Design Optimization of Ferrocement-Laminated Plate Using Genetic Algorithm

M. Rokonuzzaman, Z. Gürdal

***Abstract*—**This paper describes the design optimization of ferrocement-laminated plate made up of reinforcing steel wire mesh(es) and cement mortar. For the improvement of the designing process, the plate is modeled as a multi-layer medium, dividing the ferrocement plate into layers of mortar and ferrocement. The mortar layers are assumed to be isotropic in nature and the ferrocement layers are assumed to be orthotropic. The ferrocement layers are little stiffer, but much more costlier, than the mortar layers due the presence of steel wire mesh. The optimization is performed for minimum weight design of the laminate using a genetic algorithm. The optimum designs are discussed for different plate configurations and loadings, and it is compared with the worst designs obtained at the final generation. The paper provides a procedure for the designers in decision-making process.

***Keywords*—**Buckling, Ferrocement-Laminated Plate, Genetic Algorithm, Plate Theory.

I. Introduction

OVER the last few decades, laminated composites have found usage in aerospace, automotive, marine, civil, and sport equipment applications. This popularity is due to excellent mechanical properties of composites as well as their amenability to tailoring of those properties. The finding of an efficient laminated composite is achieved not only by sizing the cross-sectional areas and the member thickness, but also tailoring the material properties through the selective choice of orientation, number, and stacking sequence of the layers that makes up the composite laminate.

Ferrocement-laminated plate is a thin-walled type composite structure, which comprises of cement mortar and closely spaced small diameter steel wires. Due to its better weight to stiffness ratio and better post cracking behavior over traditional reinforced concrete, it is being extensively used in the construction of shell, folded plates, and thin web structures. However, to our knowledge, until now very few researchers have performed design optimization formulation and automation of this type of composite structures, even though by tailoring different properties of this type of material, more efficient and economical structure can be obtained. It is worth mentioning that researchers have been optimizing the unidirectional composites used in the aerospace structures heavily [3]-[6], where the materials are different from ferrocement-laminated composites.

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At present time, among various optimization techniques, genetic algorithm (GA) is one of the most popular methods. Since the formal introduction of GA in 1975 by John Holland [1] at the University of Michigan, this optimization tool has been increasingly applied to the solution of combinatorial problems in artificial systems. Search and optimization by GA is a process based on natural selection, where the individuals (designs) continue generation-to-generation based on Darwinian survival of fitness. A genetic algorithm performs the evolution process by means of random genetic changes using some probabilistic operators and an environment (fitness function) defined by the user. Since the introduction of GA, it is being used in different fields like engineering, medicine, and business. Goldberg [2] did an excellent introduction in search and optimization. Actually, laminated composite design optimization is often formulated as a continuous optimization problem using the thickness and orientation angles used as variables, but many times the thickness, orientation angles and materials are limited to a set of discrete values for the manufacturing and structural point of view. Design of a composite laminate involves a set of variables like ply material type and ply orientation, and therefore is well suited to genetic algorithm (GA) for design optimization problem. For this reason genetic algorithm is more suitable for this type problem. The genetic algorithm is being used extensively for the design optimization of laminated plate made of, basically, ductile matrix and synthetic unidirectional fibers [3]-[5] used in aerospace engineering.

As pointed out above that optimization problems for ferrocement like material made of brittle mortar matrix and ductile steel wire mesh having orientational effect can be solved effectively by GA. The target problem of this study is to increase the stiffness to weight ratio of ferrocement laminated plates. The ferrocement plate is assumed to be laminated by the layers of two different materials: mortar and ferrocement. The mortar layers comprise plane mortar alone, which is assumed to be homogenous. The ferrocement layers have a single layer of reinforcing mesh placed centrally in the mortar matrix. For given different in plane loading conditions, tailoring the stacking sequence and material, the performance of the ferrocement plate is to be improved.

II. Analysis of Ferrocement-Laminated Plate

Thin ferrocement-laminated structures assumed to be made up by thin layers can carry loads through two different types of mechanisms. One mechanism involves stretching of the laminate in its own plane, which is called membrane action, and another one is by bending. In the present study, all loadings and deformations are limited to the plane of the plate. However, buckling of the plate under compressive loads is considered, leading to bending stiffness design as well as the in-plane stiffnesses. The ferrocement plate is assumed to be laminated by the mortar and ferrocement layers. This kind of layering approach for analysis was followed in another work by Pankaj et al. [10]. Here, the mortar layers comprise plain mortar alone. The ferrocement layers have a single layer of reinforcing mesh placed centrally in the mortar matrix. The mortar layers are considered as isotropic homogeneous layers. The ferrocement layers are considered as orthotropic i.e. that they have two planes of symmetry in the plane of loading. To analyze the ferrocement-laminated plate, classical laminated plate theory (CLPT) is use. In Fig. 1, a cross-section of the composite laminate representing the mortar layers and ferrocement layers are shown.

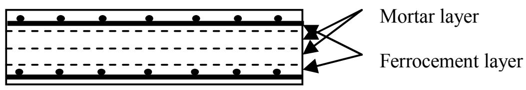


Fig. 1 Section of a four-layer ferrocement laminate

In Fig. 2, the lamina's material coordinate (1,2) and laminate's global coordinate (X, Y) are shown for the mesh steel fiber reinforced composite plate. For the material axes, generally the stronger mesh direction is given 1 and the perpendicular direction is given 2. The loading arrangement on the laminate is shown in Fig. 2, and the axial loads are applied in the mid plane of the laminate, where compression is negative.

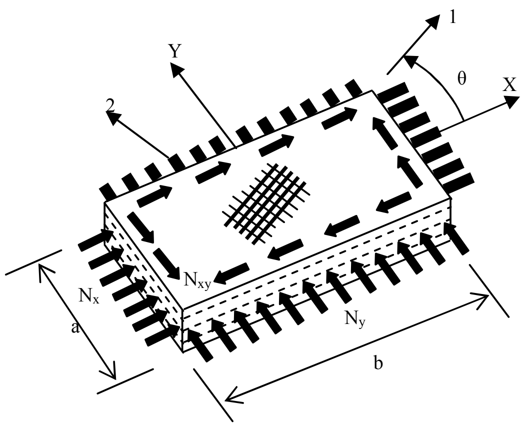


Fig. 2 Coordinate system and loading diagram of the laminate

Based on Kirchhoff's hypothesis, the force and moment resultants (N_x, N_y, N_{xy}, M_x, M_y, and M_{xy}) can be written in terms of mid-plane strains ($\varepsilon_x^0, \varepsilon_y^0, \text{and}\, \varepsilon_{xy}^0$) and curvatures ($k_x$, k_y, and k_{xy}) as [7]:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{ij}\, B_{ij} \\ B_{ij}\, D_{ij} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} \tag{1}$$

where, extension stiffnesses,

$$A_{ij} = \sum_{K=1}^{N} \left(\overline{Q_{ij}}\right)_K (Z_K - Z_{K-1}), \tag{2}$$

the Z is shown in Fig. 3;

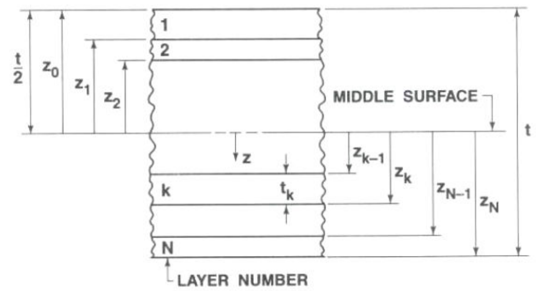


Fig. 3 Geometry of an N-layered laminate

transformed reduced stiffnesses in global coordinate,

$$\left[\overline{Q}\right] = [T]^{-1} [Q] [T]^{-T} \tag{3}$$

transformation matrix,

$$[T] = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ cs & cs & c^2 - s^2 \end{bmatrix}; \tag{4}$$

where, $c = \cos\theta$, $s = \sin\theta$, and θ is the angle between the lamina's material coordinate system and the global coordinate system, as shown in Fig. 2; reduced stiffnesses in lamina's coordinate system,

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}; \tag{5}$$

here, the components of the reduced stiffness matrix of a lamina are defined in terms of the in-plane mechanical properties of the lamina:

$$Q_{11} = \frac{E_1}{1 - \upsilon_{12}\upsilon_{21}}, \quad Q_{12} = \frac{\upsilon_{12} E_2}{1 - \upsilon_{12}\upsilon_{21}} = \frac{\upsilon_{21} E_1}{1 - \upsilon_{12}\upsilon_{21}}$$

$$Q_{22} = \frac{E_2}{1-\upsilon_{12}\upsilon_{21}}, \; Q_{66} = G_{12},$$

bending-extension coupling stiffnesses,

$$B_{ij} = \frac{1}{2}\sum_{K=1}^{N}\left(\overline{Q_{ij}}\right)_K (Z_K^2 - Z_{K-1}^2) \tag{6}$$

bending stiffnesses,

$$D_{ij} = \frac{1}{3}\sum_{K=1}^{N}\left(\overline{Q_{ij}}\right)_K (Z_K^3 - Z_{K-1}^3) \tag{7}$$

It is noted here that for this type of thin walled structure buckling is a critical phenomenon to be considered in the design. To find out critical buckling load factor of a laminated ferrocement plate, the Galerkin energy method outlined in [8] is utilized. Realizing that there is no coupling between bending and extension for symmetric laminates (i.e., [Bij] = 0), the strain energy, U, for transverse bending of a laminated plate of length (x = a) and width (y = b) shown in Fig. 2 is

$$U = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2D_{22}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 4\left(D_{16}\frac{\partial^2 w}{\partial x^2} + D_{26}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 w}{\partial x\partial y} + 4D_{66}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 \right] dxdy, \tag{8}$$

and the potential energy, V, of the biaxial and shear loads (N_x, N_y , and N_{xy}) that are applied to the plate is considered as

$$V = \frac{1}{2}\lambda\int_0^b\int_0^a \left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}\left(\frac{\partial^2 w}{\partial x\partial y}\right)\right] dxdy \tag{9}$$

The governing equation for the ferrocement plate [8]:

$$-\int\left(-\delta U - \delta V\right)dt = 0 \tag{10}$$

where δU and δV are the first variations in strain energy and potential energy due to the in-plane loads, respectively. The governing equation for the composite plate takes the form:

$$\int_0^b\int_0^a \left[D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2} + 4\left(D_{16}\frac{\partial^4 w}{\partial x^3\partial y} + D_{26}\frac{\partial^4 w}{\partial x\partial y^3}\right) + D_{22}\frac{\partial^4 w}{\partial y^4} - \lambda\left(N_x\frac{\partial^2 w}{\partial x^2} - 2N_{xy}\frac{\partial^2 w}{\partial x\partial y} - N_y\frac{\partial^2 w}{\partial y^2}\right)\right]\delta w\,dxdy + 2\oint_0^b\left[D_{26}\frac{\partial^2 w}{\partial x\partial y}\right]\frac{\partial(\delta w)}{\partial y}dx + 2\oint_0^a\left[D_{16}\frac{\partial^2 w}{\partial x\partial y}\right]\frac{\partial(\delta w)}{\partial x}dy = 0 \tag{11}$$

As the effects of the D_{16} and D_{26} terms are not neglected, the surface integrals in (11) are included in the governing equation. This allows the transverse deflection and first variation of the transverse deflection to be formulated in a double sine series:

$$\begin{aligned} w &= \sum_{i=1}^{I}\sum_{j=1}^{J} A_{ij}\sin\left(\frac{i\pi}{a}x\right)\sin\left(\frac{j\pi}{b}y\right) \\ \delta w &= \sum_{m=1}^{M}\sum_{n=1}^{N} \delta A_{mn}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right) \end{aligned} \tag{12}$$

Substituting (12) into (11) and performing the necessary integrations, the following set of algebraic equations are obtained:

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\left\{\pi^4\left[m^4 D_{11} + 2(D_{12}+2D_{66})(mn)^2 + (nR)^4 D_{22} + \left(\frac{am}{\pi}\right)^2\lambda N_x + \left(\frac{an}{\pi}\right)^2\lambda N_y\right]A_{mn} - 32mnR\pi^2\left[\sum_{i=1}^{M}\sum_{j=1}^{N} M_{ij}(i^2+m^2)D_{16} + R^2(n^2+j^2)D_{26} + \left(\frac{a}{\pi}\right)^2\lambda N_{xy}\right]A_{ij}\right\} = 0 \tag{13}$$

where

$$M_{ij} = \begin{cases} \dfrac{ij}{(m^2-i^2)(n^2-j^2)}, \text{ if } (m\pm i) \text{ and } (n\pm j) \text{ are odd} \\ 0, \text{ otherwise} \end{cases}$$

and R is defined as the plate aspect ratio (a/b). Equation (13) yields MN number of homogeneous equations that can be broken into the form of

$$[A]\{x\} - \lambda[B]\{x\} = 0 \tag{14}$$

The coefficient matrix contains terms involving N_x,N_y, and N_{xy} only. The smallest value of λ, λcr, for which the determinant of the coefficient matrix vanishes, will give the values for the critical buckling load factor.

Ferrocement falls in the family of thin laminated cementitious composites. In this study, simply, the Tsai–Wu stress based quadratic failure criterion [9], is used for well interaction between different failure modes, which stipulates that the condition for non-failure for any particular ferrocement or mortar layer under in-plane loading, *first ply-failure criteria*, as the first ply-failure criteria provides a conservative estimate of laminate failure loading:

$$F = F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + 2F_{12}\sigma_1\sigma_2 + F_1\sigma_1 + F_2\sigma_2 \le 1 \tag{15}$$

where, the stresses (σ_1, σ_2, τ_{12}) are calculated using CLPT discussed above for any layer ,and the strength parameters F11, F22, F66, F12, F1 and F2 are given by

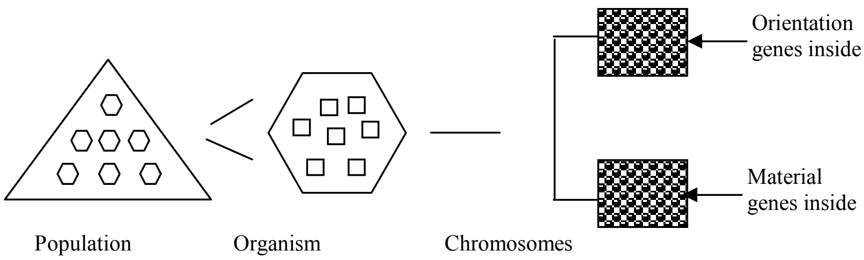
$$F_{11} = \frac{1}{X_t X_c}, \; F_{22} = \frac{1}{Y_t Y_c}, \; F_{66} = \frac{1}{S^2}, \; F_1 = \frac{1}{X_t} - \frac{1}{X_c}, \; F_2 = \frac{1}{Y_t} - \frac{1}{Y_c},$$
$$F_{12} = \frac{1}{2}\sqrt{F_{11}F_{22}} \tag{16}$$

where, X_t, X_c, Y_t, and Y_c are the tensile and compressive limit stresses of the mortar and ferrocement layer in the lamina's material directions (1, 2), up to which material behaves linearly, which are determined by simple laboratory tension and compression test data, and S is the in-plane shear strength.

III. Overview of Genetic Algorithm

Goldberg [2] stated that GA is "search procedures based on the mechanics of natural selection and natural genetics". GA applies the adaptive processes of natural system to the artificial systems. Actually, the GA is a program, which codes and decodes a design problem represented by one or more strings (chromosomes). The initial populations or designs (chromosomes) are created randomly, and after that these are subjected to different probabilistic genetic operators to search and exploit the design space. Survival of the individuals (designs) is ensured to the next generation upon the values of the fitness function defined by the user according to his problem.

For the optimization of composite ferrocement-laminated plate, GA code used in this work is described in details elsewhere [11], [12]. The main element of GA is the organism, which usually consists of a fixed number of chromosomes. Each chromosome may consist of one or many genes. Typically, each gene is coded using integer alphabets. An organism which represents a laminate design is composed of orientation and material chromosome. Each gene of the orientation chromosome represents the mesh orientation angles, and each gene of the material chromosome represents the material type used for the layers. A schematic of a typical GA structure is given in Fig. 4.



Orientation genes' code
0 = 1 empty layer
1 = 1 layer oriented at 0^0
2 = 1 layer oriented at 15^0
3 = 1 layer oriented at 30^0
4 = 1 layer oriented at 45^0
5 = 1 layer oriented at 60^0
6 = 1 layer oriented at 75^0
7 = 1 layer oriented at 90^0

Material genes' code
1=Mortar layer
2= Ferrocement layer

An arbitrary design representation of half of a laminated plate with 16-ply laminate $\left\lfloor \pm 45^0 / 0_4 / 90_2 \right\rfloor_s$:

Orientation chromosome string $[2\ 3\ 1\ 1\ 1\ 1\ 4\ 4]$

Material chromosome string $[1\ 1\ 1\ 1\ 2\ 2\ 1\ 1]$

Organism $[2\ 3\ 1\ 1\ 1\ 1\ 4\ 4\ 1\ 1\ 1\ 1\ 2\ 2\ 1\ 1]$

Fig. 4 A schematic of a typical GA structure

Size of an initial population determines the size of the population in the all-future generations. Choosing the population size is a matter of trial and error. When the string length of an organism or design will be short, then a smaller population size is justified. The first population of organisms is initialized using some type of randomized process and is termed the first generation of the search. Each organism is then placed into a common environment where it competes and breeds with other members of the population. The characteristics of an organism are provided in the gene strings of each chromosome, the most important of which is fitness. An organism's fitness shows how well it has adapted to its environment. In many GA applications, the environment is more commonly referred to as the design space or the set of all possible choices that exist for a given problem. The GA's task is to locate the area(s) in the design space that will give the best solution to the problem.

In genetic algorithms, evolution from generation to generation is simulated both by preserving the genetic information contained in the chromosome strings of fit individuals and by altering this information by means of random genetic changes. Genetic operators affect both of these goals. The goal of preserving the genetic information of fit individuals is achieved through crossover. Crossover creates child individuals by crossing over portions of two parent individuals' chromosomes. One or both of the child individuals are retained in the new child population, and the child individuals are required to be unique with respect to the other children and to the parent population.

The child population is unique but the crossover operator ensures that the genetic information of the parent population is preserved. During a one-point crossover, two parent individuals are selected at random (with selection biased towards choosing the fittest parents), and their chromosome strings are cut at a randomly determined point. A child individual then receives a chromosome string comprised of the first portion of the first parent's chromosome string and the second portion of the second parent's chromosome string and so on for crossover operations in which the parent chromosome strings are cut at more than one random point (e.g., two point crossover, uniform crossover). Thus, a unique child individual is created which includes portions of the chromosome strings of both its parents. In the following example in Fig. 5 (a), a two-point crossover is applied to two parent chromosome strings, and the resulting child chromosomes are shown. The 0 is taken to denote an absent or deleted gene. The randomly selected crossover points are denoted by the symbol "|".

The goal of introducing change to the information in the chromosome strings of individuals created by crossover is achieved with the mutation, addition, deletion, and permutation operators. The mutation operator introduces new information into the chromosome string of an individual by randomly altering one or more genes in that string. The example in Fig. 5 (b) illustrates a one-point mutation carried out on child chromosome from the above crossover operation. The mutated gene is indicated with an underscore character "_". The addition operator randomly adds a gene to the chromosome string. For example Fig. 5 (c), a randomly determined gene is added at a random point in the chromosome. The randomly selected addition point is denoted by the symbol |. In this example, addition causes the number of actual genes in the chromosome to increase from 12 to 13; as a result it increases the number of plies in the stack of laminate. The deletion operator randomly deletes a gene from the chromosome string. In the following example Fig. 5 (d), a randomly determined gene, indicated by an underscore character "_", is removed from the chromosome. In this example, deletion causes the number of actual genes in the chromosome to decrease from 13 to 12, and as a result it decreases the number of plies in the stack of laminate. If the total thickness of the laminate is to remain unchanged during optimization process, then the addition and deletion operators are not used. The permutation operator relays information from one part of the chromosome to another by inverting the order of a randomly determined sequence of genes. In the following example Fig. 5 (e), the points at which the permutation operator is applied are indicated by the symbol "|". This operator is responsible for the change in the bending stiffness of the laminate without changing its in-plane properties, so it is a very important operator in buckling load optimization. The swap operator, like the permutation operator, relays information from one part of the chromosome to the other. The swap operator switches the positions of two randomly determined genes in the chromosome. The swapped genes are indicated with an underscore character "_" in the following example, Fig. 5 (f).

Parent chromosome 1 [3 2 3 1 | 3 3 1 1 | 3 2 3 1 0 0 0]
Parent chromosome 2 [1 1 2 1 | 3 1 2 2 | 2 2 1 0 0 0 0]
Child chromosome 1 [3 2 3 1 | 3 1 2 2 | 3 2 3 1 0 0 0]
Child chromosome 2 [1 1 2 1 | 3 3 1 1 | 2 2 1 0 0 0 0]

(a) 2-point crossover

Chromosome before mutation [3 2 3 1 3 3 1 1 3 2 3 1 0 0 0]
Chromosome after mutation [3 2 3 1 3 3 2 1 3 2 3 1 0 0 0]

(b) Mutation

Chromosome before addition [3 2 3 1 3 3 2 1 3 | 2 3 1 0 0 0]
Chromosome after addition [3 2 3 1 3 3 2 1 3 3 2 3 1 0 0]

(c) Gene addition

Chromosome before deletion [3 2 3 1 3 3 2 1 3 3 2 3 1 0 0]
Chromosome after deletion [3 2 3 1 3 2 1 3 3 2 3 1 0 0 0]

(d) Gene deletion

Chromosome before permutation [3 2 3 | 1 3 2 1 3 3 | 2 3 1 0 0 0]
Chromosome after permutation [3 2 3 | 3 3 1 2 3 1 | 2 3 1 0 0 0]

(e) Permutation

Chromosome before swap [3 2 3 3 3 1 2 3 1 2 3 1 0 0 0]
Chromosome after swap [3 2 1 3 3 3 2 3 1 2 3 1 0 0 0]

(f) Gene swap

Fig. 5 Schematics of operators used in the GA

To simulate the natural selection process in the GA, a selection mechanism is mandatory. For each generation in the execution of a GA, each individual's chromosome strings are decoded by some decoding function, and the decoded individual (the phenotype) is evaluated and given a quality value, or fitness, by the objective function. Individuals are chosen for mating by randomly choosing them from the population, with selection biased towards those individuals with higher relative fitnesses. Biasing the selection process may be accomplished with, for example, roulette wheel selection, Goldberg DE [2]. The roulette wheel ascribes to each individual a probability of being selected for mating based on its relative position in the population, when the individuals are ranked and sorted according to objective function value. The fitness is part of a simulated roulette wheel, in which the fraction of the roulette wheel, f_i, associated with the i-th best individual in a ranked population of n_d designs is then

$$f_i = \frac{2(n_d + 1 - i)}{{n_d}^2 + n_d} \tag{17}$$

A uniform random variable determines which portion of the roulette wheel is selected, and the parent individual associated with that portion of the roulette wheel is selected for mating.

Thus, the selection of parents for mating and crossover is biased towards those individuals having a more optimal objective function value. Using a selection scheme instead of simply choosing parents based on their proportional fitness ensures that a highly fit individual does not dominate the population the likelihood of choosing an individual for mating is based on that individual's relative rank in the population, not on its proportional fitness. In order to ensure that optimal designs are not discarded during a GA optimization, further selection is done after the child generation has been created. The fittest individual in the parent generation may be retained in the child generation (while discarding the least fit child). This is known as elitist selection, which is used in this study.

IV. Material Properties

The elastic properties of the mortar and ferrocement lamina are deduced from in-plane axial tension and compression tests on mortar, and tension tests on reinforcing mesh, separately. For reinforcing, machine woven galvanized mesh with opening size 7.5x6.0mm and average wire diameter of 0.72 mm was considered. For deducing the elastic properties of the ferrocement composite, “rule of mixtures”, and some empirical equations for determining shear strength from the simple axial test results, was used. The unit cost is assumed here which includes both the labor and material cost. It is to mention here that, though the composite behavior of the ferrocement lamina is different in compression and tension, for the mathematical convenience, the Young’s modulus in compression is assumed same as that in tension. The detailed material properties are shown in Table I. In that table the compressive or tensile strength of the lamina is defined as the stress limit where the ferrocement composite or mortar firstly deviates from linearity and starts cracking.

TABLE I
Material Properties [10], [13]

Properties	Mortar layer	Ferrocement layer
Young's modulus (longitudinal), E_{11}	0.735×10^6 psi	0.925×10^6 psi
Young's modulus (transverse), E_{22}	0.735×10^6 psi	0.901×10^6 psi
Shear modulus, G_{12}	0.306×10^6 psi	0.351×10^6 psi
Poisson's ratio, ν_{12}	0.2	0.22
Ply thickness, t	0.25 in	0.5 in
Material density, ρ	126.10 lb/ft^3	130 lb/ft^3
Tensile strength, X_t	404 psi	654 psi
Tensile strength, Y_t	404 psi	572 psi
Compressive strength, X_c	1132 psi	1377 psi
Compressive strength, Y_c	1132 psi	1296 psi
Shear strength, S	390 psi	547 psi
Cost factor, C_f	1.0 (lb^{-1})	8.0 (lb^{-1})

V. Optimization Formulation

Mainly, the problem is to find the stacking sequence, both in terms of the through-the-thickness distribution of the orientation angles and the material types of the layers, which provides the lowest weight but does not fail due to buckling and excessive stress due to loads. For simplicity, it is assumed that the laminate is symmetric. So, the length of the GA string can be half, which reduces the search space for the GA and automatically satisfy the symmetry constraint. Balance constraint, which ensures that each ply oriented at $+\theta^0$ is complemented with another ply oriented at $-\theta^0$ throughout the stacking sequence, will be enforced using penalty parameters to eliminate the coupling between extension and shear (A_{16} and A_{26}).

From the material properties given in the Table I, it is observed that the stiffness-to-weight ratio (E_{11}/ρ) of ferrocement is greater than that of the mortar by only 18%, but it is more expensive, with a cost per pound that is 8 times greater than that of mortar. So, whatever the priority (weight or cost), always the mortar layer will be selected for its less weight and less cost. So, the present optimization problem is not a multi-objective problem so far. It is enough to optimize the design taking the weight as the objective function, because the weight controls the cost.

Optimization problem can be formulated as:

$$\text{Minimize,}\quad f=\begin{cases} W+P_f^W-w_a\times M & \text{for feasible lamiante} \\ W\times(1-M)^P+P_f^W-w_a\times M & \text{for unfeasible laminate}\end{cases} \tag{18}$$

where, W is total weight of laminate respectively, $M=\min\{\lambda_B,\lambda_F\}$; margin of safety for the critical buckling load, $\lambda_B=1-\lambda_{cr}$, using (14), margin of safety for the stress failure, $\lambda_F=1-F$, using (15), w_a is average weight over all material types for a ply in the laminate respectively, P is the constraint penalty parameter which is varied if GA generates very thin laminates, and the penalties for unbalanced laminate are

$$P_f^W=\begin{cases} 0 & \text{for balanced laminate} \\ N_{ubf}w_f & \text{for unbalanced laminate}\end{cases}$$

where, N_{ubf} is the number of unbalanced layers, and w indicates the weight of single ply, where subscript, f, indicates ferrocement layer. Here, the penalty is considered only when orthotropic ferrocement layers will be unbalanced, not for the mortar layer as it is isotropic in nature and it has not any orientation effect on shear extension coupling terms (A_{16} and A_{26}).

VI. Numerical Examples

The genetic parameters and corresponding strategies used in the optimization procedure are shown in Table II. These values are determined by trial and error in order to maximize reliability, which is the probability of reaching the optimum for a given number of function evaluations [12]. Those are used in all the examples in this paper. The design search space for GA is kept constant considering seven types of orientation angles (0-15-30-45-60-75-90) and two types of materials (ferrocement and mortar) for all examples. Optimizations are performed for the simply supported plates with various configurations (a/b ratio) shown in Fig. 6, and results are obtained with the different loadings (here, N_x/N_y is changed, rigorously, as 1, 2, and 4 to understand its effect on the stacking) for each aspect ratio shown in Table III. Here, the optimization is done for the laminates with constant number of plies (10, i.e. the maximum thickness of the laminate is limited to 5 inch. to overlook large numbers of layers) varying the stacking sequences only through the optimization process. For an example and to show how the GA converges, the improvement of fitness value and weight with generation for Plate1 (N_x=N_y=-4.0 kips/in, and N_{xy}=0.1 kips) shown in Fig. 7, and it is observed for all plates that GA converges very quickly due to small number of layers.

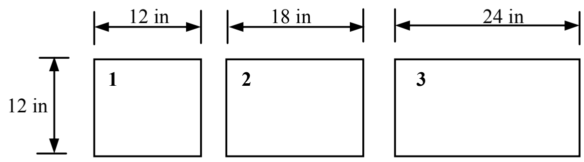


Fig. 6 Various plate configurations (number inside indicates plate designation no.)

TABLE II
GA PARAMETERS USED IN THE GENETIC SEARCH

Parameters	Value
Population size	200
Stopping criterion (maximum no of generations)	2000
Probability for crossover (Two-point crossover)	1.0
Probability for mutation (One-point mutation)	0.3
Probability for gene addition	0.00
Probability for gene deletion	0.00
Probability for gene swap	0.00
Probability for permutation	0.00

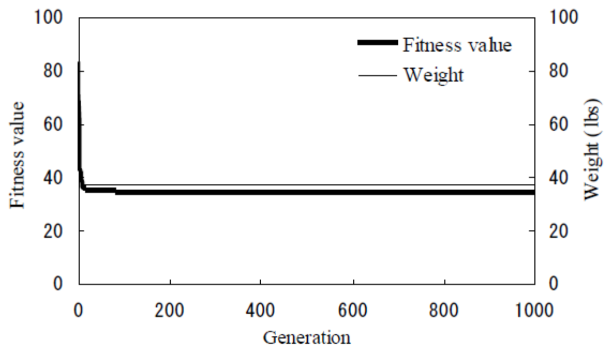


Fig. 7 Improvement in fitness value and weight with generation for Plate1 (N_x=N_y=-4.0 kips/in, and N_{xy}=0.1 kips)

TABLE III
OPTIMUM AND WORST PLATE DESIGNS

Plate (a/b ratio)	Optimum and worst stacking sequences	Loads $N_x/N_y/N_{xy}$ (kips/in,kips)	Thic (in)	Cost	Reinf. volume Fraction (%)	Weight (lbs)	Critical buckling load factor
1 (1.0)	$[+45/M_2/-45/M]_S$	-4.0/-4.0/10.1	3.5	189.07	0.49	37.41	110.49
	Worst: $[0/\pm15/90/0]_S$		5.0	433.20	0.86	54.16	334.19
	$[M/0/M/0_2]_S$	-2.0/-1.0/0.1	4.0	270.49	0.65	42.99	418.24
	Worst: $[90_5]_S$		5.0	433.20	0.86	54.16	889.02
	$[M/0_4]_S$	-4.0/-1.0/0.1	4.5	351.90	0.77	48.58	367.99
	Worst: $[0_2/90/0_2]_S$		5.0	433.20	0.86	54.16	533.48
2(0.66)	$[+45/M_2/-45/M]_S$	-4.0/-4.0/10.1	3.5	283.61	0.49	56.11	78.70
	Worst: $[+75/+15/90_2/-75]_S$		5.0	649.98	0.86	81.24	243.06
	$[M/0/M/0_2]_S$	-2.0/-1.0/0.1	4.0	405.73	0.65	64.49	346.20
	Worst: $[90_5]_S$		5.0	649.98	0.86	81.24	743.10
	$[M/0_4]_S$	-4.0/-1.0/0.1	4.5	527.86	0.77	72.87	345.15
	Worst: $[0_2/90/0_2]_S$		5.0	649.98	0.86	81.24	500.96
3(0.5)	$[+45/M_2/-45/M]_S$	-4.0/-4.0/10.1	3.5	378.15	0.49	74.82	68.59
	Worst: $[0/\pm15/90/0]_S$		5.0	866.64	0.86	108.33	209.10
	$[M/0/M/0_2]_S$	-2.0/-1.0/0.1	4.0	540.98	0.65	85.99	327.00
	Worst: $[90_5]_S$		5.0	866.64	0.86	108.33	706.61
	$[M/0_4]_S$	-4.0/-1.0/0.1	4.5	703.81	0.77	97.16	359.68
	Worst: $[0_2/90/0_2]_S$		5.0	866.64	0.86	108.33	522.85

The optimum and worst designs obtained from GA optimization are shown in the Table III for all loadings and plate configurations, where the every stacking sequence (half portion indicated by "s" after second bracket) with its corresponding thickness, cost, reinforcement volume fraction, weight and critical buckling load factor is presented. In the second bracket of the stacking sequences, the plain number designates ferrocement layer and the "M" designates the mortar-filling layer. The subscript after plain number or "M" indicates number of layers, and the left to right arrangement of the layers represents the position from top to midsection in the structure. The worst design is defined as the heaviest plate, obtained at final generation through GA procedure, that has margin of safety from buckling and stress failure, and it is observed that the optimum design has weight less than about 10 to 30% from worst one.

From the results, it is observed that due to the low weight of the mortar layer, it has domination in the stacking sequence to fill up the laminate for the moderate loading. Due to the equal biaxial loading condition, the ferrocement layers, generally, tend to orient in +/-45^0 to increase the stiffness of the laminate in the both direction, though, in the formulation, all orientation angles between –75^0 to +90^0 with 15^0 difference are used, and also when N_x/N_y increases, the strong axis of ferrocement layers tend to take acute angle (0^0) with the global X-axis of the laminate. It is advantageous to use 0^0 plies because unlike other angles (with the exception of 90^0) they do not have to come in pairs (to satisfy balance), thereby saving unnecessary additional weight or cost. The optimum stacking of each plate changes with the change in loadings, but does change very little with the plate configurations (the aspect ratio, a/b).The total thickness, the stacking and the aspect ratio of the plate affect critical buckling load factor. When the ferrocement layers are placed on the top, it increases the stiffness of the structure against bending increasing the buckling resistance of the structure. Actually, the bending stiffness of the structure is totally dominated by the position of the ferrocement layer due to its high strength when other factors are unchanged.

VII. Conclusions

The method used in this paper for solving the optimization problem is a standard genetic algorithm that has been adapted to the optimization of a ferrocement-laminated plate made up of reinforcing steel wire mesh(es) and cement mortar. The method is shown to be able to capture the optimum designs providing the designer with a very helpful tool for decision-making. Examples of different configured plates under different load combinations are designed to minimize weight and, consequently, the cost subject to constraints on buckling and stress failure. When the in-plane loads are increased, the GA evaluation process chooses the ferrocement layers, otherwise the mortar layers are selected due to its low unit weight to fill up the laminate, where the plate aspect ratio has very little effect on the stacking sequence of the plate. Also, it is revealed from the results that the optimum designs has weight less than about 10 to 30% of the worst designs found at the final generation.

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