that end with observable system failure caused by degradation, histories that end with observable sudden failure, and suspension histories. The system state process has been modeled as a 3-state hidden Markov process and a new parameter estimation procedure has been developed using the EM algorithm. It has been shown the parameter updates in each iteration of the EM algorithm have explicit formulas. Also, the explicit formula has been derived for the conditional reliability as well as for the mean residual life function of the system. A numerical example has been developed to illustrate the estimation procedure. It has been found that the procedure is both computationally efficient and converges rapidly to good parameter estimates.

Appendix A

Let F_1 and F_2 denote the cumulative distribution function of sojourn time in healthy and unhealthy states, respectively.

$$\begin{aligned} P(\xi_2 \le t) &= \int_0^t P(\xi_2 \le t|\tau_1 = u)dF_1(u) \\ &= \int_0^t P(Z_{t-u}=3|Z_u=2)dF_1(u) \\ &= 1 - e^{-\lambda_1 t} + \frac{\lambda_1(e^{-\lambda_1 t} - e^{-\lambda_2 t})}{\lambda_1-\lambda_2} \\ f_{\xi_2}(t) &= \frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}(e^{-\lambda_2 t} - e^{-\lambda_1 t}) \end{aligned}$$

and also,

$$\begin{aligned} P(\xi_2 \le t, \tau_1 \le w) &= \int_0^w P(Z_{t-u}=3|Z_u=2)dF_1(u) \\ &= \int_0^w F_2(t-u)dF_1(u) \\ &= 1 - e^{-\lambda_1 w} + \frac{\lambda_1 e^{-\lambda_2 t - \lambda_1 w}(-e^{\lambda_1 w} + e^{\lambda_2 w})}{\lambda_1-\lambda_2} \end{aligned}$$

thus,

$$f_{\xi_2,\tau_1}(t,w) = \lambda_1\lambda_2 e^{-\lambda_2 t - w(\lambda_1-\lambda_2)}$$

and for all $0 < w < t$, we define the conditional density function $f_{\tau_1|\xi_2}(w|t)$,

$$\begin{aligned} f_{\tau_1|\xi_2}(w|t) &= \frac{f_{\xi_2,\tau_1}(t,w)}{f_{\xi_2}(t)} \\ &= \frac{(\lambda_1-\lambda_2)e^{-\lambda_2 t-(\lambda_1-\lambda_2)w}}{-e^{-\lambda_1 t}+e^{-\lambda_2 t}} \end{aligned}$$

APPENDIX B

For a given single sudden failure history , the pseudo likelihood function can be derived as:

$$\begin{aligned} Q_{\bar{F}^1}(\Lambda,\Psi|\hat{\Lambda},\hat{\Psi}) &= E_{\hat{\Lambda},\hat{\Psi}}(\ln L(\Lambda,\Psi|\bar{F}^1)|F^1) \\ &= Q^{state}_{F^1}(\Lambda|\hat{\Lambda},\hat{\Psi}) + Q^{obs}_{F^1}(\Psi|\hat{\Lambda},\hat{\Psi}) \end{aligned}$$

where,

$$\begin{aligned} Q^{state}_{F^1}(\Lambda|\hat{\Lambda},\hat{\Psi}) &= \int_0^t \ln(P(\xi_2>t|\tau_1=w)f_{\tau_1}(w)f_{\xi_1}(t)) \\ &\times \frac{\hat{f}_{\vec{Y}|\tau_1,\xi_1}(\vec{y}|w,t)\hat{P}(\xi_2>t|\tau_1=w)\hat{f}_{\tau_1}(w)\hat{f}_{\xi_1}(t)}{\int_u \hat{f}_{\vec{Y}|\tau_1,\xi_1}(\vec{y}|w,t)\hat{P}(\xi_2>t|\tau_1=u)\hat{f}_{\tau_1}(u)\hat{f}_{\xi_1}(t)du}dw \\ &+ \int_t^\infty \ln(f_{\tau_1}(w)f_{\xi_1}(t)) \\ &\times \frac{\hat{f}_{\vec{Y}|\tau_1,\xi_1}(\vec{y}|w,t)\hat{f}_{\tau_1}(w)\hat{f}_{\xi_1}(t)}{\int_u \hat{f}_{\vec{Y}|\tau_1,\xi_1}(\vec{y}|w,t)\hat{P}(\xi_2>t|\tau_1=u)\hat{f}_{\tau_1}(u)\hat{f}_{\xi_1}(t)du}dw, \end{aligned}$$

where for any $w \in ((k-1)\Delta, k\Delta], k = 1,2,\ldots,T$, $W_1 = (y-\mu_1)'\Sigma_1^{-1}(y-\mu_1)$, $W_2 = (y-\mu_2)'\Sigma_2^{-1}(y-\mu_2)$

$$\begin{aligned} f_{\vec{Y}|\tau_1,\xi_1}(\vec{y}|w,t) &= g(\vec{y}|w,t) = g(\vec{y}|k\Delta,t) \qquad (6) \\ &= \frac{exp\left(\frac{-1}{2}\left(\sum_{n=1}^{k-1}W_1+\sum_{n=k}^{T}W_2\right)\right)}{\sqrt{(2\pi)^{Td}det^{k-1}(\Sigma_1)det^{T-k+1}(\Sigma_2)}} \end{aligned}$$

and for any $w > T\Delta$:

$$\begin{aligned} f_{\vec{Y}|\tau_1,\xi_1}(\vec{y}|w,t) &= g(\vec{y}|t,t) \qquad (7) \\ &= \frac{1}{\sqrt{(2\pi)^{Td}det^{T}(\Sigma_1)}}exp\Big(-\frac{1}{2}\sum_{n=1}^{T}W_1\Big) \end{aligned}$$

and the notation $\hat{f}_{\vec{Y}|\tau_1,\xi_1}(\vec{y}|w,t), \hat{P}(\xi_2>t|\tau_1=w), \hat{f}_{\tau_1}(w), \hat{f}_{\xi_1}(t)$ is used to indicate that the functions $f_{\vec{Y}|\tau_1,\xi_1}(\vec{y}|w,t), P(\xi_2>t|\tau_1=w), f_{\tau_1}(w), f_{\xi_1}(t)$ are parameterized by fixed estimates $\hat{\Lambda},\hat{\Psi}$.

To simplify the notation, for the remainder of the analysis we denote vectors $\hat{\mathbf{g}} = \big(\hat{g}(\vec{y}|\Delta,t),\ldots,\hat{g}(\vec{y}|T\Delta,t),\hat{g}(\vec{y}|t,t)\big)'$, $\ln\hat{\mathbf{g}} = \big(\ln\hat{g}(\vec{y}|\Delta,t),\ldots,\ln\hat{g}(\vec{y}|T\Delta,t),\ln\hat{g}(\vec{y}|t,t)\big)'$, $\hat{\mathbf{c}} = (\hat{c}^1,\ldots,\hat{c}^T,\hat{c}^t)$, $\hat{\mathbf{d}}_i = (\hat{d}_i^1,\ldots,\hat{d}_i^T,\hat{d}_i^t)$, $\hat{\alpha}_i = \frac{\langle\hat{\mathbf{g}},\hat{\mathbf{d}}_i\rangle}{\hat{d}}$ for $i = 1,2,3$, and for any vector $\mathbf{v},\mathbf{w}$, $\mathbf{v}'.\mathbf{w} = \langle v,w\rangle$ represents the inner product.

$$\begin{aligned} Q^{state}_{F^1}(\Lambda|\hat{\Lambda},\hat{\Psi}) &= \lambda_2\left(\hat{\alpha}_2 - t\hat{\alpha}_1\right) + \ln f_{\xi_1}(t)\left(\hat{\alpha}_1+\hat{\alpha}_3\right) \\ &+ \ln\lambda_1\left(\hat{\alpha}_1 + \frac{e^{-\hat{\lambda}_1 t}}{\hat{d}}\hat{g}(\vec{y}|t,t)\right) - \lambda_1\left(\hat{\alpha}_2+\hat{\eta}\right) \end{aligned}$$

where for $k = 1,\ldots,T$,

$$\begin{aligned} \hat{e}_1^k &= \frac{e^{(\hat{\lambda}_2-\hat{\lambda}_1)(k-1)\Delta}-e^{(\hat{\lambda}_2-\hat{\lambda}_1)k\Delta}}{\hat{\lambda}_1-\hat{\lambda}_2} \\ \hat{e}_1^t &= \frac{e^{(\hat{\lambda}_2-\hat{\lambda}_1)T\Delta}-e^{(\hat{\lambda}_2-\hat{\lambda}_1)t}}{\hat{\lambda}_1-\hat{\lambda}_2} \\ \hat{e}_2^k &= \frac{\hat{e}_1^k - k\Delta e^{(\hat{\lambda}_2-\hat{\lambda}_1)k\Delta}+(k-1)\Delta e^{(\hat{\lambda}_2-\hat{\lambda}_1)(k-1)\Delta}}{\hat{\lambda}_1-\hat{\lambda}_2} \\ \hat{e}_2^t &= \frac{\hat{e}_1^t - te^{(\hat{\lambda}_2-\hat{\lambda}_1)t}+T\Delta e^{(\hat{\lambda}_2-\hat{\lambda}_1)T\Delta}}{\hat{\lambda}_1-\hat{\lambda}_2} \\ \hat{e}_3^k &= \frac{e^{-\hat{\lambda}_1(k-1)\Delta}-e^{-\hat{\lambda}_1 k\Delta}}{\hat{\lambda}_1} \\ \hat{e}_3^t &= \frac{e^{-\hat{\lambda}_1 T\Delta}-e^{-\hat{\lambda}_1 t}}{\hat{\lambda}_1} \\ \hat{d}_1^k &= \hat{\lambda}_1 e^{-\hat{\lambda}_2 t}\hat{e}_1^k \text{ for } k = 1,...,T, \hat{d}_1^t = \hat{\lambda}_1 e^{-\hat{\lambda}_2 t}\hat{e}_1^t \\ \hat{d}_2^k &= \hat{\lambda}_1 e^{-\hat{\lambda}_2 t}\hat{e}_2^k \text{ for } k = 1,...,T, \hat{d}_2^t = \hat{\lambda}_1 e^{-\hat{\lambda}_2 t}\hat{e}_2^t \\ \hat{d}_3^k &= \hat{\lambda}_1\hat{e}_3^k \qquad \text{for } k = 1,...,T, \hat{d}_3^t = \hat{\lambda}_1\hat{e}_3^t \\ \hat{\eta} &= \frac{e^{-\hat{\lambda}_1 t}(t+\hat{\lambda}_1^{-1})}{\hat{d}}\hat{g}(\vec{y}|t,t) \\ \hat{d} &= \langle\hat{\mathbf{d}}_1,\hat{\mathbf{g}}\rangle + \hat{g}(\vec{y}|t,t)e^{-\lambda_1 t} \end{aligned}$$

Also,

$$\begin{aligned} Q^{obs}_{\bar{F}^1}(\hat{\Psi}|\hat{\Lambda},\hat{\Psi}) &= \sum_{k=1}^{T}\hat{c}^k\ln\left(g(\vec{y}|k\Delta,t)\right) + \hat{c}^t\ln\left(g(\vec{y}|t,t)\right) \\ &= \langle\hat{\mathbf{c}},\mathbf{ln\ g}\rangle \end{aligned}$$

where,

$$\begin{aligned} \hat{c}^k &= \frac{\hat{d}_1^k}{\hat{d}}\hat{g}(\vec{y}|\Delta k,t) \text{ for } k = 1,...,T, \\ \hat{c}^t &= \left(\frac{e^{-\lambda_1 t}}{\hat{d}} + \frac{\hat{\lambda}_1 e^{-\lambda_2 t}\hat{e}_1^t}{\hat{d}}\right)\hat{g}(\vec{y}|t,t) \end{aligned}$$

APPENDIX C

For a given single failure history, the pseudo likelihood function can be derived as:

$$\begin{aligned} Q_{\bar{F}^2}(\Lambda,\Psi|\hat{\Lambda},\hat{\Psi}) &= E_{\hat{\Lambda},\hat{\Psi}}(\ln L(\Lambda,\Psi|\bar{F}^2)|F^2) \\ &= Q^{state}_{F^2}(\Lambda|\hat{\Lambda},\hat{\Psi}) + Q^{obs}_{F^2}(\Psi|\hat{\Lambda},\hat{\Psi}) \end{aligned}$$

Let us assume for i=1,2, $\hat{\mathbf{a}}_i = (\hat{a}_i^1,\ldots,\hat{a}_i^T,\hat{a}_i^t)$, $\hat{\beta}_i = \frac{\langle\hat{\mathbf{a}}_i,\hat{\mathbf{g}}\rangle}{\hat{a}}$ and $\hat{\mathbf{h}} = (\hat{h}^1,\ldots,\hat{h}^T,\hat{h}^t)$. We have,

$$\begin{aligned} Q^{state}_{\bar{F}^2}(\Lambda|\hat{\Lambda},\hat{\Psi}) &= \int_0^t \ln(f_{\tau_1|\xi_2}(w|t)f_{\xi_2}(t)P(\xi_1>t)) \\ &\times \frac{g(\vec{y}|w,t)\hat{f}_{\tau_1|\xi_2}(w|t)\hat{f}_{\xi_2}(t)\hat{P}(\xi_1>t)}{\int_{u<t}g(\vec{y}|w,t)\hat{f}_{\tau_1|\xi_2}(u|t)\hat{f}_{\xi_2}(t)\hat{P}(\xi_1>t)du}dw \\ &= \lambda_2\left(\hat{\beta}_2 - t\hat{\beta}_1\right) - \lambda_1\hat{\beta}_2 + \ln\lambda_1 + \ln\lambda_2 + \ln R_{\xi_1}(t) \end{aligned}$$

where

$$\begin{aligned} \hat{a}_1^k &= e^{-\hat{\lambda}_2 t}\cdot\hat{e}_1^k \ for \ \ k = 1,\ldots,T\Delta, \hat{a}_1^t = e^{-\hat{\lambda}_2 t}\cdot\hat{e}_1^t \\ \hat{a}_2^k &= e^{-\hat{\lambda}_2 t}\cdot\hat{e}_2^k \ for \ \ k = 1,\ldots,T\Delta, \hat{a}_2^t = e^{-\hat{\lambda}_2 t}\cdot\hat{e}_2^t \\ \hat{a} &= \langle\hat{\mathbf{a}}_1,\hat{\mathbf{g}}\rangle \end{aligned}$$

and also,

$$\begin{aligned} Q^{obs}_{\bar{F}^2}(\Psi|\hat{\Lambda},\hat{\Psi}) &= \sum_{k=1}^{T}\hat{h}^k\ln\left(g(\vec{y}|k\Delta,t)\right) + \hat{h}^t\ln\left(g(\vec{y}|k\Delta,t)\right) \\ &= \langle\hat{\mathbf{h}},\mathbf{ln\ g}\rangle \end{aligned}$$

where,

$$\begin{aligned}\hat{h}^k &= \frac{e^{-\hat{\lambda}_2 t}\cdot \hat{e}_1^k}{\hat{a}}\hat{g}(\vec{y}|\Delta k,t) \text{for } k=1,...,T\\ \hat{h}^t &= \frac{e^{-\hat{\lambda}_2 t}\cdot \hat{e}_1^t}{\hat{a}}\hat{g}(\vec{y}|t,t)\end{aligned}$$

APPENDIX D

For the given suspension history, the pseudo likelihood function can be derived as:

$$\begin{aligned}Q_{\bar{S}}(\Lambda,\Psi|\hat{\Lambda},\hat{\Psi}) &= E_{\hat{\Lambda},\hat{\Psi}}(\ln L(\Lambda,\Psi|\bar{S})|S)\\ &= Q_{\bar{S}}^{state}(\Lambda|\hat{\Lambda},\hat{\Psi})+Q_{\bar{S}}^{obs}(\Psi|\hat{\Lambda},\hat{\Psi})\end{aligned}$$

Let us assume $\hat{\mathbf{r}} = (\hat{r}^1,\ldots,\hat{r}^T,\hat{r}^t)$ and for $i=1,2,3$, $\hat{\mathbf{b}}_i = (\hat{b}_i^1,\ldots,\hat{b}_i^T,\hat{b}_i^t), \hat{\gamma}_i = \frac{\langle \hat{\mathbf{g}},\hat{\mathbf{b}}_i\rangle}{\hat{b}}$. Then $Q_S^{state}(\Lambda|\hat{\Lambda},\hat{\Psi})$ and $Q_S^{obs}(\hat{\Psi}|\hat{\Lambda},\hat{\Psi})$ will be decomposed by:

$$\begin{aligned}Q_{\bar{S}}^{state}(\Lambda|\hat{\Lambda},\hat{\Psi}) &= \int_{w<t}\ln\left(P(\xi_2>t|\tau_1=w)f_{\tau_1}(w)P(\xi_1>t)\right)\\ &\times\frac{\hat{g}(\vec{y}|w,t)\hat{P}(\xi_2>t|\tau_1=w)\hat{f}_{\tau_1}(w)\hat{P}(\xi_1>t)}{\int_u \hat{g}(\vec{y}|u,t)\hat{P}(\xi_2>t|\tau_1=u)\hat{f}_{\tau_1}(u)\hat{P}(\xi_1>t)du}dw\\ &+\int_{w>t}\ln\left(P(\xi_2>t|\tau_1=w)f_{\tau_1}(w)P(\xi_1>t)\right)\\ &\times\frac{\hat{g}(\vec{y}|w,t)\hat{P}(\xi_2>t|\tau_1=w)\hat{f}_{\tau_1}(w)\hat{P}(\xi_1>t)}{\int_u \hat{g}(\vec{y}|u,t)\hat{P}(\xi_2>t|\tau_1=u)\hat{f}_{\tau_1}(u)\hat{P}(\xi_1>t)du}dw\end{aligned}$$

$$\begin{aligned}Q_{\bar{S}}^{state}(\Lambda|\hat{\Lambda},\hat{\Psi}) &= \lambda_2\left(\hat{\gamma}_2-t\hat{\gamma}_1\right)+\ln R_{\xi_1}(t)\left(\hat{\gamma}_1+\hat{\gamma}_3\right)\\ &\ln\lambda_1\left(\hat{\gamma}_1+\frac{e^{-\hat{\lambda}_1 t}}{\hat{b}}\hat{g}(\vec{y}|t,t)\right)-\lambda_1\left(\hat{\gamma}_2+\hat{\eta}\right)\end{aligned}$$

where for $k=1,\ldots,T$ and $i=1,2,3$, $\hat{b}_i^k=\hat{d}_i^k, \hat{b}_i^t=\hat{d}_i^t$ and $\hat{b}=\hat{d}$. Also,

$$\begin{aligned}Q_{\bar{S}}^{obs}(\hat{\Psi}|\hat{\Lambda},\hat{\Psi}) &= \sum_{k=1}^{T}\hat{r}^k\ln g(\vec{y}|k\Delta,t)+\hat{r}^t\ln g(\vec{y}|t,t),\\ &= \langle\hat{\mathbf{r}},\ln\mathbf{g}\rangle\end{aligned}$$

where for $k=1,\ldots,T$ and $i=1,2,3$, $\hat{r}_i^t=\hat{c}_i^t, \hat{r}_i^k=\hat{c}_i^k$.

APPENDIX E

$$\begin{aligned}R(t|\Pi_n) &= P(\xi>t+n\Delta|\xi>n\Delta,Y_1,\ldots,Y_n)\\ &= R_1(t|\xi_1>n\Delta)\cdot R_2(t|\xi_2>n\Delta,\Pi_n),\end{aligned}$$

where $R_1(t|\xi_1>n\Delta)=\frac{R_{\xi_1}(n\Delta+t)}{R_{\xi_1}(n\Delta)}$ and

$$\begin{aligned}R_2(t|\xi_2>n\Delta,\Pi_n) &= P(Z_{n\Delta+t}\neq 3|\xi_2>n\Delta,Y_1,\ldots,Y_n)\\ &= (1-\Pi_n)(1-P_{13}(t))+\Pi_n(1-P_{23}(t))\\ &= \frac{\lambda_1 e^{-\lambda_1 t}-\lambda_2 e^{-\lambda_2 t}+(\lambda_2+\lambda_1-\Pi_n\lambda_2)(e^{-\lambda_2 t}-e^{-\lambda_1 t})}{\lambda_1-\lambda_2}\end{aligned}$$

Finally,

$$\begin{aligned}&\mu_{n\Delta}=E(\xi-n\Delta|\xi>n\Delta,Y_1,\ldots,Y_n)\\ &=\int_0^\infty 1-P(\xi<n\Delta+t|\xi_1>n\Delta,\xi_2>n\Delta,y_1,\ldots,y_1)dt\\ &=\int_0^\infty R(t|\Pi_n)dt\\ &=\int_0^\infty R_1(t|\xi_1>n\Delta)\cdot R_2(t|\xi_2>n\Delta,\Pi_n)dt\end{aligned}$$

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