

# Finding Viable Pollution Routes in an Urban Network under a Predefined Cost

Dimitra Alexiou, Stefanos Katsavounis, Ria Kalfakakou

**Abstract**—In an urban area the determination of transportation routes should be planned so as to minimize the provoked pollution taking into account the cost of such routes. In the sequel these routes are cited as pollution routes.

The transportation network is expressed by a weighted graph  $G = (V, E, D, P)$  where every vertex represents a location to be served and  $E$  contains unordered pairs (edges) of elements in  $V$  that indicate a simple road. The distances / cost and a weight that depict the provoked air pollution by a vehicle transition at every road are assigned to each road as well. These are the items of set  $D$  and  $P$  respectively.

Furthermore the investigated pollution routes must not exceed predefined corresponding values concerning the route cost and the route pollution level during the vehicle transition.

In this paper we present an algorithm that generates such routes in order that the decision maker selects the most appropriate one.

**Keywords**—bi-criteria, pollution, shortest paths.

## I. INTRODUCTION

POPULATION exposure to air pollution [1], [3] is a critical component with negative impact to health issues due to industrial settlement close to inhabited urban area. Therefore the transportation that can deteriorate the pollution of the environment should be planned so as to minimize the provoked pollution and the adverse health consequences on the population located nearby the transition routes.

While determining routes it is evident to take into account certain parameters concerning the transition time or the corresponding cost.

To each road of the network two weights are assigned, representing its vehicle transition cost and its air pollution level exposure (APE), [3] respectively.

Conclusively, we have to solve a bi-criteria problem [4]. Next Section is dedicated to an analytical formulation of the stated problem. Section III is devoted to the presentation of a procedure that applies the speculations referred in Section II. Section IV gives a relevant small numerical example. The conclusions are the content of the last section.

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## II. PROBLEM FORMULATION

The transportation network is expressed by a weighted graph  $G = (V, E, T, P)$  where every vertex represents a location to be served and  $E$  contains unordered pairs (edges) of elements in  $V = (v_1, v_2, \dots, v_n)$  that indicate a simple road. The distances / cost and a weight that depict the provoked pollution by a vehicle transition at every road are assigned to each road as well. These are the items of set  $D = \{d_{i,j}, (i, j) \in E\}$  and  $P = \{p_{i,j}, (i, j) \in E\}$  respectively.

The problem presented here generates all viable routes under a predefined route cost for any given pair of vertices  $(S, T) \in V$  of a network that correspond to an urban area, where  $S$  is the source and  $T$  the destination vertex. We say that a route is viable if the sum of the provoked pollution level of its roads does not surpass a predefined summative pollution value, such a route is a feasible route [2].

Let  $SD(x, y)$  and  $SP(x, y)$  be the value of the shortest paths from  $x$  to  $y$  for every pair  $(x, y) \in V, x \neq y$  concerning the weights assigned to the edges of  $E$  as to the transit cost and to the provoked pollution separately [3]. A route is a set  $R = \{r_1, r_2, \dots, r_{n_r}\}$  of ordered adjacent vertices

of  $G$ . The route cost is  $sumr = \sum_{i=1}^{n_r-1} D(r_i, r_{i+1})$  and the associated pollution is  $sump = \sum_{i=1}^{n_r-1} P(r_i, r_{i+1})$ . The

predefined maximum allowable route cost of a route  $R$  and the corresponding pollution route gravity are the values of  $Mroute$  and  $Mpol$ , obviously the following relation must be verified.

$$Mroute > SD(x, y) \text{ and } Mpol > SP(x, y) \quad (1)$$

Next section is dedicated to the proposed algorithm we named FVP that applies the above analysis in order to Find Viable Pollution routes in an urban network under predefined cost and pollution level exposure.

## III. PROCEDURE FVP

$F$  : Set that contains the vertices included at a current

stage in a partial route.  
 $\Gamma(v)$  : Set containing the adjacent vertices of  $v$ .  
 $Q(v)$  : Set that contains the adjacent vertices of  $v$  not contained in a partial route at a current stage.  
 $n_r$  : Number of vertices included in a partial route  $R$  at a current stage.

The items of  $D = \{d_{i,j}, (i,j) \in E\}$ ,  $P = \{p_{i,j}, (i,j) \in E\}$  and  $R = \{r_1, r_2, \dots, r_{n_r}\}$  are placed and handled in the linear arrays  $D, P$  and  $R$ , respectively.

The routes are yielded in the order they are produced.

Insert  $G = (V, E, D, P)$ , Read  $S, T$

{ Compute the square arrays  $SD$  and  $SP$  using Floyd algorithm }

$v \leftarrow S, \quad n_r \leftarrow 1, \quad Q(v) \leftarrow \Gamma(v), \quad F \leftarrow v$   
 $sumr \leftarrow 0, \quad sump \leftarrow 0$

DO

If  $Q(v) = \emptyset$  then CALL BACK

Select  $y \in Q(v)$

If  $F = \emptyset$  then

$n_r \leftarrow n_r + 1, \quad R(n_r) \leftarrow y, \quad v \leftarrow y, \quad \{y = n_r\}$

$F \leftarrow F + \{v\}, \quad Q(v) \leftarrow Q(v) - \{v\}$

$sumr \leftarrow sumr + D(R(n_{r-1}), R(n_r))$

$sump \leftarrow sump + P(R(n_{r-1}), R(n_r))$

Endif

If  $sumr > Mroute$  OR  $sump > Mpol$

Then

CALL BACK

Else

If  $v = T$  then

Write  $R(i), i = 1, 2, \dots, n_r, \quad SC, SG$

End if

End if

CALL BACK

End DO

BACK:

$n_r \leftarrow n_r - 1$

If  $n_r < 0$  Then End

$F \leftarrow F - \{v\}, \quad Q(v) \leftarrow \Gamma(v), \quad v \leftarrow R(n_r)$

$sumr \leftarrow sumr - D(R(n_{r-1}), R(n_r))$

$sump \leftarrow sump - P(R(n_{r-1}), R(n_r))$

Return

#### IV. NUMERICAL EXAMPLE

Procedure FVP was applied in the network shown in Fig. 1. The distances  $d_{i,j}$  are denoted in italic nearby the edges  $(i,j)$  and next the provoked pollution  $p_{i,j}$  in bold and in parenthesis. We selected routes connecting the vertices 1 and 15,  $s=1, T=15$  for which  $SD(1,15) = 18$  and  $SP(1,15) = 11$ . We selected the predefined allowable values  $Mroute$  and  $Mpol$  to be 25% more than the corresponding shortest paths, that is  $Mroute = SD(1,15) * 1.25 = 23$  and  $Mpol = SP(1,15) * 1.25 = 14$ .

The viable feasible routes are shown in Table I and the relation between them are expressed with two perpendicular axes accordingly.

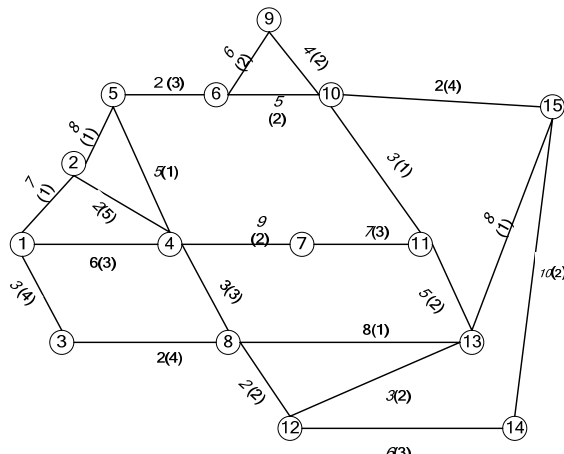


Fig. 1 Network example

Source  $s=1$ , Destination  $T=15$

$SD(s,T), Mroute, SP(s,T), Mpol$   
 $SD(s,T) = 18, Mroute = 23, SP(s,T) = 11,$   
 $Mpol = 14$

TABLE I  
VIALE FEASIBLE ROUTES

#	Routes	Sumr	Sump
1	1 2 5 6 10 15	20	11
2	1 3 8 12 14 15	19	14
3	1 3 8 13 15	21	14
4	1 4 5 6 10 15	18	13
5	1 4 8 12 13 15	22	14
6	1 4 8 12 14 15	23	13

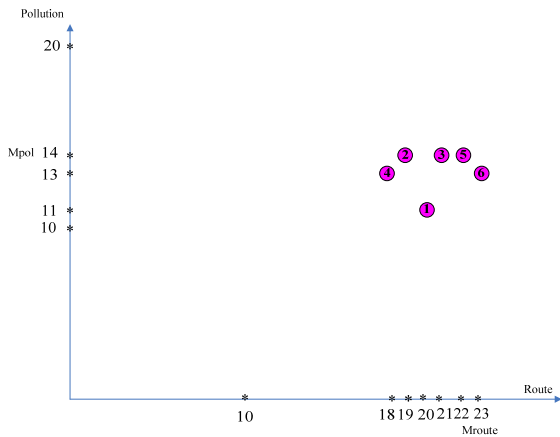


Fig. 2 Route cost vs route pollution

### V. CONCLUSIONS

A brief look at Fig. 2 indicates that the decision maker will select route 1 or 4 depending on his personal preference or on the gravity he gives to the route cost opposed to the route pollution.

The bi-criteria problem presented here is an NP-hard problem [2], although in various cases the method presented here can face successively real-world practical problems if the predefined cost and pollution values are not far away from the corresponding shortest paths, for example the network of Fig. 1 gives 18 feasible routes when the related predefined values exceeds 50% the shortest path values, since greater predefined values increase significantly the number of feasible routes.

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