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ML-Based Blind Frequency Offset Estimation Schemes for OFDM Systems in Non-Gaussian Noise Environments

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Abstract—This paper proposes frequency offset (FO) estimation schemes robust to the non-Gaussian noise for orthogonal frequency division multiplexing (OFDM) systems. A maximum-likelihood (ML) scheme and a low-complexity estimation scheme are proposed by applying the probability density function of the cyclic prefix of OFDM symbols to the ML criterion. From simulation results, it is confirmed that the proposed schemes offer a significant FO estimation performance improvement over the conventional estimation scheme in non-Gaussian noise environments.

Keywords—Frequency offset, cyclic prefix, maximum-likelihood, non-Gaussian noise, OFDM.

I. Introduction

BECAUSE of its immunity to multipath fading and high spectral efficiency, orthogonal frequency division multiplexing (OFDM) has been adopted as a modulation format in a wide variety of wireless systems such as digital video broadcasting-terrestrial (DVB-T), wireless local area network (WLAN), worldwide interoperability for microwave access (WiMAX), and long term evolution (LTE) [1]-[3]. However, OFDM is very vulnerable to the frequency offset (FO), which is caused by Doppler shift or oscillator instabilities. Thus, the FO estimation is very important in OFDM systems [1], [4]. FO estimation schemes are based on training symbols or based on the blind approach. The latter is more widely used since it does not require a high complexity compared with the former. In this paper, thus, we focus on the FO estimation based on the blind approach: Specifically, the cyclic prefix (CP) of OFDM symbols is utilized for the FO estimation.

The conventional FO estimation schemes have been proposed under the assumption that the noise distribution is a Gaussian process by the central theorem [5]. However, the ambient noise often exhibits non-Gaussian nature in wireless channels, mostly due to the impulsive nature originated from various sources such as car ignitions, moving obstacles, lightning in the atmosphere, and reflections from sea waves [6], [7]. The performance of the conventional estimation schemes developed under the Gaussian noise model can be severely degraded under such non-Gaussian noise environments.

In this paper, we propose blind FO estimation schemes robust to non-Gaussian noise environments. From the CP structure of OFDM, we first derive a maximum-likelihood (ML) FO estimation scheme in non-Gaussian noise modeled as a complex isotropic Cauchy noise. Then, a blind estimation scheme with a lower complexity is derived from the ML FO estimation scheme by reducing the set of the candidate values. From simulation results, it is shown that the proposed schemes offer a significant performance improvement over the conventional blind estimation scheme in non-Gaussian noise environments.

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II. Signal Model

The kth received OFDM sample $r(k)$ can be expressed as

$$r(k) = x(k)e^{j2\pi k\varepsilon/N} + n(k) \tag{1}$$

for $k = -G, \cdots, -1, 0, 1, \cdots, N-1$, where $x(k)$ is the kth sample of the transmitted OFDM symbol generated by the inverse fast Fourier transform (IFFT) of size N, G is the size of the CP, ε is the FO normalized to the subcarrier spacing $1/N$, and $n(k)$ is the kth sample of additive noise.

In this paper, we adopt the complex isotropic symmetric α stable (CISαS) model for the independent and identically distributed noise samples $\{n(k)\}_{k=0}^{N-1}$; this model has been widely employed due to its strong agreement with experimental data [8], [9]. The probability density function (pdf) of $n(k)$ is then given by [8]

$$f_n(\rho) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\gamma\left(u^2+v^2\right)^{\frac{\alpha}{2}} - j\Re\{\rho(u-jv)\}}dudv, \tag{2}$$

where $\Re\{\cdot\}$ denotes the real part, the dispersion $\gamma > 0$ is related to the spread of the pdf, and the characteristic exponent $\alpha \in (0, 2]$ is related to the heaviness of the tails of the pdf: A smaller value of α indicates a higher degree of impulsiveness, whereas a value closer to 2 indicates a more Gaussian behavior.

A closed-form expression of (2) is not known to exist except for the special cases of $\alpha = 1$ (complex isotropic Cauchy) and $\alpha = 2$ (complex isotropic Gaussian). In particular, we have

$$f_n(\rho) = \begin{cases} \frac{\gamma}{2\pi}\left(|\rho|^2 + \gamma^2\right)^{-\frac{3}{2}}, & \text{when } \alpha = 1 \\ \frac{1}{4\pi\gamma}\exp\left(-\frac{|\rho|^2}{4\gamma}\right), & \text{when } \alpha = 2. \end{cases} \tag{3}$$

Due to such a lack of closed-form expressions, we concentrate on the case of $\alpha = 1$: We shall see in Section IV that the estimation schemes obtained for $\alpha = 1$ are not only more robust to the variation of α, but they also provide a

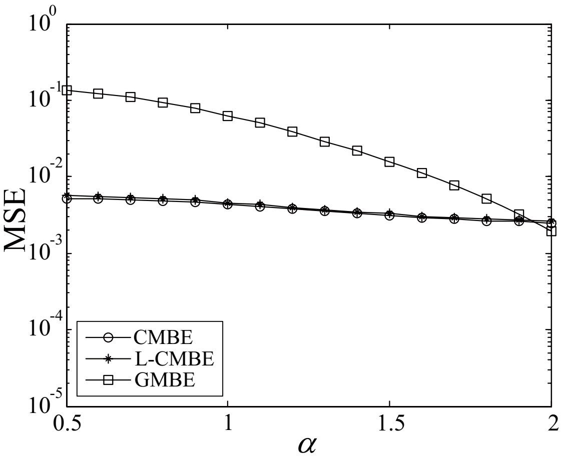

Fig. 1. The MSE performances of the CMBE, L-CMBE, and GMBE schemes as a function of α when the GSNR is 5 dB

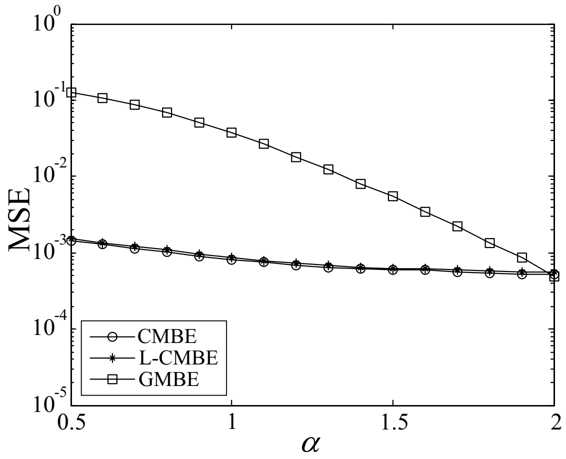

Fig. 2. The MSE performances of the CMBE, L-CMBE, and GMBE schemes as a function of α when the GSNR is 10 dB

better performance for most values of α, than the conventional estimation scheme.

III. Proposed Schemes

A. Maximum-likelihood FO Estimation Scheme

In estimating the FO, we consider a property of the CP structure of OFDM, i.e., $x(k) = x(k+N)$ for $k = -G, -G+1 \cdots, -1$ as in [5]. Then, from (1), we have

$$r(k+N) - r(k)e^{j2\pi\varepsilon} = n(k+N) - n(k)e^{j2\pi\varepsilon} \quad (4)$$

for $k = -G, -G+1 \cdots, -1$. Observing that $n(k+N) - n(k)e^{j2\pi\varepsilon}$ obeys the complex isotropic Cauchy distribution with dispersion 2γ (since the distribution of $-n(k)e^{j2\pi\varepsilon}$ is the same as that of $n(k)$), we obtain the pdf

$$f_{\boldsymbol{r}}(\boldsymbol{r}|\varepsilon) = \prod_{k=-G}^{-1} \frac{\gamma}{\pi\left(\left|r(k+N) - r(k)e^{j2\pi\varepsilon}\right|^2 + 4\gamma^2\right)^{\frac{3}{2}}} \quad (5)$$

of $\boldsymbol{r} = \{r(k+N) - r(k)e^{j2\pi\varepsilon}\}_{k=-G}^{-1}$ conditioned on ε. The ML estimation is then to choose $\hat{\varepsilon}$ such that

$$\begin{aligned}\hat{\varepsilon} &= \arg\max_{\tilde{\varepsilon}}[\log f_{\boldsymbol{r}}(\boldsymbol{r}|\tilde{\varepsilon})] \\ &= \arg\min_{\tilde{\varepsilon}} \Lambda(\tilde{\varepsilon}), \end{aligned} \quad (6)$$

where $\tilde{\varepsilon}$ denotes the candidate value of ε and the log-likelihood function $\Lambda(\tilde{\varepsilon}) = \sum_{k=-G}^{-1} \log\left\{\left|r(k+N) - r(k)e^{j2\pi\tilde{\varepsilon}}\right|^2 + 4\gamma^2\right\}$ is a periodic function of $\tilde{\varepsilon}$ with period 1: The minima of $\Lambda(\tilde{\varepsilon})$ occur at a distance of 1 from each other, causing an ambiguity in estimation. Assuming that ε is distributed equally over positive and negative sides around zero, the valid estimation range of the ML estimation scheme can be set to $-0.5 < \varepsilon \leq 0.5$, as in [5]. The estimation scheme (6) will be called the Cauchy ML blind estimation (CMBE) scheme.

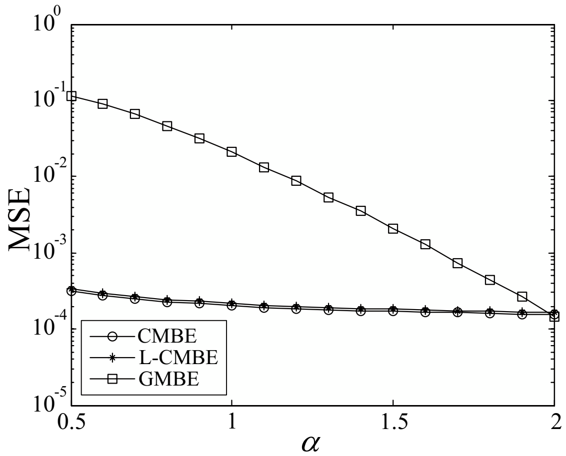

Fig. 3. The MSE performances of the CMBE, L-CMBE, and GMBE schemes as a function of α when the GSNR is 15 dB

IV. Simulation Results

A. Low-complexity FO Estimation Scheme

The CMBE scheme is based on the exhaustive search over the whole estimation range ($|\varepsilon| < 0.5$), which requires high computational complexity. Thus, we propose a low-complexity FO estimation scheme with the reduced set of the candidate values.

In order to obtain the reduced set of the candidate values, we exploit the fact that $\varepsilon = \frac{1}{2\pi}\angle\{x^*(k)x(k+N)\} = \frac{1}{2\pi}\angle\{r^*(k)r(k+N)\}$ for $k = -G, -G+1 \cdots, -1$ in the absence of noise. Based on this property, we obtain the set of the candidate values

$$\bar{\varepsilon}(k) = \frac{1}{2\pi}\angle\{r^*(k)r(k+N)\}, \text{ for } k = -G, -G+1 \cdots, -1. \quad (7)$$

Exploiting the set of the candidate values in (7), the FO estimate $\hat{\varepsilon}_L$ can be obtained as follows

$$\hat{\varepsilon}_L = \arg\min_{\bar{\varepsilon}(k)} \Lambda\left(\bar{\varepsilon}(k)\right), \text{ for } k = -G, -G+1 \cdots, -1. \quad (8)$$

In the following, (8) is denoted as the low-complexity CMBE (L-CMBE) scheme. Using only $N/2$ candidate values, the L-CMBE scheme can offer an almost same performance as the CMBE scheme with the exhaustive search, which is verified by simulation results in Section IV.

In this section, the proposed CMBE and L-CMBE schemes are compared with the Gaussian ML blind estimation (GMBE) scheme in [5] in terms of the mean squared error (MSE). We assume the following parameters: The IFFT size $N = 64$, FO $\varepsilon = 0.25$, the search spacing of 0.001 for the CMBE scheme, and a multipath Rayleigh fading channel with length $L = 8$ and an exponential power delay profile of $\mathbf{E}[|h(l)|^2] = \exp(-l/L)/\{\sum_{l=0}^{L-1}\exp(-l/L)\}$ for $l = 0, 1, \cdots, 7$, where $h(l)$ is the lth channel coefficient of a multipath channel and $\mathbf{E}[\cdot]$ denotes the statistical expectation. Since CISαS noise with $\alpha < 2$ has an infinite variance, the standard signal-to-noise ratio (SNR) becomes meaningless for such a noise. Thus, we employ the geometric SNR (GSNR) defined as $\mathbf{E}[|x(k)|^2]/(4C^{-1+2/\alpha}\gamma^{2/\alpha})$, where $C = \exp\{\lim_{m\to\infty}(\sum_{i=1}^{m}\frac{1}{i} - \ln m)\} \simeq 1.78$ is the exponential of the Euler constant [10]. The GSNR indicates the relative strength between the information-bearing signal and the CISαS noise with $\alpha < 2$. Clearly, the GSNR becomes the standard SNR when $\alpha = 2$. Since γ can be easily and exactly estimated using only the sample mean and variance of the received samples [11], it may be regarded as a known value: Thus, γ is set to 1 without loss of generality.

Figs. 1-3 show the MSE performances of the CMBE, L-CMBE, and GMBE schemes as a function of α when the GSNR is 5, 10, and 15 dB, respectively. From the figures, we can clearly observe that the proposed schemes not only outperform the conventional scheme for most values of α, except for those close to 2, but also provide a robustness to the variation of the value of α. Another important observation is that the estimation performance of the L-CMBE scheme is almost same as that of the CMBE scheme. From this observation, it is confirmed that the candidate values for the L-CMBE scheme is reasonable.

V. Conclusion

In this paper, we have proposed FO estimation schemes using CP in non-Gaussian noise environments. Considering the pdf of the CP of OFDM symbols, we have first derived the ML FO estimation scheme in non-Gaussian noise modeled as a complex isotropic Cauchy noise. Subsequently, we also have derived a simplified FO estimation scheme by decreasing the set of the candidate values. From simulation results, it has been confirmed that the proposed schemes offer not only the similar FO estimation performance compared with the conventional scheme in Gaussian noise environment but also a robustness and a substantial FO estimation performance improvement over the conventional estimation scheme in non-Gaussian noise environments.

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