

# CFD Prediction of the Round Elbow Fitting Loss Coefficient

Ana Paula P. dos Santos, Claudia R. Andrade, Edson L. Zaparoli

**Abstract**—Pressure loss in ductworks is an important factor to be considered in design of engineering systems such as power-plants, refineries, HVAC systems to reduce energy costs. Ductwork can be composed by straight ducts and different types of fittings (elbows, transitions, converging and diverging tees and wyes). Duct fittings are significant sources of pressure loss in fluid distribution systems. Fitting losses can be even more significant than equipment components such as coils, filters, and dampers. At the present work, a conventional 90° round elbow under turbulent incompressible airflow is studied. Mass, momentum, and  $k-\varepsilon$  turbulence model equations are solved employing the finite volume method. The SIMPLE algorithm is used for the pressure-velocity coupling. In order to validate the numerical tool, the elbow pressure loss coefficient is determined using the same conditions to compare with ASHRAE database. Furthermore, the effect of Reynolds number variation on the elbow pressure loss coefficient is investigated. These results can be useful to perform better preliminary design of air distribution ductworks in air conditioning systems.

**Keywords**— Duct fitting, Pressure loss, Elbow.

## I. INTRODUCTION

IN an air duct system, there are two types of resistance against the airflow: frictional losses and dynamic losses. Frictional losses result mainly from the shearing stress between the fluid layers of the laminar sub layer, which is adjacent to the surface of the duct wall. Friction also exists when the fluid particles in the turbulent flow bump against the protuberances of the duct wall. These lead to the production of eddies and energy loss. Friction losses occur along the entire length of an air duct. On the other hand, when air flows through duct fittings, such as, elbows, tees, diffusers, contractions, entrances and exits, a change in velocity or direction of flow may occur. Such a change leads to flow separation and the formation of eddies and disturbances in that area. The energy loss resulting from these eddies and disturbances are called dynamic loss. Although a duct fitting is fairly short, the disturbances that it produces may persist over a considerable downstream distance, Wang [1].

Pressure loss in ductworks is an important factor to be considered in design of engineering systems such as coils,

filters, dampers, power-plants, refineries, HVAC (Heating, Ventilation and Air Conditioning) systems to reduce energy costs. Ductwork can be composed by straight ducts and different types of fittings. Duct fittings are significant sources of dynamic pressure loss in fluid distribution systems due to changes in the airflow direction and/or duct area (transitions), Meyer [2].

ESDU [3] presents an extensive pressure-loss database for flow through single bends in curved ducts. Results are presented in terms of a non-dimensional static-pressure loss coefficient, but the majority of the results are presented using a graphical solution.

Another well-known approach is to express the effect of duct fittings in terms of an approximate equivalent length [4]. This methodology is used to obtain pressure loss coefficients for duct with rectangular sections based on its value for pipes (circular cross sections).

Even though technical literature bring important pressure loss database available to designers, such as provided by Sauer et al. [5], there are also many engineering cases where these fitting loss coefficients do not provide required information for ductwork sizing or balancing, specially with non-conventional cross section ducts. Hence, the CFD (Computational Fluid Dynamics) tool can be useful to attain accurate solutions, as presented in the works Moujaes and Deshmukh [6] and Gallegos-Muñoz et al [7].

A numerical investigation has been performed into the accuracy of using CFD for the prediction of pressure loss in HVAC duct fittings, and the factors affecting its accuracy, Shao and Riffat [8]. It was found that the pressure loss predicted by the Reynolds Stress Model (RSM) is more than double that predicted by the  $k-\varepsilon$  model, and the prediction based on power law discretization/interpolation scheme is much higher than that based on the higher order QUICK scheme. The most accurate prediction achieved in this study (relative error of about 10%) was based on the  $k-\varepsilon$  model and the higher order QUICK scheme.

Mumma et al. [9] analyzed the loss pressure coefficients in several ductwork fittings using a commercially-available CFD code, where one duct fitting studied was 90° round elbow. According to those authors the computational determination of duct fitting loss coefficients appears to be a viable alternative to laboratory testing, reducing design time consumption.

Wang and Shirazi [10] also performed a numerical investigating for one 90° round elbow and 180° bends using a CFD commercial code, but their main scope was to evaluate the flow and mass transfer coefficients in short- and long-radius elbows for usage in conjunction with a mechanistic

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model for calculating CO<sub>2</sub> corrosion rates in oil and gas pipelines. Thus, a standard k-ε model and a low-Reynolds version of the k-ε model were used to predict turbulent flow and mass transfer in the elbows analyzed by them. The authors developed an equation for estimating the maximum mass transfer coefficients for 90° elbows.

Numerical predictions of air flow and pressure distribution in elbows with and without turning vanes were analyzed in the work of Moujaes and Aekula [11]. The pressure drop difference between vaned and unvaned elbows were determined and showed good agreement with experimental data.

At this context, this paper addresses a numerical analysis of the 90° round elbow duct fitting pressure drop coefficients employing CFD tool. For the numerical model validation, the ASHRAE database [12] was taken as pressure-loss coefficient benchmark values. The main objective is to validate the numerical tool using this well-known conventional fitting element, allowing analyzes the airflow when the duct is fairly short, with abrupt expansions or contractions and a non-conventional cross-section (different from circular and rectangular). These last configurations are mainly encountered in air distribution ductworks of aircraft air conditioning systems which possess weight and space restrictions and will be investigated in future work.

In the following sections the mathematical modeling, problem description, computational strategy and results will be presented. It will be shown that the pressure loss coefficient values decays as a function of the Reynolds number within the studied range ( $1 \cdot 10^4 \leq Re \leq 1 \cdot 10^6$ ) allowing to propose a correlation based on a curve fitting over the numerical results.

## II. PROBLEM DEFINITION AND MATHEMATICAL MODELING

The present problem focuses on the Reynolds number influence in pressure loss coefficient for a 90° round elbow named as CD3-1 (see ASHRAE database, [12]) with  $r/D=1.5$  and  $D=76$  mm, as shown in Fig. 1. Two straight pipes with 10D length were connected to assure a fully developed flow in the elbow inlet cross section and to visualize the curvature effects (airstream swirl) at the elbow outlet.

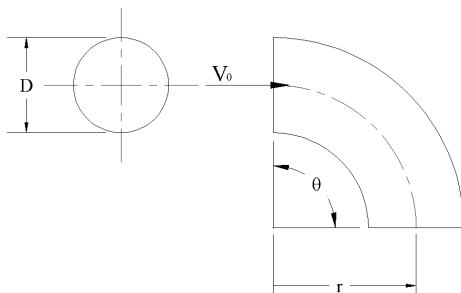


Fig. 1 Schematic drawing of the 90° round elbow with  $r/D=1.5$  and  $D=76$ mm

The mathematical modeling of a turbulent air flow inner a ductwork under constant properties and steady-state

conditions, are describe by continuity, momentum and turbulence model equations, as follow:

$$\nabla \cdot \rho \vec{V} = 0 \quad (1)$$

$$\rho [(\vec{V} \cdot \nabla) \vec{V}] = -\nabla P + \bar{F} - \nabla \times [\mu_{\text{eff}} (\nabla \times \vec{V})] \quad (2)$$

where  $\mu_{\text{eff}} = \mu + \mu_t$

The realizable k-ε is an isotropic turbulence model [13] based on eddy viscosity ( $\mu_t$ ) approach which is computed from:

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \quad (3)$$

where  $C_\mu$  is no longer constant (as occurs in the standard k-ε turbulence model) and is determined as:

$$C_\mu = \frac{1}{A_0 + A_S \frac{kU^*}{\varepsilon}} \quad (4)$$

$$A_0 = 4.04; A_S = \sqrt{6} \cos \phi; U^* \equiv \sqrt{S:S}$$

More details about the realizable k-ε model equations can be found in Fluent [14]. The near wall treatment employed was standard wall function, where the law-of-the-wall for mean velocity is given by:

$$U^+ = \frac{1}{\kappa} \ln(E y^+) = \frac{U_p C_\mu^{1/4} k_p^{1/2}}{\tau_w / \rho} \quad (5)$$

$$\text{with } y^+ = \frac{\rho C_\mu^{1/4} k_p^{1/2} y_p}{\mu}$$

The Reynolds numbers used were based on the average inflow velocity ( $V_0$ ) and duct diameter ( $D$ ) calculated as:

$$Re_D = \frac{\rho V_0 D}{\mu} \quad (6)$$

Different values for a uniform velocity profile are prescribed at the first straight pipe inlet to simulate each desired Reynolds number, maintaining constant the pipe diameter. A turbulence level at 10% and the pipe hydraulic diameter are also imposed to determine the boundary condition for the turbulent quantities [15].

At the second straight pipe outlet, null values are specified for pressure and normal derivatives for remaining variables. Nomenclature herein used is presented in Table I.

TABLE I  
NOMENCLATURE

Symbol	Quantity
Pressure	P
Body force density	F
molecular dynamic viscosity	$\mu$
turbulent dynamic viscosity	$\mu_t$
empirical constant	$E = 9.793$
turbulence kinetic energy	k
turbulence kinetic energy at point P	$k_p$
mean strain rate	$S = \frac{1}{2} \left[ (\nabla \vec{V}) + (\nabla \vec{V})^T \right]$
mean velocity	$U^+$
mean velocity of the fluid at point P	$U_p$
velocity vector	$\vec{V}$
distance from point P to the wall	$y_p$
dissipation rate	$\varepsilon$
Von Kármán constant	$\kappa = 0.4187$
fluid density	$\rho$

### III. COMPUTATIONAL STRATEGY

The numerical simulations have been performed using a CFD commercial code based on finite volume method. This procedure comprises three steps: pre-processor, solver and post-processor, described as:

- (i) Pre-processor: drawing of geometric domain (space occupied by the fluid flow); grid generation (domain subdivision in small finite volumes) and physics, mathematical and numerical setups (mathematical model, fluid properties, numerical approach, boundaries and initial conditions);
- (ii) Solver: computational solution of the algebraic equations systems obtained after the governing equation discretization (employing iterative algorithms);

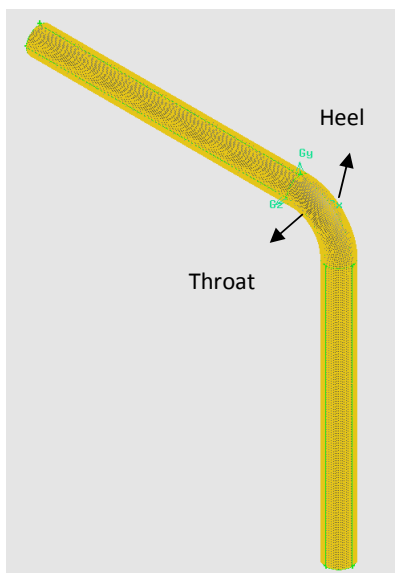
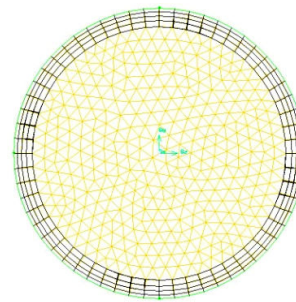


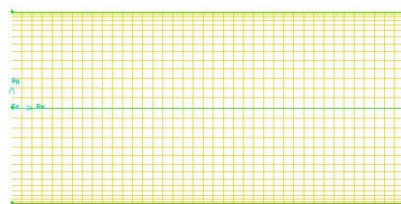
Fig. 2 Computational 3-D domain and mesh for the 90° round elbow

- (iii) Post-processor: quantitative and qualitative analysis (visualization) of the obtained results.

Mathematical model for turbulent flow inner a duct fitting was solved using a steady-state formulation. As, a strategy of velocity-pressure coupling was utilized the SIMPLE algorithm with a segregated formulation. The numerical scheme evaluated was second order discretization for the advective terms and pressure field. A successive mesh refinement study was performed using meshes composed by hybrid elements (hexahedral and tetrahedral) and a boundary layer close to the elbow wall, as shown in Figs. 2 and 3.



(a)



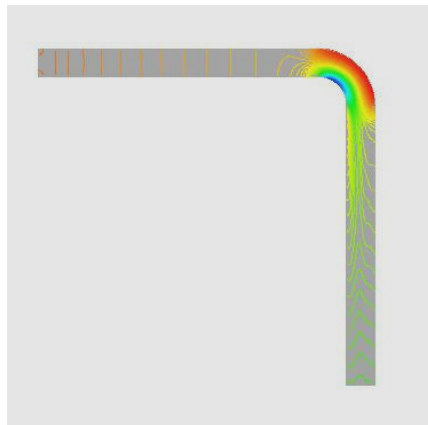
(b)

Fig. 3 (a) Mesh detail at the elbow circular cross-section and (b) mesh detail at the pipe wall

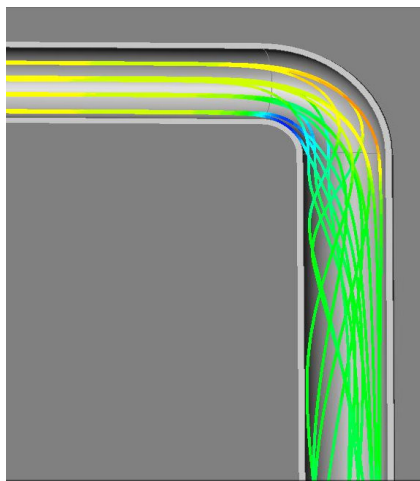
The selected mesh satisfies a trade-off study between accuracy and computational effort and is comprised by 392,700 elements. All simulations were carried out until the normalized maximum residuals of the continuity, momentum and turbulence quantities reached a value of  $10^{-6}$ .

### IV. RESULTS

Static pressure distribution and streamlines at a plane through the elbow centerline are illustrated in Fig. 4 at  $Re=1 \cdot 10^5$ . Note that when the air flows from the straight part of the pipe towards the curved part, it is accompanied by a build-up in pressure (elbow heel in Fig. 4 (a)) and a decrease along the outer elbow wall (throat). This occurs because the air at the elbow heel gradually turns, while the airstream close to the throat tends to travel in a straight line.



(a)



(b)

Fig. 4 (a) Static pressure contours and (b) Streamlines details at  $Re=1 \cdot 10^5$

Besides, it can be observed that the pressure contours are practically parallel along the first straight pipe and are disrupted when the airstream encounters the elbow. The elbow curvature influence isn't restricted to its outlet but affects the airflow downstream along the second straight pipe resulting in completely distorted pressure contours and spiraled streamlines (caused by the secondary flow, Fig. 5).

Therefore, the 10D pipe length is sufficient to allow a fully developed flow at the elbow inlet but does not allow the airflow redistribution along the second straight pipe. The secondary flow kinetic energy (see Fig. 5) could be totally dissipated by the viscous effects if a more extended straight pipe would be connected to the elbow outlet.

In the present work, a 10D pipe length was used only to allow an accurate comparison with literature numerical results provided by Mumma et al [5].

It is important to remember that an ideal condition would be to construct a ductwork with fittings making gradual turns and transitions as long as possible. However, in the majority of practical applications, the air distribution system has space

and/or weight restrictions which do not allow keeping the pressure loss as small as desirable.

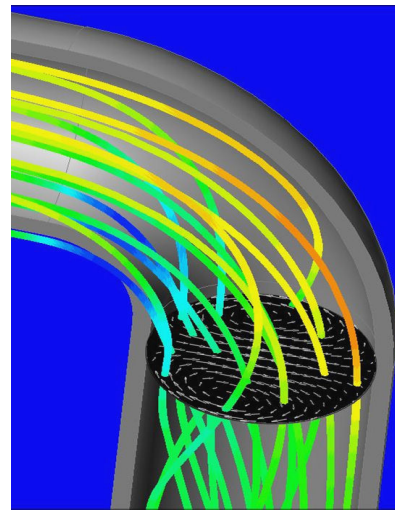


Fig. 5 Secondary flow patterns at the elbow cross section

It is well-known that pressure losses in a ductwork system are typically categorized as major and minor losses. Major (or frictional) losses are the direct result of fluid friction in the ductwork system and the pressure loss are usually computed through the use of friction factors that have been compiled for both laminar and turbulent flows. On the other hand, minor (or dynamic) losses are characterized as any losses which are due to pipe inlets and outlets, fittings and bends, valves, expansions and contractions, filters, dampers, etc.

For any given duct fitting, its pressure loss is most often determined using the concept of a loss coefficient,  $C_0$ . In the present work, this localized head loss due to the elbow resistance is calculated by comparing the pressure drop in a piping with elbow and a straight piping (without elbow) but with the same centerline length. At this case the pressure contours remain parallel along the pipe axial direction, as shown in Fig. 6.

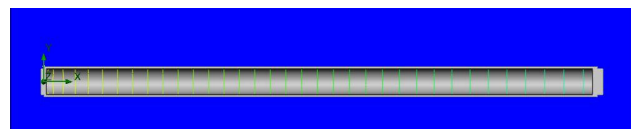


Fig. 6 Static pressure contours for a straight pipe (centerline length equal to the elbow shown in Fig. 2)

Thus, the minor pressure-loss coefficient is determined as:

$$C_0 = \frac{\Delta P_{te} - \Delta P_{tp}}{P_v} \quad (7)$$

where:

$\Delta P_{te}$  = total pressure drop in piping with elbow

$\Delta P_{tp}$  = total pressure drop in straight pipe

$P_v = 0.5\rho V_0^2$  = velocity pressure at the inlet of the piping with elbow.

For a 90° round elbow as schematized in Fig. 1 ( $r/D=1.5$  and  $D=76\text{mm}$ ) the ASRHAE [12] recommended value is  $C_0 = 0.15$  but no corrections are presented for different turbulent Reynolds number. Mumma et al. [5] obtained  $C_0 = 0.14$  at  $Re = 1 \cdot 10^5$  and other  $Re$  values are not investigated. Table II presents some numerical results obtained when the Reynolds number varies in the range  $1 \cdot 10^4 \leq Re \leq 1 \cdot 10^6$ .

TABLE II  
PRESENT WORK NUMERICAL RESULTS

Re	$1 \cdot 10^4$	$1 \cdot 10^5$	$1 \cdot 10^6$
$\Delta P_{te}$ (Pa)	3.45	130.3	8518.31
$P_v$ (Pa)	2.2515	226.0673	22614.137
$\Delta P_{tp}$ (Pa)	2.66552	97.9	6628.164
$C_0$	0.348	0.143	0.083

Note that the loss coefficient ( $C_0$ ) value at  $Re = 1 \cdot 10^5$  is in good agreement with ASRHAE [12] database with a difference less than 5%. This deviation is within typical experimental uncertainty values.

Fig. 7 exhibits the loss coefficient ( $C_0$ ) values as a function of the Reynolds number. A curve fitting over numerical results can be performed, resulting that:

$$C_0 = \frac{264.56}{126.23 + Re^{0.7345}} + 0.079 \quad (8)$$

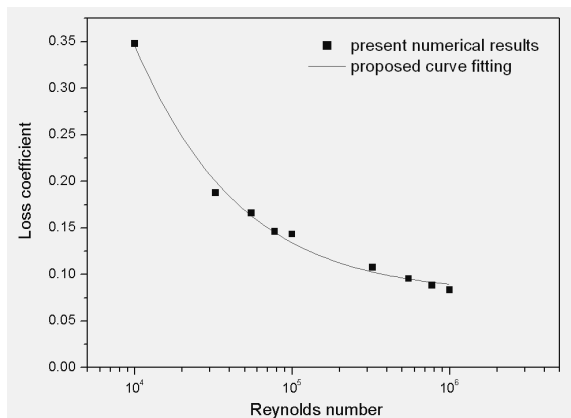


Fig. 7 Loss coefficient ( $C_0$ ) as a function of the Reynolds number

As occurs in the case of experimental data for elbows with rectangular cross-sections (Sauer et al. [2]), the  $C_0$  value decays as the  $Re$  number increases. An accurate determination of the pressure-loss coefficient (associated with each duct fitting) is fundamental to perform the ductwork sizing and/or balancing procedures.

## V. CONCLUSION

At the present paper, a conventional 90° round elbow under turbulent incompressible airflow was numerically studied. The loss coefficient attributed to the elbow resistance was determined at different Reynolds number with the main objective of validate the numerical tool. It had been shown that these effects aren't restricted to the elbow outlet but can be captured at a considerable downstream distance in agreement with the work of Gallegos-Muñoz et al. [7].

This study represented a preliminary step to analyze duct fittings encountered in typical aircraft air distribution systems in future work. In these configurations, the air conditioning ductwork is commonly composed by short flattened accessories with non-conventional cross-sections which loss coefficients are not available in the literature, justifying the CFD tool usage.

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