

# Design of a Reduced Order Robust Convex Controller for Flight Control System

S. Swain, P. S. Khuntia

**Abstract**—In this paper an optimal convex controller is designed to control the angle of attack of a FOXTROT aircraft. Then the order of the system model is reduced to a low-dimensional state space by using Balanced Truncation Model Reduction Technique and finally the robust stability of the reduced model of the system is tested graphically by using Kharitonov rectangle and Zero Exclusion Principle for a particular range of perturbation value. The same robust stability is tested theoretically by using Frequency Sweeping Function for robust stability.

**Keyword**—Convex Optimization, Kharitonov Stability Criterion, Model Reduction, Robust Stability.

## I. INTRODUCTION

THE main problems of flight control system are due to the nonlinear dynamics, modeling uncertainties and parametric variations. Generally an aircraft moves in a three dimensional plane by controlling the three control surfaces aileron, rudder and elevator. These three control surfaces control the motion of the aircraft about the roll, pitch and yaw axes. The elevators of an aircraft control the orientation of the aircraft by changing the pitch and the angle of attack of the aircraft. Though a lot of works have been done to control the angle of attack, still it is an open issue which is discussed in the present work. Not only the designed controller is required to offer satisfactory performance in terms of controlling the angle of attack, but also the system model has to be robust stable for a wide range of change in parametric values of closed loop transfer function. Alireza Karimi, Hamid Khatibi and Roland Longchamp synthesize the robust control of linear time-invariant SISO polytopic systems using the polynomial approach [1]. Kin Cheong Sou, Megretski, A. and Daniel, L. proposed a Quasi-Convex optimization approach to Parameterized Model Order Reduction (MOR) framework [2]. V. L. Kharitonov in 1978 found out asymptotic stability of a family of systems for an equilibrium position with help linear differential equations [3]. Kharitonov theorem also provides the necessary and sufficient conditions for checking the robust stability of dynamic system with fractional order interval systems [4]-[6]. Fu. M. developed a simple approach which unifies and generalizes a class of weak Kharitonov regions for robust stability of linear uncertain systems [7]. Jie Chen

considered robust stability problem for interval plants in the case of single input (multi-output) or single output (multi-input) systems using a generalization of V.L. Kharitonov's theorem [8]. Bevrani, H. designed a robust proportional-integral-derivative (PID) feedback compensator for better stability and robust performance of a radio-frequency amplifier with wide range parameter variation [9]. R.J. Bhiwani and B.M. Patre analyse the robust stability feedback controller synthesis can be tested using Kharitonov's theorem for fuzzy parametric uncertain systems [10]. Toscano and Lyonnet synthesized a feedback controller to obtain robust static feedback using evolutionary algorithm [11].

In this paper an optimal convex controller is designed using convex optimization technique to control the angle of attack [12], [13] of FOXTROT aircraft. Then the order of the system model is reduced by using Balanced Truncation technique to a low dimensional state space [14], [15]. Finally the robust stability [16]-[18] of the reduced model is tested graphically by using Kharitonov rectangle & Zero Exclusion Principle for a parametric perturbation ' $\mu$ ' and theoretically by using Frequency Sweeping function for robust stability. In this work ' $\mu$ ' is allowed to increase up to a particular value below which the system model is found to be robust stable by establishing the Kharitonov polynomials to be Hurwitz. Increasing beyond this value of ' $\mu$ ' further the system model is not robust stable resulting non Hurwitz Kharitonov polynomials. It is also shown that the Kharitonov rectangle does not include zero within it thereby verifying the interval polynomial family to be robust stable for all frequencies  $\omega \geq 0$  resulting  $H(\omega)$  to be positive real [19]

## II. KHARITONOV INTERVAL POLYNOMIALS

Consider an n-th order polynomial [3] of the form given by  $p(s) = s^n + a_{n-1}s^{n-1} + \dots + a_0$  for all  $a_0, \dots, a_{n-1}$  such that  $\underline{a}_k = a_k - \mu, \bar{a}_k = a_k + \mu$ , where  $\mu$  = the perturbation in parametric values.

Let the polynomials be defined as

$$g_1(s) = \underline{a}_0 + \bar{a}_2 s^2 + \underline{a}_4 s^4 + \dots = \sum_{k=0, \text{even}}^n j^k \cdot \min\{j^k \underline{a}_k, j^k \bar{a}_k\} \cdot s^k$$

$$g_2(s) = \bar{a}_0 + \underline{a}_2 s^2 + \bar{a}_4 s^4 + \dots = \sum_{k=0, \text{even}}^n j^k \cdot \max\{j^k \underline{a}_k, j^k \bar{a}_k\} \cdot s^k$$

$$h_1(s) = \underline{a}_1 + \bar{a}_3 s^3 + \underline{a}_5 s^5 + \dots = \sum_{k=1, \text{odd}}^n j^{k-1} \cdot \min\{j^{k-1} \underline{a}_k, j^{k-1} \bar{a}_k\} \cdot s^k$$

S. Swain is working as Associate Professor in the Electrical Engineering Department, Konark Institute of Science and Technology, Bhubaneswar, Orissa, India (e-mail:swainsrinibash@gmail.com).

P.S. Khuntia is working as Professor in the Department of Electronics and Telecommunication Engineering, Konark Institute of Science and Technology, Bhubaneswar, Orissa, India, Pin: 752050 (e-mail: parthsarathi\_k@yahoo.com).

$$h_2(s) = \bar{a}_1 s + \bar{a}_3 s^3 + \bar{a}_5 s^5 + \dots = \sum_{k=1, \text{odd}}^n j^{k-1} \cdot \max\{j^{k-1} \bar{a}_k, j^{k-1} \bar{a}_k\} \cdot s^k$$

Now, the Kharitonov polynomials are given by

$$k_{kl}(s) = g_k(s) + h_l(s)$$

where  $k, l = 1, 2$

For  $k = 1$  and  $l = 1$

$$k_{11}(s) = g_1(s) + h_1(s) \quad (1)$$

For  $k = 1$  and  $l = 2$

$$k_{12}(s) = g_1(s) + h_2(s) \quad (2)$$

For  $k = 2$  and  $l = 1$

$$k_{21}(s) = g_2(s) + h_1(s) \quad (3)$$

For  $k = 2$  and  $l = 2$

$$k_{22}(s) = g_2(s) + h_2(s) \quad (4)$$

The above set of polynomials  $k_{11}(s), k_{12}(s), k_{21}(s)$  and  $k_{22}(s)$  are said to be Hurwitz if and only if its every member is Hurwitz. These polynomials are called Kharitonov Interval polynomials.

### III. ANGLE OF ATTACK

Angle of attack [12], [13] specifies the angle between the chord line of the wing of a fixed-wing aircraft and the vector representing the relative motion between the aircraft and the atmosphere. The angle of attack ( $\alpha$ ) of an aircraft is controlled by the deflection in control surface (Elevator). Fig. 1 below shows the description of angle of attack of an aircraft.

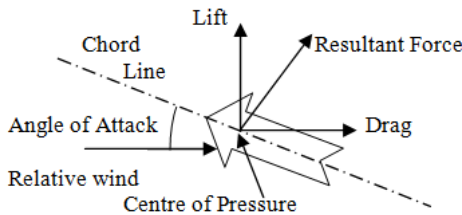


Fig. 1 Description of angle of attack of an aircraft

#### A. Block Diagram of Angle of Attack Control System

The block diagram for angle of attack is shown in Fig. 2 below in which the input is the deflection of elevator ( $\delta_E$ ) as commanded by the pilot and the output is the desired angle of attack ( $\alpha$ ).

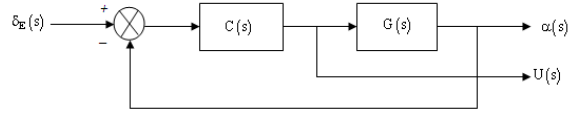


Fig. 2 Block Diagram of Angle of Attack Control System

In Fig. 2,

$\delta_E(s)$  = Deflection of elevator as commanded by the pilot

$\alpha(s)$  = The desired angle of attack of the aircraft

$G(s)$  = Transfer function between  $\delta_E$  and  $\alpha$

$C(s)$  = Convex controller to be designed

$U(s)$  = Output of controller

#### B. Transfer Functions between $\delta_E$ and $\alpha$

The short period approximation [12] consists of assuming any variations in speed ( $u$ ) of the aircraft which arise in air speed as a result of control surface deflection and atmospheric turbulence is so small that any terms in the equation of motion involving  $u$  are negligible. In other words, the approximation assumes that short period transients are of sufficiently short duration resulting constant speed  $U_0$  of the aircraft i.e.  $u=0$ . Thus, the equations of longitudinal motion in terms of stability may now be written as:

$$\dot{w} = Z_w w + U_0 q + Z_{\delta_E} \delta_E \quad (5)$$

$$\begin{aligned} \dot{q} &= w M_w + \dot{w} M_w + q M_q + M_{\delta_E} \delta_E \\ &= (M_w + M_{\dot{w}} Z_w) w + (M_q + U_0 M_{\dot{w}}) q + (M_{\delta_E} + Z_{\delta_E} M_{\dot{w}}) \delta_E \end{aligned} \quad (6)$$

If the state vector for short period motion is  $x \triangleq \begin{bmatrix} w \\ q \end{bmatrix}$  and the control vector 'u' is taken as the deflection of the elevator  $\delta_E$ , then (5) and (6) may be written as a state equation:

$$\dot{x} = Ax + Bu \quad (7)$$

In (7), the values of A and B are

$$\begin{aligned} A &= \begin{bmatrix} Z_w & U_0 \\ (M_w + M_{\dot{w}} Z_w) & (M_q + U_0 M_{\dot{w}}) \end{bmatrix} \\ B &= \begin{bmatrix} Z_{\delta_E} \\ (M_{\delta_E} + Z_{\delta_E} M_{\dot{w}}) \end{bmatrix} \\ \therefore [sI - A] &= \begin{bmatrix} s - Z_w & -U_0 \\ -(M_w + M_{\dot{w}} Z_w) & s - (M_q + U_0 M_{\dot{w}}) \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \Delta_{SP}(s) &= \det[sI - A] \\ &= s^2 - [Z_w + M_q + U_0 M_{\dot{w}}]s + [Z_w M_q - U_0 M_w] \end{aligned}$$

$$= s^2 + 2\xi_{SP}\omega_{SP}s + \omega_{SP}^2 \quad (8)$$

$$\text{In (8), } 2\xi_{SP}\omega_{SP} = Z_w + M_q + U_0 M_{\dot{w}}$$

$$\omega_{SP} = \left[ Z_w M_q - U_0 M_{\dot{w}} \right]^{\frac{1}{2}}$$

On simplifying the above equations, the transfer functions between  $w$  and  $\delta_E$  is given by

$$\begin{aligned} \frac{w(s)}{\delta_E(s)} &= \frac{(U_0 M_{\delta_E} - M_q Z_{\delta_E}) \left\{ 1 + \frac{s Z_{\delta_E}}{U_0 M_{\delta_E} - M_q Z_{\delta_E}} \right\}}{\Delta_{SP}(s)} \\ &\Rightarrow \frac{w(s)}{\delta_E(s)} = \frac{K_w (1 + s T_1)}{\Delta_{SP}(s)} \end{aligned} \quad (9)$$

$$\text{In (9), } T_1 = \frac{Z_{\delta_E}}{K_w}, K_w = U_0 M_{\delta_E} - M_q Z_{\delta_E}$$

$$\text{Again, } \alpha = \frac{w}{U_0}, \alpha(s) = \frac{w(s)}{U_0} \text{ and } w(s) = U_0 \alpha(s)$$

Substituting the value of  $w(s)$  in (9), we get

$$\frac{\alpha(s)}{\delta_E(s)} = \frac{K_w (1 + s T_1)}{U_0 \Delta_{SP}(s)} \quad (10)$$

Using the values of the stability derivatives [12] as shown in Appendix-I and substituting these values in (10), the transfer function  $G(s)$  between  $\delta_E$  and  $\alpha$  for the flight condition-1 is given by

$$G(s) = \frac{2.0302s + 102.8}{s^2 + 0.901s + 0.5633} = \frac{3.604s + 182.5}{1.775s^2 + 1.598s + 1} \quad (11)$$

#### IV. DESIGN OF OPTIMAL CONVEX CONTROLLER

It is a controller that uses convex optimization for controlling a linear system. The analysis and design of linear control systems is based on numerical convex optimization [20] over closed-loop maps. Convexity makes numerical solution effective i.e. it determines whether a given set of specifications is achieved by the controller or not.

The internal stability of a system is a closed loop affine constraint i.e. if the controllers  $K$  and  $\hat{K}$  each stabilize the plant  $P$  and yield closed-loop transfer matrices  $H$  and  $\hat{H}$  respectively, then for each  $\lambda \in \mathbb{R}$  there is a controller given by

$$H_\lambda(s) = \lambda H(s) + (1-\lambda) \hat{H}(s) \quad (12)$$

Here two PI (Proportional-Integral) controllers  $K(s) = 60 + 90/s$  and  $\hat{K}(s) = 18 + 81/s$  are considered which

individually stabilize the plant  $G$ . With ' $K$ ' in the loop, the transfer matrix from  $\delta_E$  to  $[\alpha, u]^T$  is given by

$$H(s) = \begin{bmatrix} \frac{216.2s^2 + 11270s + 1.6420}{1.775s^3 + 217.8s^2 + 1.1280s + 1.6420} \\ \frac{106.5s^3 + 255.6s^2 + 203.8s + 90}{1.775s^3 + 217.8s^2 + 1.1280s + 1.6420} \end{bmatrix} \quad (13)$$

Similarly, with  $\hat{H}$  in the loop the closed loop transfer matrix is given by

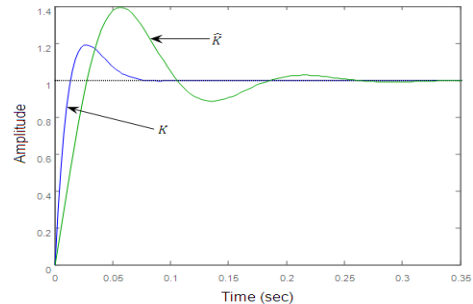
$$\hat{H}(s) = \begin{bmatrix} \frac{64.87s^2 + 3577s + 1.4780}{1.775s^3 + 66.47s^2 + 3578s + 1.4780} \\ \frac{31.95s^3 + 172.6s^2 + 147.4s + 81}{1.775s^3 + 66.47s^2 + 3578s + 1.4780} \end{bmatrix} \quad (14)$$

Substituting the value of  $H(s)$  from (13) and the value of  $\hat{H}(s)$  from (14) in (12), we get

$$\begin{aligned} H_\lambda(s) &= \lambda H(s) + (1-\lambda) \hat{H}(s) = \lambda \begin{bmatrix} \frac{216.2s^2 + 11270s + 1.6420}{1.775s^3 + 217.8s^2 + 1.1280s + 1.6420} \\ \frac{106.5s^3 + 255.6s^2 + 203.8s + 90}{1.775s^3 + 217.8s^2 + 1.1280s + 1.6420} \end{bmatrix} \\ &+ (1-\lambda) \begin{bmatrix} \frac{64.87s^2 + 3577s + 1.4780}{1.775s^3 + 66.47s^2 + 3578s + 1.4780} \\ \frac{31.95s^3 + 172.6s^2 + 147.4s + 81}{1.775s^3 + 66.47s^2 + 3578s + 1.4780} \end{bmatrix} \end{aligned} \quad (15)$$

#### A. Closed Loop Step Response for Flight Condition-1

The step responses from  $\delta_E$  to  $\alpha$  for  $K$  and  $\hat{K}$  are shown in Fig. 3 (a) and the corresponding step response from  $\delta_E$  to  $u$  are shown in Fig. 4 (a). Fig. 3 (b) shows the closed-loop step responses from  $\delta_E$  to  $\alpha$  with six different values of  $\lambda$  generated by  $K$  and  $\hat{K}$ . Similarly, Fig. 4 (b) shows the closed-loop step responses from  $\delta_E$  to  $u$  with six different values of  $\lambda$ .



(a)

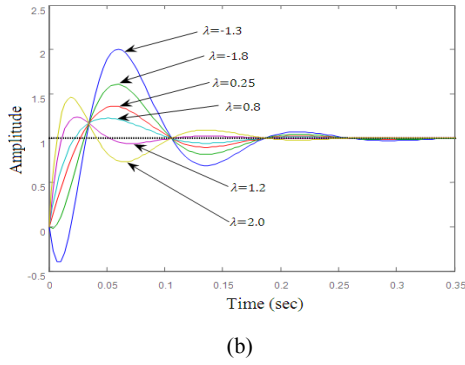


Fig. 3 Closed-loop step response achieved by one family of stabilizing controllers from  $\delta_E$  to  $\alpha$

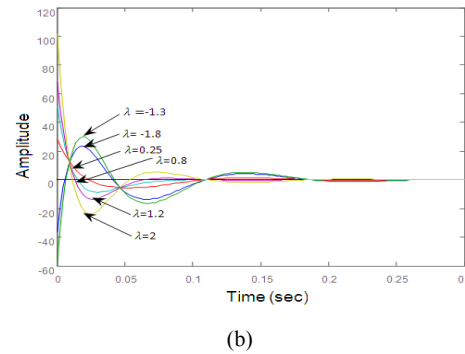
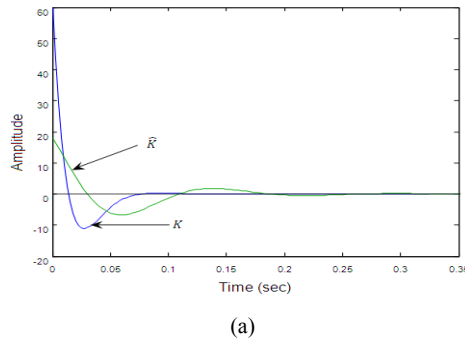


Fig. 4 Closed-loop step response achieved by one family of stabilizing controllers from  $\delta_E$  to  $\alpha$

From Fig. 4, the optimum value of ' $\lambda$ ' for the angle of attack ' $\alpha$ ' is found to be  $\lambda_{\text{opt}\alpha} = 1.2$  and that for controller output ' $u$ ' is found to be  $\lambda_{\text{opt}u} = 0.25$ . The closed-loop transfer matrix for the angle of attack ' $\alpha$ ' is given by

$$H_{\lambda_{\text{opt}\alpha}}(s) = \lambda_{\text{opt}\alpha} H(s) + (1 - \lambda_{\text{opt}\alpha}) \hat{H}(s) \quad (16)$$

Similarly, the closed-loop transfer matrix for the controller output ' $u$ ' is given by

$$H_{\lambda_{\text{opt}u}}(s) = \lambda_{\text{opt}u} H(s) + (1 - \lambda_{\text{opt}u}) \hat{H}(s) \quad (17)$$

Substituting the value of  $\lambda_{\text{opt}\alpha} = 1.2$  in (16), we get

$$\begin{aligned} H_{\lambda_{\text{opt}\alpha}}(s) &= H_{1.2}(s) = 1.2 \times H(s) + (1 - 1.2) \hat{H}(s) \\ &= 1.2 \left[ \frac{216.2s^2 + 11270s + 1.6420}{1.775s^3 + 217.8s^2 + 1.1280s + 1.6420} \right] \\ &\quad + (1 - 1.2) \left[ \frac{64.87s^2 + 3577s + 1.4780}{1.775s^3 + 66.47s^2 + 3578s + 1.4780} \right] \\ &= \frac{437.6s^5 + 37170s^4 + 1555000s^3 + 44630000s^2 + 225400000s + 242800000}{3.152s^6 + 504.7s^5 + 40850s^4 + 1584000s^3 + 44650000s^2 + 225400000s + 242800000} \\ &\Rightarrow H_{\lambda_{\text{opt}\alpha}}(s) = H_{1.2}(s) \\ &= \frac{437.6s^5 + 37170s^4 + 1555000s^3 + 44630000s^2 + 225400000s + 242800000}{3.152s^6 + 504.7s^5 + 40850s^4 + 1584000s^3 + 44650000s^2 + 225400000s + 242800000} \quad (18) \end{aligned}$$

Again, substituting the value of  $\lambda_{\text{opt}u} = 0.25$  in (17), we get

$$\begin{aligned} H_{\lambda_{\text{opt}u}}(s) &= H_{0.25}(s) = 0.25 \times H(s) + (1 - 0.25) \hat{H}(s) \\ &= 0.25 \left[ \frac{106.5s^3 + 255.6s^2 + 203.8s + 90}{1.775s^3 + 217.8s^2 + 1.1280s + 1.6420} \right] \\ &\quad + (1 - 0.25) \left[ \frac{31.95s^3 + 172.6s^2 + 147.4s + 81}{1.775s^3 + 66.47s^2 + 3578s + 1.4780} \right] \\ &= \frac{215.6s^6 + 7587s^5 + 398500s^4 + 2503000s^3 + 4514000s^2 + 3335000s + 1330000}{3.152s^6 + 504.7s^5 + 40850s^4 + 1584000s^3 + 44650000s^2 + 225400000s + 242800000} \\ &\therefore H_{\lambda_{\text{opt}u}}(s) = H_{0.25}(s) \\ &= \frac{215.6s^6 + 7587s^5 + 398500s^4 + 2503000s^3 + 4514000s^2 + 3335000s + 1330000}{3.152s^6 + 504.7s^5 + 40850s^4 + 1584000s^3 + 44650000s^2 + 225400000s + 242800000} \quad (19) \end{aligned}$$

For every  $\lambda \in \mathbb{R}$ , there is a controller  $K_\lambda$  that yields closed loop transfer matrix  $H_\lambda$  is given by

$$K_\lambda = \frac{\lambda [K(s) - \hat{K}(s)] + \hat{K}(s) [1 + G_1(s)K(s)]}{[1 + G_1(s)K(s)] + \lambda G_1(s) [\hat{K}(s) - K(s)]} \quad (20)$$

For  $\lambda = \lambda_{\text{opt}\alpha}$ , the convex controller for the angle of attack is given by

$$K_{\lambda_{\text{opt}\alpha}} = \frac{\lambda_{\text{opt}\alpha} [K(s) - \hat{K}(s)] + \hat{K}(s) [1 + G_1(s)K(s)]}{[1 + G_1(s)K(s)] + \lambda_{\text{opt}\alpha} G_1(s) [\hat{K}(s) - K(s)]}$$

For  $\lambda_{\text{opt}\alpha} = 1.2$ ,

$$K_{\lambda_{\text{opt}\alpha}} = K_{1.2} = \frac{121.4s^4 + 4164s^3 + 220700s^2 + 1209000s + 1330000}{1.775s^4 + 36.16s^3 + 2034s^2 + 14454s} \quad (21)$$

Therefore, the optimum value of  $\lambda$  which yields closed loop transfer matrix  $H_\lambda$  is  $\lambda_{\text{opt}\alpha} = 1.2$ .

## V. MODEL ORDER REDUCTIONS

There are many advantages to work with models with low-dimensional state space. Low-dimensional models are easier to analyze, much faster to simulate and requires lesser hard works for synthesis of controller [14]. Model reduction methods have been used successfully to solve large-scale problems in areas such as control engineering, signal processing, image compression, fluid mechanics, and power systems. From (18),

$$H_{qm}(s) = H_2(s) = \frac{437.6s^5 + 37170s^4 + 1555000s^3 + 44630000s^2 + 225400000s + 242800000}{3.152s^6 + 5047s^5 + 40850s^4 + 1584000s^3 + 44660000s^2 + 225400000s + 242800000} \quad (22)$$

By using Balanced Truncation Model Reduction Technique, the above model  $H_{1,2}$  in (22) may be reduced to a new model given by

$$H_1(s) = \frac{-0.0008368s^4 + 139.3s^3 + 10600s^2 + 408800s + 10600000}{s^4 + 1521s^3 + 11650s^2 + 409700s + 10600000} \quad (23)$$

## VI. ROBUST STABILITY OF REDUCED SYSTEM MODEL

The characteristic equation for FC-1 is obtained from (23) as  $p_1(s) = 1 + G(s)C(s) = s^4 + 1521s^3 + 11650s^2 + 409700s + 10600000$ . The perturbation in parametric value of  $p(s)$  i.e. ' $\mu$ ' is allowed to increase from up to 20% and the Kharitonov polynomials for FC-1 are found out using (1) to (4) are as follows:

$$\left. \begin{aligned} K_{11}(s) &= 8480000s^4 + 327760s^3 + 13980s^2 + 182.52s + 0.8 \\ K_{12}(s) &= 8480000s^4 + 491640s^3 + 13980s^2 + 121.68s + 0.8 \\ K_{21}(s) &= 12720000s^4 + 327760s^3 + 9320s^2 + 182.52s + 1.2 \\ K_{22}(s) &= 12720000s^4 + 491640s^3 + 9320s^2 + 121.68s + 1.2 \end{aligned} \right\} \quad (24)$$

These above polynomials are tested for Hurwitz using Routh Hurwitz Criteria and found out to be Hurwitz Polynomials by establishing the coefficients in first column are positive. If the perturbation is further allowed beyond the above value of ' $\mu$ ' the polynomials are found not to be Hurwitz resulting the coefficients to be negative. Thus it is concluded that the designed controller along with the plant transfer function (angle of attack) discussed here is robust stable up to the perturbation range of 20%.

## A. Kharitonov Rectangle and Zero Exclusion Principle for Interval Families (Graphical Testing of Robust Stability)

An interval polynomial family having invariant degree and at least one stable variable is robustly stable if and only if the origin of the complex plane is excluded from the Kharitonov rectangle at all non-negative frequencies i.e. for all frequencies  $\omega \geq 0$ . The four vertices of Kharitonov rectangle  $K_{11}(j\omega_0)$ ,  $K_{12}(j\omega_0)$ ,  $K_{21}(j\omega_0)$  and  $K_{22}(j\omega_0)$  are obtained

by substituting  $s = j\omega_0$  in (24) for FC-1, at a fixed frequency  $\omega_0 = 2$ . The Kharitonov rectangles for FC-1 at  $\omega_0 = 2$  is shown in Fig. 5 below.

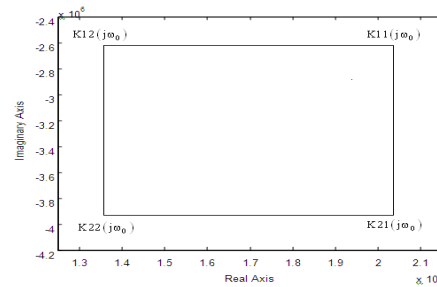


Fig. 5 Kharitonov rectangle for FC-1 at  $\omega_0 = 2$

However, the size and the position of the Kharitonov rectangle may change with  $\omega$  but the sides of the rectangle remain parallel to the respective real and imaginary axis.

## B. Frequency Sweeping Function for Robust Stability

An interval polynomial family is robustly stable if and only if  $H(\omega) \geq 0$  for all frequencies  $\omega \geq 0$  where

$$H(\omega) = \max \begin{Bmatrix} \text{Re } K_{11}(j\omega) \\ -\text{Re } K_{12}(j\omega) \\ \text{Im } K_{21}(j\omega) \\ -\text{Im } K_{22}(j\omega) \end{Bmatrix}$$

## For FC-1

Substituting  $s = j\omega$  in (24), we get

$$K_{11}(j\omega) = 8480000\omega^4 - 327760j\omega^3 - 13980\omega^2 + 182.52j\omega + 0.8$$

$$K_{12}(j\omega) = 8480000\omega^4 - 491640j\omega^3 - 13980\omega^2 + 121.68j\omega + 0.8$$

$$K_{21}(j\omega) = 12720000\omega^4 - 327760j\omega^3 - 9320\omega^2 + 182.52j\omega + 1.2$$

$$K_{22}(j\omega) = 12720000\omega^4 - 491640j\omega^3 - 9320\omega^2 + 121.68j\omega + 1.2$$

Again,

$$\text{Re } K_{11}(j\omega) = 8480000\omega^4 - 13980\omega^2 + 0.8$$

$$\text{Re } K_{12}(j\omega) = 8480000\omega^4 - 13980\omega^2 + 0.8$$

$$\text{Im } K_{21}(j\omega) = -327760\omega^3 + 182.52\omega$$

$$\text{Im } K_{22}(j\omega) = -491640\omega^3 + 121.68\omega$$

It is clear from the above equations that, for any frequencies  $\omega \geq 0$ , the value of  $H(\omega) \geq 0$  and the family of interval polynomial is robustly stable. Thus it is concluded that the designed controller not only offers the desired angle of attack but also produce robust stability.

## APPENDIX

TABLE I  
STABILITY DERIVATIVES OF LONGITUDINAL DYNAMICS OF FOXTROT  
AIRCRAFT

Stability Derivatives	Flight Condition(FC)		
	FC-1	FC-2	FC-3
$U_0 (\text{ms}^{-1})$	70	265	350
$X_u$	-0.012	-0.009	-0.0135
$X_w$	0.14	0.016	0.006
$Z_u$	-0.117	-0.088	0.0125
$Z_w$	-0.452	-0.547	-0.727
$Z_q$	-0.76	-0.88	-1.25
$M_u$	0.0024	-0.008	0.009
$M_w$	-0.006	-0.03	-0.08
$M_{\dot{w}}$	-0.002	-0.001	-0.001
$M_q$	-0.317	-0.487	-0.745
$X_{\delta_E}$	1.83	0.69	0.77
$Z_{\delta_E}$	-2.03	-15.12	-27.55
$M_{\delta_E}$	-1.46	-11.14	-20.07

## REFERENCES

- [1] Alireza Karimi, Hamid Khatibi and Roland Longchamp, Robust control of polytopic systems by convex optimization Automatica, Vol.43, No.8, 2007, pp.1395-1402.
- [2] Kin Cheong Sou, Megretski, A. and Daniel, L., A Quasi-Convex Optimization Approach to Parameterized Model Order Reduction, IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, Vol.27, No.3, 2008, pp.456-469.
- [3] V.L Kharitonov, Asymptotic stability of an equilibrium position of a family of systems of linear differential equations, *Differentsial'nye Uravnenia*, vol. 14, 1978, pp.2086-2088.
- [4] H.Chapellat, S.P. Bhattacharyya, A generalization of Kharitonov's Theorem; Robust stability of interval plants, IEEE Transactions on Automatic Control, Vol. 34, No.3,1989,pp. 306 – 311.
- [5] V. Yogesh, J.R.P Hote, Gupta, D. Roy Choudhury, Kharitonov's theorem and Routh Criterion for stability margin of interval systems, International Journal of Control, Automation and Systems, Vol. 8, No.3,2010,pp.647-654.
- [6] K.A. Moornani, M. Haeri, Robust stability testing function and Kharitonov-like theorem for fractional order interval systems, IET, Control Theory & Applications, Vol. 4, No.10, 2010,pp. 2097- 2108.
- [7] Fu M, "A class of weak Kharitonov regions for robust stability of linear uncertain systems", IEEE transactions on Automatic control, Vol. 36, No. 8, 1991, pp. 975-978.
- [8] Jie Chen, S.-I. Niculescu, Fu Peilin, Robust Stability of Quasi-Polynomials: Frequency-Sweeping Conditions and Vertex Tests, IEEE Transactions on Automatic Control, Vol.53, No.5, 2008,pp.1219 – 1234.
- [9] H. Bevrani, S. Shokoohi, Robust stabilizer feedback loop design for a radio-frequency amplifier, IEEE International Conference on Control Applications (CCA),2010, pp. 2250-2255.
- [10] R.J. Bhiwani, B.M. Patre, Stability analysis of fuzzy parametric uncertain systems, ISA transactions, Vol. 50, No.4, 2011, pp. 538–547.
- [11] R. Toscano, P. Lyonnet, Robust static output feedback controller synthesis using Kharitonov's theorem and evolutionary algorithms, Information, Vol. 180, No.10, 2010, pp. 2023–2028.
- [12] Donald McLean, *Automatic flight control system*, Prentice- Hall, 1990.
- [13] Gracey, William, Summary of methods of Measuring Angle of Attack on Aircraft, NACA Technical Note (NASA Technical Reports), 1958 (NACA-TN-4351): pp.1–30.
- [14] Sootla, A., Rantzer A. and Kotsalis, G, Multivariable Optimization-Based Model Reduction, IEEE Transactions on Automatic Control, Vol.54, No.10, 2009, pp.2477-2480.
- [15] Jan Hasenauer, Martin Löhning, Mustafa Khammash and Frank Allgöwer, Dynamical optimization using reduced order models: A method to guarantee performance, Journal of Process Control, Vol.22, No.8, 2012, pp. 1490-1501.
- [16] S.P. Bhattacharya, H. Chapellat and L.H. Keel, *Robust control, The Parametric Approach*, Prentice Hall, 1995.
- [17] Issam Amri, Dhaou Soudani and Mohamed Benrejeb, Robust Exponential Stabilization of Uncertain Perturbed Systems with Multiple Time Varying Delays in State and Control Input, International Review of Automatic Control, Vol. 4, No.2, 2011, pp.191-203.
- [18] Ala Eldin Abdallah Awouda, Mohsen Habibi Tehrani and Rosbi Bin Mamat, Robust PID Tuning Rule Using Prediction Algorithm with Optimization Method, International Review of Automatic Control, Vol. 3, No.2, 2010, pp.100-105.
- [19] R.J. Minnichelli, J.J. Anagnost and C.A. Desoer, IEEE Transaction on Automatic Control, Vol.34, No.9, 1989, pp.995-998.
- [20] Stephen Boyd, Craig Barratt and Stephen Norman, Linear Controller Design: Limits of Performance via Convex Optimization, Proceedings of the IEEE, Vol.78, No.3, 1990, pp. 529-574.

**Mr. S. Swain** was born on 21<sup>st</sup> May 1970 in Odisha, India. He completed his ME degree in Power Systwm Engg. in 2001 from VSSUT, Burla, Odisha India. Now he is continuing his Ph.D degree in BPUT, Rourkela, Odisha, India. His major field of study is Control System analysis and design.

He is now working as Associate Professor in Konark Institute of Science and Technology, BBSR since last seven years. He has published two papers in International Journals. His current research interest is Soft Computing Application in the field of Control system.

Mr. Swain is a life member in ISTE. He is also a senior member in Institution of Engineers, India.

**Dr. P. S. Khuntia** was born on 06<sup>th</sup> oct. 1969 in Odisha, India. He completed his ME degree in Automatic Control System and Robotics from MS University, Varodara in 1999. He has completed his Ph.D degree from ISM Dhanbad in 2011. India. His major field of study is Intelligent Control, Digital Signal Processing and Soft computing.

He is now working as Professor in Konark Institute of Science and Technology, BBSR. He has published many papers in National and International Journals. His current research interest is Advanced Control Theory and its application.

Dr. Khuntia is a life member in ISTE. He is also a senior member in Institution of Engineers, India.