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Effect of Conjugate Heat and Mass Transfer on MHD Mixed Convective Flow past Inclined Porous Plate in Porous Medium

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***Abstract*—**This analysis is performed to study the momentum, heat and mass transfer characteristics of MHD mixed convective flow past inclined porous plate in porous medium, including the effect of fluid suction. The fluid is assumed to be steady, incompressible and dense. Similarity solution is used to transform the problem under consideration into coupled nonlinear boundary layer equations which are then solved numerically by using the Runge-Kutta sixth-order integration scheme together with Nachtsheim-Swigert shooting iteration technique. Numerical results for the various types of parameters entering into the problem for velocity, temperature and concentration distributions are presented graphically and analyzed thereafter. Moreover, expressions for the skin-friction, heat transfer co-efficient and mass transfer co-efficient are discussed with graphs against streamwise distance for various governing parameters.

***Keywords*—**Fluid suction, heat and mass transfer, inclined porous plate, MHD, mixed convection, porous medium.

I. Introduction

THE study of MHD flow for an electrically conducting fluid past a heated surface has attracted the interest of many researchers in view of its important applications in many engineering fields such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, crystal growth and so on. Moreover, this study is largely concerned with the flow and heat transfer characteristics in various physical situations. An analysis of heat and mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and concentration, taking into consideration the effects of ohmic heating and viscous dissipation is investigated by Chen et al. [1]. Alam et al. [2] studied the problem of combined free-forced convection and mass transfer flow over a vertical porous flat plate, in presence of heat generation and thermal diffusion. The effects of thermophoresis and chemical reaction on unsteady hydromagnetic free convection and mass transfer flow past an impulsively started infinite inclined porous plate in the presence of heat generation/absorption has been examined by Alam and Rahman [3]. Alam et al. [4] also investigated the effects of thermophoresis and the homogeneous chemical reactions of first order on magnetohydrodynamics mixed convective flow past a heated inclined permeable flat plate in the presence of heat generation or absorption considering the viscous dissipation and joule heating. The authors, Alam et al. [4] considered the chemical reaction in title but the chemical reaction term has been neglected from the momentum equation. Also the similarity solutions were presented neglecting the Grashof number or buoyancy parameter effects for mixed convection. MHD mixed convective heat transfer about a semi-infinite inclined plate in the presence of magneto and thermal radiation effects has been examined by Aydin and Kaya [5]. Reddy and Reddy [6] analyzed a steady two-dimensional MHD free convection and mass transfer flow past an inclined semi-infinite vertical surface in the presence of heat generation in a porous medium. The authors, Reddy and Reddy [6] have considered the porous medium but the fluid suction effects was neglected to analyze the problem.

Therefore, it is more reasonable to include the chemical reaction term on the momentum equation to explore the impact of the momentum, heat and mass transfer characteristics with a transverse applied magnetic field. Therefore, in the light of above literatures, the aim of the present work is to investigate the problem of conjugate heat and mass transfer on magnetohydrodynamic mixed convective flow along an inclined flat plate in a porous medium, including the effect of fluid suction. The velocity, temperature, and concentration distributions are presented for various governing parameters such as magnetic parameter, local mass Archimedes number, and fluid suction parameter. Also the effect of various governing parameters on the local skin friction, the Nusselt number, and the Sherwood number are presented against the streamwise distance.

II. Model and Mathematical Formulation

A steady two-dimensional MHD laminar mixed convective flow of a viscous, incompressible fluid along a semi-infinite inclined porous plate with an acute angle α to the vertical is considered. The physical coordinates (x,y) are chosen such that x is measured from the leading edge in the streamwise direction and y is measured normal to the surface of the plate. The velocity components in the directions of x and y are u and v respectively. A magnetic field of uniform strength B_0 is applied normal to the direction of flow and the gravitational force g acts in the vertically downward direction. The external flow with a uniform velocity U_∞ takes place in the direction to the inclined plate. It is assumed that T and C are the

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temperature and concentration of the fluid which are the same, everywhere in the fluid. The surface is maintained at a constant temperature T_w, which is higher than the constant temperature T_∞ of the surrounding fluid and the concentration C_w, is greater than the constant concentration C_∞. The schematic view of flow configuration and coordinates system is shown in Fig. 1.

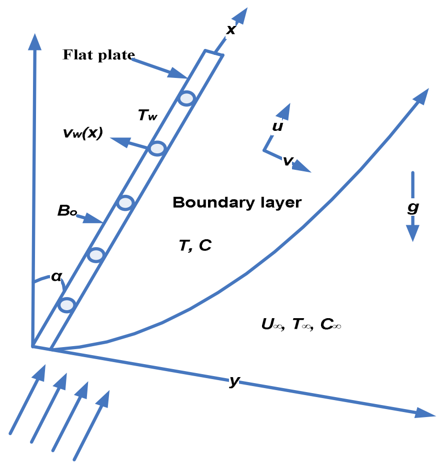

Fig. 1 The flow configuration and coordinate system

Under the usual Boussinesq approximation, the governing equations for steady, laminar, two-dimensional boundary layer flow under above assumptions can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)\cos\alpha+ g\beta^*(C-C_\infty)\cos\alpha-\left(\frac{\sigma B_0^2}{\rho}+\frac{\nu}{K^*}\right)u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty) \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \quad (4)$$

In the above equations, ν is the kinematic viscosity, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with mass fraction, σ is the electrical conductivity, K^* is permeability of the porous medium, ρ is the density of the fluid, k is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure, Q_0 is the heat generation constant and D is the mass diffusivity. The appropriate boundary conditions for the velocity, temperature and concentration of this problem are as follows:

$$u=0, v=-v_w(x), T=T_w, C=C_w \quad \text{at } y=0 \quad (5.1)$$

$$u=U_\infty, T=T_\infty, C=C_\infty \quad \text{as } y\to\infty \quad (5.2)$$

In addition, U_∞ is the free stream velocity and $v_w(x)$ represent the permeability of the porous plate where its sign indicates suction (< 0), subscripts w and ∞ refer to the wall and boundary layer edge, respectively. To facilitate the analysis, the governing differential equations are to be made nondimensional with suitable transformations and the following dimensionless variables are introduced by Cebeci and Bradshaw [7] as follows:

$$\eta=y\sqrt{\frac{U_\infty}{\nu x}}, \psi=\sqrt{\nu x U_\infty}f(\eta), \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \quad (6)$$

where, ψ (x, y) is the stream function defined by $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, such that the continuity equation (1) is satisfied automatically. In terms of these new variables, the velocity components can be expressed as:

$$u=U_\infty f'(\eta) \quad (7.1)$$

$$v=\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}(\eta f'-f) \quad (7.2)$$

Here, the prime stands for ordinary differentiation with respect to similarity variable η. Using dimensionless variables, the transformed momentum, energy, and concentration equations together with the boundary conditions can be written as:

$$f'''+\frac{1}{2}ff''+Ar_t\theta\cos\alpha+Ar_m\phi\cos\alpha-(M+K)f'=0 \quad (8)$$

$$\theta''+\frac{1}{2}\Pr f\theta'+\Pr Q\theta=0 \quad (9)$$

$$\phi''+\frac{1}{2}Sc\, f\phi'=0 \quad (10)$$

With the boundary conditions:

$$f=f_w, f'=0, \theta=1, \phi=1 \quad \text{at} \quad \eta=0 \quad (11.1)$$

$$f'\to 1, \theta\to 0, \phi\to 0 \quad \text{as} \quad \eta\to\infty \quad (11.2)$$

where, $f_w=-v_w(x)\sqrt{x/(\nu U_\infty)}$ is the nondimensional wall mass transfer coefficient such that $f_w>0$ indicates wall suction and $f_w<0$ indicates wall injection or blowing. The corresponding dimensionless groups that appear in the nondimensional form of governing equations are defined as:

$$Ar_t = \frac{Gr_t}{\mathrm{Re}_x^2}, Ar_m = \frac{Gr_m}{\mathrm{Re}_x^2}, Gr_t = \frac{g\beta(T_w - T_\infty)x}{U_\infty^2},$$

$$Gr_m = \frac{g\beta^*(C_w - C_\infty)x}{U_\infty^2}, \mathrm{Re}_x = \frac{xU_\infty}{\nu}, Sc = \frac{\nu}{D},$$

$$M = \frac{\sigma B_0^2 x}{\rho U_\infty}, K = \frac{\nu x}{K^* U_\infty}, \mathrm{Pr} = \frac{\nu\rho c_p}{k}, Q = \frac{Q_0 x}{\rho c_p U_\infty} \quad (12)$$

where, Ar_t is the local thermal Archimedes number, Ar_m is the local mass Archimedes number, Gr_t is the local thermal Grashof number, Gr_m is the local mass Grashof number, Re_x is local Reynolds number, M is the magnetic parameter, K is the permeability parameter, Pr is the Prandtl number, Q is the heat generation parameter and Sc is the Schmidt number.

By employing definition of wall shear stress $\tau_w = \mu\left(\partial u/\partial y\right)_{y=0}$, along with Fourier's law $q_w = -k\left(\partial T/\partial y\right)_{y=0}$ and Fick's law $J_s = -D\left(\partial C/\partial y\right)_{y=0}$, the nondimensional forms of local skin-friction coefficient is $C_f = 2\mathrm{Re}^{-1/2} f''(0)$, Nusselt number is $N_u = -2\mathrm{Re}^{1/2}\theta'(0)$, and Sherwood number is $S_h = -2\mathrm{Re}^{1/2}\phi'(0)$, where $\mathrm{Re}_x = xU_\infty/\nu$ is denoting the local Reynolds number.

III. Numerical Procedure

The system of transformed nonlinear ordinary differential equations (8)-(10), together with the boundary conditions (11.1) and (11.2), has been solved numerically using Nachtsheim-Swigert shooting iteration technique along with sixth order Runge-Kutta initial value solver. The numerical methods are not described details for the sake of brevity, referring to Nachtsheim and Swigert [8].

IV. Code Validation

For the accuracy of the numerical results, the present study is compared with the previous study Reddy and Reddy [6] as shown in Fig. 2. It is observed that the present results are in good agreement with that of Reddy and Reddy [6]. This favorable comparison leads confidence in the numerical results to be reported in the next section.

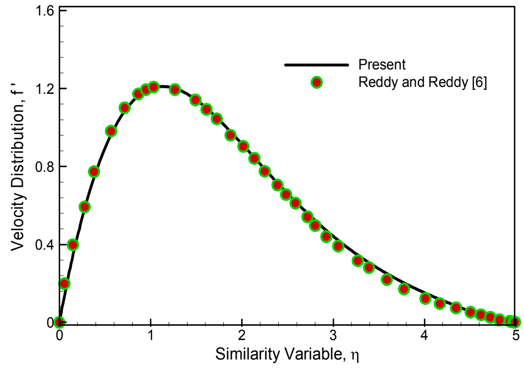

Fig. 2 Comparison of velocity distribution for $Ar_t = 2.0$, $Ar_m = 2.0$, $M = 0.5$, $Q = 0.5$, $K = 0.5$, $\alpha = 30^0$, Pr = 0.71, $f_w = 0.0$ and $Sc = 0.6$

V. Results and Discussions

The aim of the present research is to investigate the flow, heat and mass transfer characteristics for the MHD mixed convective flow over an inclined porous plate in a porous medium. A spacious set of numerical results is shown graphically. The numerical calculations have been carried out for different values of the various physical parameters on the nonlinear ordinary differential equations. The set of considered values of the various physical parameters for numerical investigation are $Ar_t = 1.0$, $Ar_m = 1.0$, $M = 0.01$, $Q = 0.25$, $K = 0.01$, $\alpha = 30^0$, Pr = 0.71, $f_w = 1.0$, $Sc = 0.78$ and $U_\infty / \nu = 1.0$ unless otherwise specified. The value of Prandtl number Pr is taken to be 0.71 which correspond physically to air and the values of Schmidt number Sc are taken 0.78 for ammonia (NH_3). Effect of magnetic parameter M (M = 0.0, 0.01, 0.02) on velocity, temperature and concentration distributions are shown in Figs. 3 (a)-(c) while $Ar_t = 1.0$, $Ar_m = 1.0$, $Q = 0.25$, $K = 0.01$, $\alpha = 30^0$, Pr = 0.71, $f_w = 1.0$ and $Sc = 0.78$. The existence of magnetic field, normal to the flow of electrically conducting fluid generates a Lorentz force, which acts against the flow. Thus the velocity decreases with increasing of the magnetic field parameter as observed in Fig. 3 (a). However boundary layer thickness decreases with the increase in the magnetic field parameter. Fig. 3 (b) illustrates the distribution of temperature, the temperature distribution increases with the increase of the magnetic parameter M. This is because of the applied magnetic field which tends to heat the fluid, thus reduces the heat transfer from the wall. On the basis of the concentration distribution in Fig. 3 (c), the effect of applied magnetic field is found which tends to increase the concentration, and hence it yields on reduction in the concentration gradient at the wall. Figs. 4 (a)-(c) depict the influence of local mass Archimedes number Ar_m (Ar_m = 0.0, 1.0, 2.0) on the velocity, temperature and concentration distributions for the value of parameters $Ar_t = 1.0$, $M = 0.01$, $Q = 0.25$, $K = 0.01$, $\alpha = 30^0$, Pr = 0.71, $f_w = 1.0$ and $Sc = 0.78$, respectively. The local mass Archimedes number Ar_m ascertains the ratio of local mass Grashof number to the local Reynolds number Re_x. As expected, the fluid velocity increases due to increase in the species buoyancy force. From Fig. 4 (a), it is observed that the velocity distribution increases with increasing the local mass Archimedes number Ar_m. On the other hand, from Figs. 4 (b) and (c), the temperature and concentration distributions decrease with increasing the local mass Archimedes number Ar_m. Figs. 5(a)-(c) reveal the effects of suction parameter f_w (f_w = 0.0, 1.0, 2.0) on the velocity, temperature and concentration distributions, respectively for the value of parameters Ar_t = 1.0, Ar_m = 1.0, M = 0.01, Q = 0.25, K = 0.01, α = 30^0, Pr = 0.71 and Sc = 0.78. It is observed from Fig. 5 (a) that, imposition of wall fluid suction ($f_w > 0$) has the effect of reducing the velocity boundary layer causing the fluid velocity tends to decreases. Also from Figs. 5 (b), (c), we observe that the temperature distribution as well as concentration distribution decrease with the increase of wall suction parameter f_w. We also notice that, fluid suction causing a lower thermal boundary layer which corresponds to a higher temperature gradient. As a result the temperature reaches a minimum value and similarly for concentration distribution.

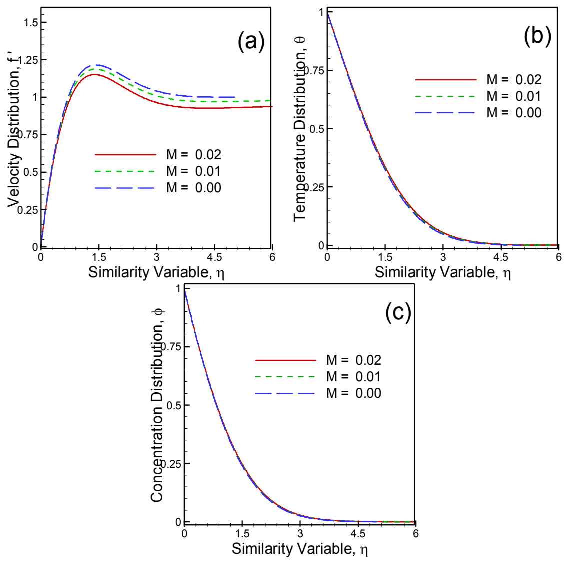

Fig. 3 Representative (a) velocity; (b) temperature; (c) concentration distributions for different values of magnetic parameter M

Figs. 6 (a)-(c) shows the variation of the local skin friction coefficient, the local Nusselt number, and the local Sherwood number in the boundary layer while Ar_t = 1.0, Ar_m = 1.0, Q = 0.25, K = 0.01, α = 30^0, Pr = 0.71, f_w = 1.0, Sc = 0.78 and U_∞ / ν = 1.0. In Fig. 6 (a), the local skin friction coefficient is found to decreases due to increase in the magnetic parameter M. The reason for this that, the applied magnetic field tends to impede the flow motion and thus to reduce the surface friction force. It is noted that the local Nusselt number as well as the local Sherwood number decrease with the increase of the magnetic parameter M, are displayed in Figs. 6 (b) and (c). Effect of the local mass Archimedes number Ar_m with the streamwise distance x on the local skin friction coefficient, the local Nusselt number, and the local Sherwood number are displayed in Figs. 7 (a)-(c) while Ar_t = 1.0, M = 0.01, Q = 0.25, K = 0.01, α = 30^0, Pr = 0.71, f_w = 1.0, Sc = 0.78 and U_∞ / ν = 1.0. From these Figs. 7 (a)-(c), it is observed that the local skin friction coefficient, the local Nusselt number, and the local Sherwood number increase with the increase of local mass Archimedes number Ar_m.

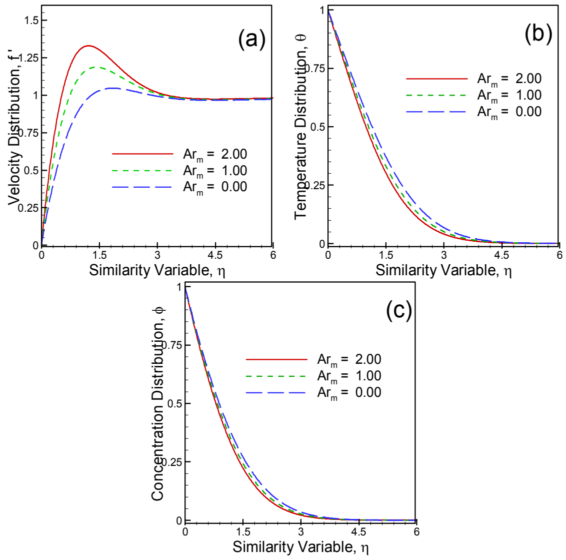

Fig. 4 Representative (a) velocity; (b) temperature; (c) concentration distributions for different values of local mass Archimedes number Ar_m

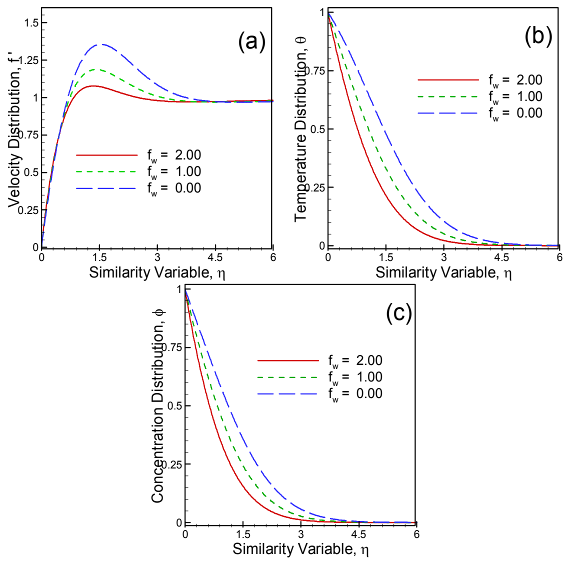

Fig. 5 Representative (a) velocity; (b) temperature; (c) concentration distributions for different values of permeability parameter of porous plate f_w

Figs. 8 (a)-(c), present the effect of the suction parameter f_w with the streamwise distance x on the local skin friction coefficient, the local Nusselt number, and the local Sherwood number while Ar_t = 1.0, Ar_m = 1.0, M = 0.01, Q = 0.25, K = 0.01, α = 30^0, Pr = 0.71, Sc = 0.78 and U_∞ / ν = 1.0 respectively. It is clear from Figs. 8 (a)-(c) that the local skin friction coefficient, the local Nusselt number and the local Sherwood number increase with the increase of wall suction parameter f_w.

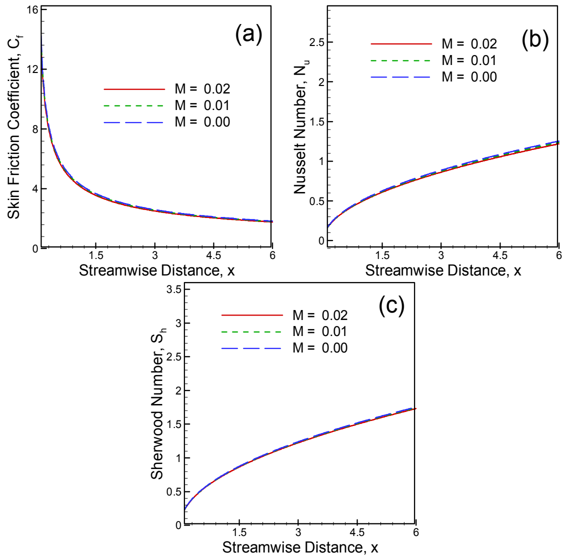

Fig. 6 Effects of magnetic parameter M on (a) local skin friction coefficient; (b) local Nusselt number; (c) local Sherwood number at the wall

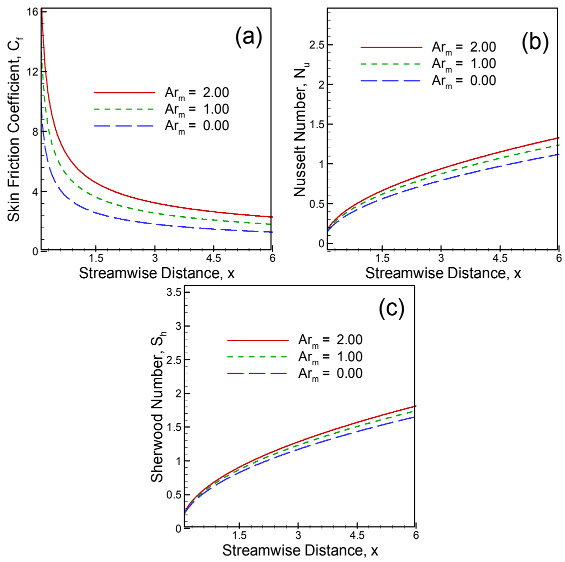

Fig. 7 Effects of local mass Archimedes number Ar_m on (a) local skin friction coefficient; (b) local Nusselt number; (c) local Sherwood number at the wall

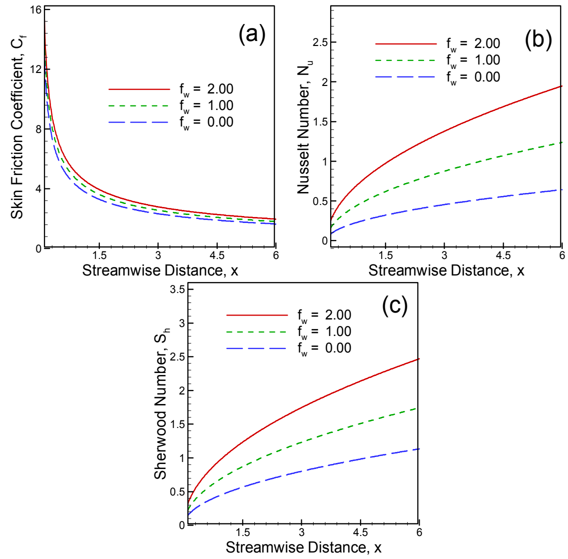

Fig. 8 Effects of permeability parameter of porous plate f_w on (a) local skin friction coefficient; (b) local Nusselt number; (c) local Sherwood number at the wall

VI. Conclusion

In the present problem, a mathematical model for conjugate heat and mass transfer on MHD mixed convective flow along inclined porous plate in porous medium has been developed. The nonlinear formulation governing equations and their associated boundary conditions have been obtained. The resulting transformed governing equations are solved numerically using Nachtsheim-Swigert shooting iteration technique with sixth order Runge-Kutta integration Scheme. The influence of the magnetic parameter M, the local mass Archimedes number Ar_m, and the fluid suction parameter f_w on velocity, temperature as well as concentration distributions are presented graphically. The local skin friction, the Nusselt number, and the Sherwood number are presented against streamwise distance x for various governing parameters. From the present investigation, the following conclusions can be drawn:

- Due to increase of the magnetic parameter, the velocity tends to decreases and the temperature as well as the concentration increase. Moreover, the local skin friction, the Nusselt number, and the Sherwood number decrease with increasing the magnetic parameter.
- The velocity and the temperature increase but the concentration decreases due to increase in local mass Archimedes number. Also the local skin friction coefficient, the local Nusselt number, and the local Sherwood number increase with the increase of local mass Archimedes number.
- As the fluid suction parameter increases, the velocity, temperature and concentration tend to decrease. On the other hand, for the suction parameter increases, the local skin friction coefficient, the local Nusselt number, and the local Sherwood number increase.

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