Performance Evaluation of Universities as Groups of Decision Making Units

Ali Payan, Bijan Rahmani Parchicolaie

Abstract—Universities have different offices such as educational, research, student, administrative, and financial offices. This paper considers universities as groups of decision making units (DMUs) in which DMUs are their offices. This approach gives us with a more just evaluation of universities instead of separate evaluation of the offices of universities. The proposed approach to evaluate group performance of universities is based on common set of weights method in DEA. The suggested method not only can compare groups and measure their efficiencies, but also can calculate the efficiency of units within group and efficiency spread of groups. At last, the suggested method is applied for the analysis of the performance of universities in 14th district of Islamic Azad University as groups under evaluation.

Keywords—Common set of weights, group efficiency, performance analysis, spread efficiency.

I. INTRODUCTION

SINCE the introduction of data envelopment analysis (DEA) by Charnes et al. [1] and its consequent development by Banker et al. [2], new branches have been developed in this particular scientific field most of which were abstract concepts from other sciences whose transition to measurable mathematical expression was made possible by DEA. Ranking, return to scale, productivity, cost efficiency, group or program performance, etc. are some examples of the above mentioned abstract concepts. For further information refer to Cook and Seiford [3]. In another paper, Liu et al., [4] provided a statistical study of literature on DEA.

One of the important and applicable issues on DEA includes the evaluation of university performance. A great number of studies have been done on the performance of universities. Technical and scale efficiencies of Australian Universities was measured by Avkiran [5]. A similar study was performed by Abbott and Doucouliagos [6] to evaluate the universities of Australian. Johnes [7] measured teaching efficiency of UK Universities. Casu and Thanassoulis [8] calculated cost efficiency in central administrative services in the UK universities. Cost efficiency was evaluated in Australian universities by Horne and Hu [9]. Productivity change in Australian universities among 1998 to 2003 was estimated by Worthington and Lee [10]. Chu and Li [11] calculated productivity growth in Chinese universities during the post-reform period. We can mention another study by

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Katharaki and Katharakis [12] to comparatively assess the efficiency of Greek universities. As a recent research, Ahmadi and Keshavarzi [13] Compared Educational Efficiency among Male and Female Literature Teachers Graduated from Islamic Azad University, State University and Payam-e-noor University.

Despite the number of extensive studies done on the performance evaluation of universities through DEA, regarding the fact that university systems consist of various offices such as educational, research, student, administrative, and financial offices, the performance of universities has been done without considering these offices or considering each office separately. To have a more just evaluation of universities, it is suggested to measure the overall efficiency of universities based on the performance of different offices. To achieve this objective, we will consider universities as groups of decision making units in which DMUs within them are teaching, research and student offices.

Teaching, research and student offices as DMUs within each group are not homogenous units, while in the proposed methods of group performance [14]–[16] units within the groups are considered as homogenous DMUs. Thus, we cannot evaluate the universities as groups of DMUs by using the methods in the literature of group performance. This paper introduces a new structure for groups of DMUs in which DMUs within them are not homogenous. Universities are good samples for this structure. Then, a method is presented to evaluate groups with the new structure.

In this paper, we firstly introduce groups of DMUs with a new structure while units within them do not have homogenous assumption. Subsequently, a method based on common set of weights method in DEA is suggested to measure the efficiency of groups with new structure. The proposed method also has the capability to calculate the efficiency of units within each group. Moreover, a new method is suggested to measure the spread efficiency or well-balanced management of groups. In order to solve the presented method, a linear programming process is described that can provide an efficient solution for the proposed multi-objective programming problem. At last, an empirical example of universities in 14th district of Islamic Azad University is presented to show the abilities of the proposed method.

II. COMMON SET OF WEIGHTS METHOD TO EVALUATE GROUP EFFICIENCY

Consider g groups in which units within them are not homogenous. Number of DMUs within groups is equal to n. $DMU_j (j = 1, \dots, n)$ within each group consume m_j inputs

$x_{ij}^h (i = 1, \dots, m_j)$ to produce s_j outputs $y_{rj}^h (r = 1, \dots, s_j)$. Group G_h can be considered as follows:

$$G_h = \{DMU_1^h, \dots, DMU_n^h\}, h = 1, \dots, g,$$
$$DMU_j^h = \begin{pmatrix} x_j^h \\ y_j^h \end{pmatrix},$$
$$x_j^h = (x_{1j}^h, \dots, x_{m_j j}^h), y_j^h = (y_{1j}^h, \dots, y_{s_j j}^h)$$

With the above introduced structure, jth units $(j = 1, \dots, n)$ in different groups are homogenous.

Assume $v_j = (v_{1j}, \dots, v_{m_j j})$ and $u_j = (u_{1j}, \dots, u_{s_j j})$ are weight vectors of inputs and outputs. Efficiency of $DMU_j (j = 1, \dots, n)$ in group h $(h = 1, \dots, g)$ is defined as follows:

$$E_j^h = \frac{u_j y_j^h}{v_j x_j^h} = \frac{\sum_{r=1}^{s_j} u_{rj} y_{rj}^h}{\sum_{i=1}^{m_j} v_{ij} x_{ij}^h}, j = 1, \dots, n, h = 1, \dots, g \quad (1)$$

Overall efficiency of group h $(h = 1, \dots, g)$ is defined as a convex combination of the efficiency of units within the group, so we have:

$$E_{G_h} = \sum_{j=1}^n w_j^h E_j^h, h = 1, \dots, g \quad (2)$$

In other words, the performance of group h $(h = 1, \dots, g)$ is the resultant of all the activities within the group. From the statistical point of view, E_{G_h} is the weighted average of the efficiency of units within group h $(h = 1, \dots, g)$. From the economical point of view, w_j^h can be considered as the contribution of $DMU_j (j = 1, \dots, n)$ in the performance group h $(h = 1, \dots, g)$.

We propose the following multi-objective non-linear programming problem as a common set of weights model to measure the performance of groups of DMUs as:

$$Max E_{G_h}, h = 1, \dots, g, s.t. E_j^h \le 1, j = 1, \dots, n, h = 1, \dots, g,$$
$$v_j \ge 0, j = 1, \dots, n, u_j \ge 0, j = 1, \dots, n \quad (3)$$

A method is considered to solve the model (3) in the next section. Let v_j^* and $u_j^* (j = 1, \dots, n)$ are optimal weights of problem (3). Thus, overall efficiency of group h $(h = 1, \dots, g)$and the efficiency of $DMU_j (j = 1, \dots, n)$ within group h $(h = 1, \dots, g)$ are calculated as follows:

$$E_j^{h^*} = \frac{u_j^* y_j^h}{v_j^* x_j^h}, j = 1, \dots, n, h = 1, \dots, g, \quad (4)$$
$$E_{G_h}^* = \sum_{j=1}^n w_j^h E_j^{h^*}, h = 1, \dots, g \quad (5)$$

III. PERFORMANCE ANALYSIS

A. Within Group Analysis

An effective management is a well-balanced one having a high performance. According to model (3), performance of each group can be calculated by estimating the score of its efficiency, but in order to evaluate a well-balanced management it should be examined whether different parts of a group (DMUs of groups) have similar performance or not. If the answer is affirmative, then it can be claimed that the performance of the group is balanced, otherwise if some units of group have a higher level of performance while other units show low performance, the overall group performance is not balanced. Such a situation results in consequential damages which in many cases are irrecoverable. Compensation for the weak performance of an inefficient unit with low performance requires immense time and expenditure, but if it is done in an appropriate manner, costs could be reduced.

According to the points outlined above, dispersion value of efficiency of units within a group denotes on a well-balanced or unbalanced management system. The lower the dispersion level, the more balanced the levels of management and implementation. The following index to determine the level of management balance which was called within-group efficiency spread by Camanho and Dyson [14] is presented.

Assume that $E_1^{h^*}, \dots, E_n^{h^*}$ signifies the efficiency of units in group h, then $e_{kj}^h = |E_k^{h^*} - E_j^{h^*}|$ indicates the performance gap of unit k and unit j. The number of such gaps is as:

$$R_h = n - 1 + n - 2 + \cdots + 2 + 1 = \sum_{t=1}^{n-1} t = \frac{n(n-1)}{2}$$

Efficiency spread of group h can be explained by using:

$$e^h = \frac{1}{R} \sum_{k=1}^n \sum_{j=k+1}^n e_{kj}^h \quad (6)$$

In this relation, e^h is the average of efficiency gaps of units in group h. It is obvious that if e^h has the tendency to move towards zero, then the efficiencies of DMUs in group h is extremely close. The larger e^h becomes, the bigger the efficiency gaps for units in group h turn will be. In fact the smaller the efficiency spread is, the higher will be the uniformity and balance of performance. Formula (6) is comparable to IE index in Camanho and Dyson's method [14].

B. Among Group Analysis

We are aware that jth units in different groups are homogenous, so we can provide a comparison among the groups from the point of view of $DMU_j (j = 1, \dots, n)$. If jth units in different groups have similar performance, then it can be claimed that the performance of the groups in $DMU_j (j = 1, \dots, n)$ is balanced, otherwise if jth unit of some groups has a higher level of performance while in other groups has a low performance, performance of the groups in $DMU_j (j = 1, \dots, n)$ will not be balanced. In other words, similar performance of groups in $DMU_j (j = 1, \dots, n)$ is shown as the well-balanced management in part j of different groups. Therefore, the following index is presented as a criterion to compare different groups based on the performance of $DMU_j (j = 1, \dots, n)$. Assume $e_j^{hl} = |E_j^{h^*} - E_j^{l^*}|$ is the distance between the performance of $DMU_j (j = 1, \dots, n)$ in groups h and l. The average of these distances will be as:

$$e_j = \frac{1}{RJ} \sum_{h=1}^{g} \sum_{l=h+1}^{g} e_j^{hl} \qquad (7)$$

It indicates well-balanced management in part j of different groups. It is notable that

$$RJ = g - 1 + g - 2 + \cdots + 2 + 1 = \frac{g(g-1)}{2}$$

C. Determine Common Set of Weights

Now, consider model (3). This model can be rewritten as:

$$Max \sum_{j=1}^{n} w_j^h \frac{u_j y_j^h}{v_j x_j^h}, \quad h = 1, \ldots, g, s.t. \frac{u_j y_j^h}{v_j x_j^h} \le 1, \quad j = 1, \ldots, n, \quad h = 1, \ldots, g,$$
$$v_j \ge 0, \quad j = 1, \ldots, n,$$
$$u_j \ge 0, \quad j = 1, \ldots, n \qquad (8)$$

Let,

$$w_j^h = \frac{v_j x_j^h}{\sum_{k=1}^{n} v_k x_k^h}, \quad j = 1, \ldots, n, \quad h = 1, \ldots, g \qquad (9)$$

So, we have

$$E_{G_h} = \sum_{j=1}^{n} w_j^h E_j^h = \sum_{j=1}^{n} w_j^h \frac{u_j y_j^h}{v_j x_j^h} = \sum_{j=1}^{n} \frac{v_j x_j^h}{\sum_{k=1}^{n} v_k x_k^h} \frac{u_j y_j^h}{v_j x_j^h} = \frac{\sum_{j=1}^{n} u_j y_j^h}{\sum_{j=1}^{n} v_j x_j^h} \qquad (10)$$

We present three analyze of (10) as:

1- E_{G_h} is a weighted sum of outputs to inputs which is the same as the concept of efficiency in DEA. In other words, group h is considered as a unit which its inputs and outputs are all the inputs and outputs of all DMUs within group h, respectively.

2- E_{G_h} is a linear fractional function. Properties of optimization problems with linear fractional objectives are relatively similar to problems with linear objective functions. Thus, problem (8) is converted to a multi-objective linear fractional programming problem in which there are authentic techniques to solve it.

3- w_j^h is the contribution of $DMU_j (j = 1, \ldots, n)$ in overall efficiency of group h. Denominator of w_j^h for different units is equal, and so the difference of contribution of units is in their weighted sum of inputs. The more the value of weighed sum of inputs for a unit, the more contribution of the unit in overall performance of group will be. On the other hand, the weighed sum of inputs of a DMU in economic is the cost of the unit. Thus, when the cost of a unit in group is high, then the unit must have a high contribution in the performance of group.

Model (8) can be converted to the following multi-objective linear fractional programming problem as:

$$Max \frac{\sum_{j=1}^{n} u_j y_j^h}{\sum_{j=1}^{n} v_j x_j^h}, h = 1, \ldots, g,$$
$$s.t \frac{u_j y_j^h}{v_j x_j^h} \le 1, j = 1, \ldots, n, h = 1, \ldots, g,$$
$$v_j \ge 0, j = 1, \ldots, n, u_j \ge 0, j = 1, \ldots, n \qquad (11)$$

We now intend to present a method to obtain an efficient solution of model (11). The obtained solution is considered as a set of weights to calculate the overall efficiency of groups and the efficiency of DMUs within groups, by using (4) and (5).

We have

$$E_j^h \le 1, j = 1, \ldots, n, h = 1, \ldots, g \Rightarrow w_j^h E_j^h \le w_j^h$$
$$\Rightarrow \sum_{j=1}^{n} w_j^h E_j^h \le \sum_{j=1}^{n} w_j^h \Rightarrow E_{G_h} \le 1, h = 1, \ldots, g$$

Therefore, number one can be considered as a goal for hth objective function. Thus,

$$E_{G_h} = \frac{\sum_{j=1}^{n} u_j y_j^h}{\sum_{j=1}^{n} v_j x_j^h} \le 1 \Rightarrow \sum_{j=1}^{n} u_j y_j^h \le \sum_{j=1}^{n} v_j x_j^h, h = 1, \ldots, g$$

In this situation, we have:

$$\sum_{j=1}^{n} u_j y_j^h \le \theta_h, \theta_h \le \sum_{j=1}^{n} v_j x_j^h, h = 1, \ldots, g$$

So, there are $d_h^+, d_h^- \ge 0 (h = 1, \ldots, g)$,

$$\sum_{j=1}^{n} u_j y_j^h + d_h^- = \theta_h, \sum_{j=1}^{n} v_j x_j^h - d_h^+ = \theta_h$$

If $d_h^+ + d_h^- = 0$, then E_{G_h} is equal to unity. Thus, group h is reached the highest possible efficiency. But, if d_h^- or d_h^+ is more than zero, $d_h^+ + d_h^-$ more than zero, then E_{G_h} will not have the highest performance, and so group h will be inefficient. We can minimize $d_h^+ + d_h^-$ to calculate the minimum distance of the efficiency of group h to number one or equivalently the maximum efficiency of group h. In this direction, problem (11) is transformed to a multi-objective linear programming problem as:

$$min \ d_h^+ + d_h^-, h = 1, \ldots, g,$$
$$s.t. \sum_{j=1}^{n} u_j y_j^h + d_h^- = \theta_h, h = 1, \ldots, g,$$
$$\sum_{j=1}^{n} v_j x_j^h - d_h^+ = \theta_h, h = 1, \ldots, g,$$
$$u_j y_j^h - v_j x_j^h \le 0, j = 1, \ldots, n, h = 1, \ldots, g,$$
$$v_j \ge 0, j = 1, \ldots, n, u_j \ge 0, j = 1, \ldots, n, \qquad (12)$$

which can be transformed to the following linear programming problem as:

$$min \sum_{h=1}^{g} d_h^+ + d_h^-, s.t. \sum_{j=1}^{n} u_j y_j^h + d_h^- = \theta_h, h = 1, \ldots, g,$$
$$\sum_{j=1}^{n} v_j x_j^h - d_h^+ = \theta_h, h = 1, \ldots, g,$$

$$u_j y_j^h - v_j x_j^h \leq 0, j = 1, \ldots, n, \quad h = 1, \ldots, g,$$

$$\sum_{h=1}^{g} \sum_{j=1}^{n} v_j x_j^h = g$$

$$v_j \geq 0, \quad j = 1, \ldots, n, u_j \geq 0, \quad j = 1, \ldots, n \qquad (13)$$

We have added $\sum_{h=1}^{g} \sum_{j=1}^{n} v_j x_j^h = g$ as a normalize constraint in order to avoid zero weights.

Let $(v^*, u^*) = (v_1^*, \ldots, v_n^*, u_1^*, \ldots, u_n^*)$ be an optimal vector of weights in which the optimal value of the objective function of model (13) is z^*.

Theorem: If z^* is equal to zero, then (v^*, u^*) will be complete solution of model (11).

Proof: Objective function of model (13) is a summation of non-negative variables. If the optimal value of the objective function is equal to zero, then all of the variables on objective function will be zero. Therefore $, d_h^{+*} = 0, d_h^{-*} = 0$ $(h = 1, \ldots g)$. Thus,

$$\sum_{j=1}^{n} u_j^* y_j^h = \theta_h, \sum_{j=1}^{n} v_j^* x_j^h = \theta_h, h = 1, \ldots g$$

$$\Rightarrow \frac{\sum_{j=1}^{n} u_j^* y_j^h}{\sum_{j=1}^{n} v_j^* x_j^h} = 1, h = 1, \ldots g$$

On the other hand, number one is an upper bound for objective functions of model (11). So, solution (v^*, u^*) is a complete solution for model (11).

If optimal value of the objective function of model (13) is more than zero, then we can use the following model to efficiency test of the optimal weights of model (13). This model is firstly suggested by Hosseinzadeh Lotfi et al., [17] to efficiency test of multi-objective linear fractional programming problem. The model is as:

$$max \sum_{h=1}^{g} d_h^+ + d_h^-,$$

$$s.t \sum_{j=1}^{n} u_j y_j^h - d_h^- = \theta_h m_h, \qquad h = 1, \ldots, g,$$

$$\sum_{j=1}^{n} v_j x_j^h + d_h^+ = \theta_h n_h, \qquad h = 1, \ldots g,$$

$$u_j y_j^h - v_j x_j^h \leq 0, \quad j = 1, \ldots, n, \quad h = 1, \ldots, g,$$

$$\sum_{h=1}^{g} \sum_{j=1}^{n} v_j x_j^h = g,$$

$$v_j \geq 0, \quad j = 1, \ldots, n, u_j \geq 0, \quad j = 1, \ldots, n, \qquad (14)$$

where

$$m_h = \sum_{j=1}^{n} u_j^* y_j^h, \qquad n_h = \sum_{j=1}^{n} v_j^* x_j^h, \qquad h = 1, \ldots, g$$

If the optimal value of the objective function of model (14) is equal to zero, (v^*, u^*) will be an efficient solution [17]. Otherwise, let $(\tilde{v}, \tilde{u})$ is the optimal weights of model (14) with positive objective function. Now, we can use model (14) to

efficiency test of $(\tilde{v}, \tilde{u})$. Suppose (v^1, u^1) ‹(v^2, u^2) ... (v^k, u^k) are a sequence of the obtained weights from solving model (14) in which the optimal value of the objective function of model (14) for each of them is more than zero. Consider (v^t, u^t), and put

$$m_h^t = \sum_{j=1}^{n} u_j^t y_j^h, \qquad n_h^t = \sum_{j=1}^{n} v_j^t x_j^h, \qquad h = 1, \ldots, g$$

By solving model (14), optimal weights (v^{t+1}, u^{t+1}) are obtained in which the optimal value of the objective function is positive. So, we have:

$$\left. \begin{array}{l} \sum_{j=1}^{n} u_j^{t+1} y_j^h - d_h^{t+1^-} = \theta_h^{t+1} m_h^t, h = 1, \ldots, g, \\ \sum_{j=1}^{n} v_j^{t+1} x_j^h + d_h^{t+1^+} = \theta_h^{t+1} n_h^t, h = 1, \ldots g, \end{array} \right\} \Rightarrow \left. \begin{array}{l} \sum_{j=1}^{n} u_j^{t+1} y_j^h \geq \theta_h^{t+1} m_h^t, h = 1, \ldots, g, \\ \sum_{j=1}^{n} v_j^{t+1} x_j^h \leq \theta_h^{t+1} n_h^t, h = 1, \ldots, g, \end{array} \right\}$$

$$\Rightarrow \frac{\sum_{j=1}^{n} u_j^{t+1} y_j^h}{\sum_{j=1}^{n} v_j^{t+1} x_j^h} \geq \frac{m_h^t}{n_h^t} = \frac{\sum_{j=1}^{n} u_j^t y_j^h}{\sum_{j=1}^{n} v_j^t x_j^h}$$

Optimal value of the Objective function of model (14) is more than zero in each iteration, thus at least one of the variables $d_h^{t+1^+}$ or $d_h^{t+1^-}$ $(h = 1, \ldots g)$ is positive. As a result,

$$\frac{\sum_{j=1}^{n} u_j^{t+1} y_j^h}{\sum_{j=1}^{n} v_j^{t+1} x_j^h} \geq \frac{\sum_{j=1}^{n} u_j^t y_j^h}{\sum_{j=1}^{n} v_j^t x_j^h}, h = 1, \ldots g,$$

and strict inequality is held for at least an h ($h = 1, \ldots g$). Thus, (v^t, u^t) is not an efficient solution for model (11), but at least the value of one of the objective functions of model (11) is increased. On the other hand, $\frac{\sum_{j=1}^{n} u_j y_j^h}{\sum_{j=1}^{n} v_j x_j^h}$ $(h = 1, \ldots g)$ is bounded to number one. Thus, by repetition this procedure in the worst situation a solution as $(\tilde{v}, \tilde{u})$ is obtained in which $\frac{\sum_{j=1}^{n} \tilde{u}_j y_j^h}{\sum_{j=1}^{n} \tilde{v}_j x_j^h} = 1$. Therefore, $(\tilde{v}, \tilde{u})$ is a complete solution for multi-objective linear fractional programming problem (11). As a result, model (14) is convergent to an efficient or complete solution, in a finite number of iterations.

The following algorithm is suggested to solve multi-objective linear fractional programming problem (11) in order to find an efficient solution.

Step 1. Solve model (13) and consider (v^1, u^1) and z_1^* as optimal solution and optimal value of objective function, respectively.

Step 2. If $z_1^* = 0$, then (v^1, u^1) is complete solution, otherwise, put $t = 1$ and go to step 3.

Step 3. Solve problem (14) in which $m_h^t = \sum_{j=1}^{n} u_j^t y_j^h$, $n_h^t = \sum_{j=1}^{n} v_j^t x_j^h$ $(h = 1, \ldots, g)$. Consider (v^{t+1}, u^{t+1}) and z_{t+1}^* as optimal solution and optimal value of objective function, respectively.

Step 4. If $z_{t+1}^* = 0$, then (v^{t+1}, u^{t+1}) is efficient solution, otherwise, put $t = t + 1$ and return to step 3.

IV. EXAMPLE

The aim of this section is to evaluate Islamic Azad Universities in 14th district of Iran. Based on the structure of Islamic Azad Universities, universities in each district called branches in the related district. Each branch in this district is considered as a group of units in which DMUs within each of them are teaching (E), research (R) and student (S) offices. Groups are Iranshahr branch (B_1), Khash branch (B_2), Chabahar branch (B_3), Zabl branch (B_4), Zahedan branch (B_5), Saravan branch (B_6) and Nikshahr branch (B_7). Teaching indices are considered as the number of staff (EI_1), the number of educational departments (EI_2), the number of students (EI_3), the number of full time professors (EO_1), and the number of educated students (EO_2) in which the first three indices are inputs and the two last are outputs. The data related to education are presented in Table I.

TABLE I
DATA OF TEACHING OFFICE

Teaching	EI_1	EI_2	EI_3	EO_1	EO_2
E_1(Iranshahr)	7	22	3501	54	1097
E_2(Khash)	4	6	933	13	315
E_3(Chabahar)	6	10	2854	14	477
E_4(Zabol)	12	5	2659	48	307
E_5(Zahedan)	76	27	10351	126	2631
E_6(Saravan)	4	8	1853	22	542
E_7(Nikshahr)	2	6	1124	7	600

For the research office, the number of research staff (RI_1) and the number of full time professors (RI_2) are considered as inputs, and the number of published papers (RO_1), the number of published books (RO_2) and the number of completed research proposals (RO_3) are considered as outputs. The related information are reported in Table II. Considering that the number of published books is zero for some branches, to contribute the effect of this index in evaluate performance of all branches, we replace small number $\varepsilon = 0.1$ by 0.

TABLE II
DATA OF RESEARCH OFFICE

Research	RI_1	RI_2	RO_1	RO_2	RO_3
R_1	3	54	19	1	17
R_2	3	13	30	ε	13
R_3	2	14	3	ε	5
R_4	8	48	20	ε	15
R_5	23	126	145	5	12
R_6	3	22	3	ε	2
R_7	2	7	18	3	1

In student office, we present two indices of the number of student staff (SI_1) and area (SI_2) as inputs and two indices of the number of students (SO_1) and the number of loans (SO_2) as outputs. The data are provided in Table III. Similarly, to contribute the effect of indices with zero value in evaluate the performance of all branches, we consider number $\varepsilon = 0.1$ as a replacement.

TABLE III
DATA OF STUDENT OFFICE

Student	SI_1	SI_2	SO_1	SO_2
S_1	2	620	176	230
S_2	1	260	20	3
S_3	2			
S_4	6			188
S_5	27	8222	1600	1224
S_6	3	235	25	65
S_7	1			

By using data in the above Tables I-III, we evaluate branches in 14th district as groups of DMUs. The obtained results are summarized in Tables IV and V.

TABLE IV
PERFORMANCE EVALUATION OF OFFICES

Student	Weights of Offices	Efficiency of Offices
E_1	0.5304260	1.000000
R_1	0.2012463	0.6545765
S_1	0.2683277	1.000000
E_2	0.2977012	0.9725570
R_2	0.4213793	1.000000
S_2	0.2809196	0.2272725
E_3	0.4887668	0.4614041
R_3	0.1704111	0.1530640
S_3	0.3408220	0.5681816E-03
E_4	0.1986654	1.000000
R_4	0.3205339	0.2502845
S_4	0.4808007	0.1897137E-03
E_5	0.2059884	0.9346766
R_5	0.2371723	0.6437910
S_5	0.5568393	0.6734003
E_6	0.3001114	0.9110779
R_6	0.2332963	0.1020412
S_6	0.4665923	0.9469708E-01
E_7	0.3792217	1.000000
R_7	0.3103891	1.000000
S_7	0.3103891	0.1136363E-02

TABLE V
PERFORMANCE EVALUATION OF UNIVERSITIES

Group	Group Efficiency	Spread Efficiency
B_1	0.9304848	0.4605646
B_2	0.7747559	1.030303
B_3	0.2517965	0.8200079
B_4	0.2789813	1.832891
B_5	0.7201977	0.5620316
B_6	0.3414156	1.627865
B_7	0.6899636	1.331818

The second column of Table IV shows the weights of teaching, research and students offices to calculate the efficiency of branches. Efficiency of different offices of each group is presented in the third column of Table IV. Based on the suggested method in this paper, the efficiency of each group is the weighted sum of the efficiency of DMUs within it which these efficiencies in this example are reported in the second column of Table V. The last column of Table V shows within efficiency spread of different branches. The values are

shown the difference of performances among teaching, research and student offices.

Referring again to the second column of Table IV, we will find out that the values in this column indicate the effect of different offices on the performance of the related branch. Based on the values in this column, the effect of teaching office of Iranshahr, Chabahar, and Nikshahr branches on their performances is more than the other two offices. The performance of Zabol, Zahedan and Saravan branches is more affected of their student offices. Khash is the only branch that its performance is affected by its research office. Using the information in the third column of Table IV, we can compare the performance different offices within each branch, and similar offices in different branches. Iranshahr branch has the best performance in teaching and student offices, while the best performance in Nikshahr branch is related to teaching and research offices. Research office in Khash branch is the best one among its three offices. In other branches, the best performance is related to teaching office. We can also rank seven branches based on the performance of each of offices whose the results are as:

$$E_1 \approx E_4 \approx E_7 \succ E_2 \succ E_5 \succ E_6 \succ E_3$$
$$R_2 \approx R_7 \succ R_1 \succ R_5 \succ R_4 \succ R_3 \succ R_6$$
$$S_1 \succ S_5 \succ S_2 \succ S_6 \succ S_7 \succ S_3 \succ S_4$$

Using the efficiency of branches in the second column of Table V which is obtained by values of the two before columns, the branches are ranked as:

$$B1 \succ B2 \succ B5 \succ B7 \succ B6 \succ B4 \succ B3$$

Therefore, Iranshahr and Khash have the first and second ranks, respectively. It is evident from the last column of Table V that Iranshahr has the lowest spread efficiency rate; therefore, the efficiency of its different offices will be similar. Based on another study, number 1.832891 indicates that the efficiency spread of Zabol is the highest efficiency spread among the branches in 14th district. Thus, there is no well-balanced management in the offices of Zabol branch. The numerical results in the third column of Table V about Zabol city confirms this fact, because the teaching office of this branch is efficient in which the performance of its student office is very low.

One of the capabilities of the proposed method is to estimate the efficiency spread of similar offices in different branches. Within group spread efficiency for teaching, research and student offices are equal to 0.1770434, 0.4353554 and 0.4563848, respectively. We can see that the performance of teaching offices in different branches of the 14th district of Islamic Azad University is closer to each other than the other two offices. This shows that the bylaws and regulations of the teaching office are exerted with more preciseness than the other two offices.

V. CONCLUSION

In this paper, a new structure was presented to evaluate the performance of universities as decision making groups. We applied common set of weights method to evaluate universities with new structure. The method can evaluated the relative performance of universities and at the same time measures the performance of different offices of universities. Another capability of this method is to estimate the efficiency spread of universities. Common set of weights is used to evaluate the efficiency of universities which is obtained from linear programming problems and that is an efficient solution of the proposed multi-objective linear fractional programming problem. Referring to the solved problem above, this method is cost-effective from the computational point of view, because the method only uses linear programming problems to obtained common set of weights. It should be noted that none of DEA methods are capable to calculate all these features of university evaluation and therefore, the proposed method can perform a more comprehensive evaluation compared to other methods. In a future study, we will research on estimating productivity of decision making groups based on the proposed method in this paper.

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