

Improved Stability Criteria for Neural Networks with Two Additive Time-Varying Delays

Miaomiao Yang, Shouming Zhong

Abstract—This paper studies the problem of stability criteria for neural networks with two additive time-varying delays. A new Lyapunov-Krasovskii function is constructed and some new delay dependent stability criterias are derived in the terms of linear matrix inequalities(LMI), zero equalities and reciprocally convex approach. The several stability criterion proposed in this paper is simpler and effective. Finally, numerical examples are provided to demonstrate the feasibility and effectiveness of our results.

Keywords—Stability, Neural networks, Linear Matrix Inequalities (LMI), Lyapunov function, Time-varying delays.

I. INTRODUCTION

RECURRENT years neural networks have been studied extensively and have been widely applied within various engineering fields such as associative memories, neuro-biology, population dynamics, and computing technology[1-5]. Existing stability criteria can be classified into two categories, that is, delay-independent ones and delay-dependent ones. It is well known that delay-independent ones are usually more conservative than the delay-dependent ones, so much attention has been paid in recent years to the study of delay-dependent stability conditions[6-8]. It should be pointed out that the stability results mentioned are based on systems with one single delay in the state.

In this paper, we consider the stabilization of the system described by

$$\dot{x}(t) = Ax(t) + Bx(t - \tau(t)) \quad (1)$$

where $\tau(t)$ is a time delay in the state $x(t)$, $A \in \mathbb{R}^{n \times n}$, and $B \in \mathbb{R}^{n \times n}$ are known real constant matrices, $0 \leq \tau(t) \leq \tau$, and $\dot{\tau}(t) \leq u$. In recent years, there are many people propose the model with multiple additive time-delays, the model as following:

$$\dot{x}(t) = Ax(t) + Bx(t - \sum_{i=1}^n \tau_i(t))$$

The paper in[5, 9] analysis the stability of system with two additive time-varying delay components. which is

$$\dot{x}(t) = Ax(t) + Bx(t - \tau_1(t) - \tau_2(t))$$

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The stability of the system (2) was studied in [5], and a delay-dependent stability criterion was obtained. An improved stability criterion was derived in [9] by construct a Lyapunov functional to employ the information of the marginally delayed state $x(t - \tau)$, where $u = u_1 + u_2$. However, another marginally delayed state $x(t - \tau_1)$ was not considered, do not make full use of the information about $\tau(t)$, $\tau_1(t)$, $\tau_2(t)$, which would be inevitably conservative to some extent. What is more, the purpose of reducing conservatism is still limited due to the existence of multiple coefficients the number of the LMIs decision variables, from a theoretical point of view, still remains challenging.

In this paper, we first consider delay-dependent stability for the system (2) by constructing a new Lyapunov functional which employs information of the marginally delays state $x(t - \tau_1)$ as well as $x(t - \tau)$. By construction a new Lyapunov Krasovskii functional, obtain the identical maximum allowable delay bounds, we derived a new and less conservative delay dependent stability condition for a system with two additive delay components. Finally a numerical examples given to illustrate the effectiveness of the proposed methods.

Notation: Throughout this paper, the superscripts $' - 1'$ and $'T'$ stands for inverse and transpose of matrix, respectively; \mathbb{R}^n denotes an n-dimensional Euclidean space; $\mathbb{R}^{m \times n}$ is the set of all $m \times n$ real matrices; $P > 0$ means that the matrix P is symmetric positive definite, $\text{diag}(\cdot, \cdot, \cdot)$ denotes a block diagonal matrix. In block symmetric matrix or long matrix expression, we use $(*)$ to represent a term that is induced by symmetry, I is an appropriately dimensional identity matrix.

II. PROBLEM STATEMENT.

Consider the following neural networks system with two additive time-varying delays:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bx(t - \tau_1(t) - \tau_2(t)) \\ x(t) &= \phi(t), t \in [-\tau, 0] \end{aligned} \quad (2)$$

where $\tau_1(t)$, $\tau_2(t)$ is a time delay in the state $x(t)$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n}$ are known real system constant matrices of appropriate dimensions corresponding to non-delayed and delayed. $\phi(t)$ is the initial condition on the segment $[-\tau, 0]$.

$$\begin{aligned} 0 \leq \tau_1(t) \leq \tau_1, 0 \leq \tau_2(t) \leq \tau_2, \dot{\tau}_1(t) \leq u_1, \dot{\tau}_2(t) \leq u_2 \\ \tau(t) = \tau_1(t) + \tau_2(t), \dot{\tau}(t) = u \\ \tau = \tau_1 + \tau_2, u = u_1 + u_2 \end{aligned} \quad (3)$$

where τ_1, τ_2, u_1, u_2 are constants.

Lemma 1.[10].For any positive constant matrix $Z = Z^T > 0$, $Z \in \mathfrak{R}^{n \times n}$, scalars $h_1 > h_2 > 0$ such that the following integrations are well defined, then

$$\begin{aligned}
 & -(h_2 - h_1) \int_{h_2}^{h_1} x^T(s)Zx(s)ds \\
 & \leq - \int_{h_2}^{h_1} x^T(s)dsZ \int_{h_2}^{h_1} x(s)ds \\
 & -\frac{1}{2}(h_2^2 - h_1^2) \int_{h_2}^{h_1} \int_{t+\theta}^t x^T(s)Zx(s)dsd\theta \\
 & \leq - \int_{h_2}^{h_1} \int_{t+\theta}^t x^T(s)dsZ \int_{h_2}^{h_1} \int_{t+\theta}^t x(s)ds
 \end{aligned}
 \tag{4}$$

Lemma 2.[11].For any constant matrix $X \in \mathfrak{R}^{n \times n}$, $Y \in \mathfrak{R}^{n \times n}$, $\Omega = \begin{bmatrix} R & X^T \\ X & R \end{bmatrix}$, scalars $0 \leq \tau_0 \leq \tau(t) \leq \tau_M$, and vector function $\dot{x} : [-\tau_M, -\tau_0] \rightarrow \mathfrak{R}^n$, such that the following integrations are well defined, then

$$\begin{aligned}
 & - \int_{t-\tau_M}^{t-\tau_0} \dot{x}^T(s)R\dot{x}(s)ds \leq - \begin{bmatrix} x^T(t-\tau_M) - x^T(t-\tau(t)) \\ x^T(t-\tau(t)) - x^T(t-\tau_0) \end{bmatrix}^T \\
 & \times \Omega \begin{bmatrix} x^T(t-\tau_M) - x^T(t-\tau(t)) \\ x^T(t-\tau(t)) - x^T(t-\tau_0) \end{bmatrix}
 \end{aligned}
 \tag{5}$$

III. MAIN RESULTS

Theorem 1.For given scalars $0 \leq \tau < \infty$, $0 \leq \tau_1 < \infty$, and $0 \leq \tau_2 < \infty$, $u > 0$, $u_1 > 0$, $u_2 > 0$, then the system(2) is asymptotically stable with delays $\tau(t)$, $\tau_1(t)$, $\tau_2(t)$, if exist positive-definite matrices $Q_i (i = 1, 2, \dots, 6)$, $\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}$, $R_i (i = 1, 2, 3, 4, 5)$ and P , for any matrices X_1, X_2, X_3 , with appropriate dimension such that the following LMIs hold:

$$E = \begin{bmatrix} e_{11} & e_{12} & X_1 & e_{14} & X_5 & e_{16} & X_3 & e_{18} & e_{19} \\ * & e_{22} & e_{23} & 0 & 0 & 0 & 0 & e_{28} & e_{29} \\ * & * & e_{33} & 0 & 0 & 0 & 0 & e_{38} & e_{39} \\ * & * & * & e_{44} & e_{45} & 0 & 0 & 0 & 0 \\ * & * & * & * & e_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & e_{66} & e_{67} & 0 & 0 \\ * & * & * & * & * & * & e_{77} & 0 & 0 \\ * & * & * & * & * & * & * & e_{88} & 0 \\ * & * & * & * & * & * & * & * & e_{99} \end{bmatrix}$$

$$\begin{aligned}
 e_{11} &= PA + AP^T + \sum_{i=1}^6 Q_i + \tau^2 A^T R_1 A + \tau_1^2 A^T R_2 A \\
 &+ \tau_2^2 A^T R_3 A - R_1 - R_2 - R_3 + \tau^2 R_4 + \frac{\tau^4}{4} A^T R_5 A \\
 e_{12} &= PB + \tau^2 A^T R_1 B + \tau_1^2 A^T R_2 B + \tau_2^2 A^T R_3 B + R_1 \\
 &+ \frac{\tau^4}{4} A^T R_5 B - X_1 \\
 e_{14} &= R_2 - X_2, e_{16} = R_3 - X_3 \\
 e_{18} &= P_{11} - P_{12} - P_{12} A, e_{19} = P_{12} - P_{22} - P_{22} A \\
 e_{22} &= -(1-u)Q_1 + \tau^2 B^T R_1 B + \tau_1^2 B^T R_2 B + 2X_1 \\
 &+ \tau_2^2 B^T R_3 B + \frac{\tau^4}{4} B^T R_5 B - 2R_1
 \end{aligned}$$

$$\begin{aligned}
 e_{23} &= R_1 - X_1, e_{28} = P_{12} B, e_{29} = P_{22} B \\
 e_{33} &= -R_1 - Q_2, e_{38} = P_{12} - P_{11}, e_{39} = P_{22} - P_{12} \\
 e_{44} &= -(1-u_1)Q_3 - 2R_2 + 2X_2, e_{45} = R_2 - X_2 \\
 e_{55} &= -Q_4 - R_2, e_{67} = R_3 - X_3 \\
 e_{66} &= -(1-u_2)Q_5 - 2R_3 + 2X_3 \\
 e_{77} &= -Q_6 - R_3, e_{88} = -R_4, e_{99} = -R_5
 \end{aligned}$$

Proof: Construct a Lyapunov function as follows:

$$V(x_t) = \sum_{i=1}^6 V_i(x_t)$$

where

$$V_1(x_t) = x^T(t)Px(t)$$

$$\begin{aligned}
 V_2(x_t) &= \int_{t-\tau(t)}^t x^T(s)Q_1x(s)ds + \int_{t-\tau}^t x^T(s)Q_2x(s)ds \\
 &+ \int_{t-\tau_1(t)}^t x^T(s)Q_3x(s)ds + \int_{t-\tau_1}^t x^T(s)Q_4x(s)ds \\
 &+ \int_{t-\tau_2(t)}^t x^T(s)Q_5x(s)ds + \int_{t-\tau_2}^t x^T(s)Q_6x(s)ds
 \end{aligned}$$

$$\begin{aligned}
 V_3(x_t) &= \tau \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s)R_1\dot{x}(s)dsd\theta \\
 &+ \tau_1 \int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}^T(s)R_2\dot{x}(s)dsd\theta \\
 &+ \tau_2 \int_{-\tau_2}^0 \int_{t+\theta}^t \dot{x}^T(s)R_1\dot{x}(s)dsd\theta
 \end{aligned}$$

$$V_4(x_t) = \tau \int_{-\tau}^0 \int_{t+\theta}^t x^T(s)R_4x(s)dsd\theta$$

$$V_5(x_t) = \frac{\tau^2}{2} \int_{-\tau}^0 \int_{t+\theta}^t \int_{t+\lambda}^t \dot{x}^T(s)R_5\dot{x}(s)dsd\theta d\lambda$$

$$\begin{aligned}
 V_6(x_t) &= \begin{bmatrix} \int_{t-\tau}^t x(s)ds \\ \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}(s)dsd\theta \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \\
 &\times \begin{bmatrix} \int_{t-\tau}^t x(s)ds \\ \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}(s)dsd\theta \end{bmatrix}
 \end{aligned}$$

The time derivative of $V(x_t)$ along the trajectory of system (2) is given by

$$\dot{V}(x_t) = \sum_{i=1}^6 \dot{V}_i(x_t)$$

where

$$\dot{V}_1(x_t) = 2x^T(t)Px(t) \tag{6}$$

$$\begin{aligned}
 \dot{V}_2(x_t) &= x^T(t) \sum_{i=1}^6 Q_i x(t) - x^T(t-\tau)Q_2x(t-\tau) \\
 &- x^T(t-\tau_2)Q_6x(t-\tau_2) - x^T(t-\tau_1)Q_4x(t-\tau_1) \\
 &- (1-u)x^T(t-\tau(t))Q_1x(t-\tau(t))
 \end{aligned}$$

$$\begin{aligned} & - (1 - u_2)x^T(t - \tau_2(t))Q_5x(t - \tau_2(t)) \\ & - (1 - u_1)x^T(t - \tau_1(t))Q_3x(t - \tau_1(t)) \end{aligned} \quad (7)$$

$$\begin{aligned} \dot{V}_3(x_t) & \leq \dot{x}^T(t)[\tau^2R_1 + \tau_1^2R_2 + \tau_2^2R_3]\dot{x}(t) \\ & - \tau \int_{t-\tau}^t \dot{x}^T(s)R_1\dot{x}(s)ds \\ & - \tau_1 \int_{t-\tau_1}^t \dot{x}^T(s)R_2\dot{x}(s)ds \\ & - \tau_2 \int_{t-\tau_2}^t \dot{x}^T(s)R_3\dot{x}(s)ds \end{aligned}$$

Based on the lemma 1 ,we have

$$\begin{aligned} & - \tau \int_{t-\tau}^t \dot{x}^T(s)R_1\dot{x}(s)ds \leq - \left[\begin{matrix} x^T(t - \tau) - x^T(t - \tau(t)) \\ x^T(t - \tau(t)) - x^T(t) \end{matrix} \right]^T \\ & \times \begin{bmatrix} R_1 & X_1^T \\ X_1 & R_1 \end{bmatrix} \begin{bmatrix} x^T(t - \tau) - x^T(t - \tau(t)) \\ x^T(t - \tau(t)) - x^T(t) \end{bmatrix} \end{aligned} \quad (9)$$

$$\begin{aligned} & - \tau_1 \int_{t-\tau_1}^t \dot{x}^T(s)R_2\dot{x}(s)ds \leq - \left[\begin{matrix} x^T(t - \tau_1) - x^T(t - \tau_1(t)) \\ x^T(t - \tau_1(t)) - x^T(t) \end{matrix} \right]^T \\ & \times \begin{bmatrix} R_2 & X_2^T \\ X_2 & R_2 \end{bmatrix} \begin{bmatrix} x^T(t - \tau_1) - x^T(t - \tau_1(t)) \\ x^T(t - \tau_1(t)) - x^T(t) \end{bmatrix} \end{aligned}$$

$$\begin{aligned} & - \tau_2 \int_{t-\tau_2}^t \dot{x}^T(s)R_3\dot{x}(s)ds \leq - \left[\begin{matrix} x^T(t - \tau_2) - x^T(t - \tau_2(t)) \\ x^T(t - \tau_2(t)) - x^T(t) \end{matrix} \right]^T \\ & \times \begin{bmatrix} R_3 & X_3^T \\ X_3 & R_3 \end{bmatrix} \begin{bmatrix} x^T(t - \tau_2) - x^T(t - \tau_2(t)) \\ x^T(t - \tau_2(t)) - x^T(t) \end{bmatrix} \end{aligned} \quad (10)$$

$$\begin{aligned} \dot{V}_4(x_t) & = \tau^2x^T(t)R_4x(t) - \tau \int_{t-\tau}^t x^T(s)R_4x(s)ds \\ & \leq \tau^2x^T(t)R_4x(t) - \int_{t-\tau}^t x^T(s)dsR_4 \int_{t-\tau}^t x(s)ds \end{aligned} \quad (12)$$

$$\begin{aligned} \dot{V}_5(x_t) & \leq \frac{\tau^4}{4}\dot{x}^T(t)R_5\dot{x}(t) - \left(\int_{-\tau}^0 \int_{t+\theta}^t \dot{x}(s)dsd\theta \right)^T R_5 \\ & \times \left(\int_{-\tau}^0 \int_{t+\theta}^t \dot{x}(s)dsd\theta \right) \end{aligned} \quad (13)$$

$$\begin{aligned} \dot{V}_6(x_t) & = 2 \begin{bmatrix} \int_{t-\tau}^t x(s)ds \\ \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}(s)dsd\theta \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \\ & \begin{bmatrix} x(t) - x(t - \tau) \\ \tau\dot{x}(t) - x(t) + x(t - \tau) \end{bmatrix} \end{aligned} \quad (14)$$

Then,from (6)-(14),we can obtain

$$\dot{V}(x_t) \leq g^T(t)Eg(t) \quad (15)$$

where,

$$\begin{aligned} g^T(t) & = [x(t), x(t - \tau(t)), x(t - \tau), x(t - \tau_1(t)), x(t - \tau_1), \\ & x(t - \tau_2(t)), x(t - \tau_2), \int_{t-\tau}^t x^T(s)ds, \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s)ds] \end{aligned} \quad (16)$$

(8) **Corollary 1.**For given scalars $0 \leq \tau < \infty, 0 \leq \tau_1 < \infty,$ and $0 \leq \tau_2 < \infty,$ then the system(2) is asymptotically stable with delays $\tau(t), \tau_1(t), \tau_2(t)$,if there exist positive-definite matrices $P, Q_2, Q_4, Q_6, R_i (i = 1, 2, 3, 4, 5),$ $\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix},$ for any matrices $X_1, X_2, X_3,$ with appropriate dimension such that the following LMIs hold:

$$E = \begin{bmatrix} e_{11} & e_{12} & X_1 & e_{14} & X_5 & e_{16} & X_3 & e_{18} & e_{19} \\ * & e_{22} & e_{23} & 0 & 0 & 0 & 0 & e_{28} & e_{29} \\ * & * & e_{33} & 0 & 0 & 0 & 0 & e_{38} & e_{39} \\ * & * & * & e_{44} & e_{45} & 0 & 0 & 0 & 0 \\ * & * & * & * & e_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & e_{66} & e_{67} & 0 & 0 \\ * & * & * & * & * & * & e_{77} & 0 & 0 \\ * & * & * & * & * & * & * & e_{88} & 0 \\ * & * & * & * & * & * & * & * & e_{99} \end{bmatrix}$$

$$\begin{aligned} (10) \quad e_{11} & = PA + AP^T + \sum_{i=1}^6 Q_6 + \tau^2A^TR_1A + \tau_1^2A^TR_2A \\ & + \tau_2^2A^TR_3A - R_1 - R_2 - R_3 + \tau^2R_4 + \frac{\tau^4}{4}A^TR_5A \\ e_{12} & = PB + \tau^2A^TR_1B + \tau_1^2A^TR_2B + \tau_2^2A^TR_3B + R_1 \\ & + \frac{\tau^4}{4}A^TR_5B - X_1 \\ e_{14} & = R_2 - X_2, e_{16} = R_3 - X_3 \\ e_{18} & = P_{11} - P_{12} - P_{12}A, e_{19} = P_{12} - P_{22} - P_{22}A \\ e_{22} & = \tau^2B^TR_1B + \tau_1^2B^TR_2B + 2X_1 + \tau_2^2B^TR_3B \\ & + \frac{\tau^4}{4}B^TR_5B - 2R_1 \\ e_{23} & = R_1 - X_1, e_{28} = P_{12}B, e_{29} = P_{22}B \\ e_{33} & = -R_1 - Q_2, e_{38} = P_{12} - P_{11}, e_{39} = P_{22} - P_{12} \\ e_{44} & = -2R_2 + 2X_2, e_{45} = R_2 - X_2 \\ e_{55} & = -Q_4 - R_2, e_{67} = R_3 - X_3 \\ e_{66} & = -2R_3 + 2X_3 \\ e_{77} & = -Q_6 - R_3, e_{88} = -R_4, e_{99} = -R_5 \end{aligned}$$

Proof: Choosing $Q_1 = 0, Q_3 = 0$ and $Q_5 = 0$ in Theorem 1, one can easily obtains this result. ■

IV. NUMERICAL EXAMPLES

In this section, we provide the simulation of examples to illustrate the effectiveness of our method.

Example 1. Considering the system (2) with the following parameters:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -9 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

and we suppose that $\dot{\tau}_1(t) \leq 0.1, \dot{\tau}_2(t) \leq 0.8$.
 First, the maximum delay bounds τ_2 are shown under different τ_1 are list in Table I.
 Then, the maximum delay bounds τ_1 are shown under different τ_2 are list in Table II.
 The maximum delay bounds τ_2 are shown under different τ_1 about corollary 1 are list in Table III.

TABLE I
 ALLOWABLE UPPER BOUND OF τ_2 WITH VARIOUS τ_1

Method	$\tau_1 = 1.0$	$\tau_1 = 1.1$	$\tau_1 = 1.2$	$\tau_1 = 1.5$
[12]	0.180	0.080	-	-
[13]	0.378	0.278	0.178	-
[14]	0.415	0.376	0.340	0.248
[15]	0.512	0.457	0.406	0.283
[16]	0.519	0.486	0.453	0.378
this works	0.810	0.710	0.610	0.310

TABLE II
 ALLOWABLE UPPER BOUND OF τ_1 WITH VARIOUS τ_2

Method	$\tau_2 = 0.3$	$\tau_2 = 0.4$	$\tau_2 = 0.5$
[12]	0.880	0.780	0.680
[13]	1.078	0.978	0.878
[14]	1.324	1.039	0.806
[15]	1.453	1.214	1.021
this works	1.510	1.410	1.310

TABLE III
 ALLOWABLE UPPER BOUND OF τ_2 WITH VARIOUS τ_1

Method	$\tau_1 = 0.3$	$\tau_1 = 0.5$	$\tau_1 = 1.0$
Corollary	1.088	0.888	0.388

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